Review:
Quantum state: A preparation of a physical system on which we can perform measurements.

Wavefunction: Expression of the state's state in terms of position.
Operator: A function that takes a quantum state and outputs another quantum state. \( \hat{\mathbf{p}}, \hat{\mathbf{x}}, \hat{\mathbf{A}} \)

Eigen state (function, vector):
Solution \( \psi \) to the equation \( \hat{\mathbf{A}} \psi = \alpha \psi \)
where \( \alpha \) is an eigenvalue.

Hermitian operator: An operator with all real eigenvalues. All physical observables are Hermitian.
Ex: \( \hat{\mathbf{p}}, \hat{\mathbf{x}}, \hat{\mathbf{A}} \).

What are eigenfunctions of \( \hat{\mathbf{p}} \)?
\[-i\hbar \frac{\partial}{\partial x} \psi(x) = p \psi(x), \quad \psi(x) = \frac{1}{\sqrt{2\pi}} e^{ipx/\hbar} \]

Note: If \( p \) is complex, \( \psi \to \infty \) at \( +\infty \) or \( -\infty \), so \( p \) must be real. \( \therefore \) \( \hat{\mathbf{p}} \) Hermitian.

Degeneracy: When an operator has more than one solution \( \psi \) to \( \hat{\mathbf{A}} \psi = \alpha \psi \) for a fixed value \( \alpha \).
Ex: 3-D box. Need more info!
\[ E_n(x) = -\frac{\hbar^2}{2m} \left[ n^2 \frac{\partial^2}{\partial x^2} + \frac{\hbar}{\sqrt{2m}} \frac{\partial}{\partial x} \right] + \frac{\hbar}{\sqrt{2m}} \partial_{\sqrt{n}}(x) \]

For localized states:

\[ 0 \leq n \leq \frac{\hbar^2}{2m} \]

Classically forbidden:

Finite square well:

\[ V_0 > 0 \]

Three regions:

For infinite well, \( V(x) = 0 \):
Recall II:

\[- \psi_{n}''(x) = k^{2} \psi_{n}(x) \quad k = \frac{1}{\hbar} \sqrt{2mE} \]

general soln: \[\psi_{n}(x) = Ce^{ikx} + De^{-ikx} \quad k \to k_{n}\]

In II:

\[\psi_{n}'' = \alpha^{2} \psi_{n}(x) \quad \alpha = \frac{1}{\hbar} \sqrt{2m(V-E_{n})}\]

Look:

\[\psi_{L}(x) = Ae^{\alpha x} \quad \text{x negative: } x < 0\]

\[\psi_{RM}(x) = Be^{-\alpha(x-L)} \quad x > L\]

Finding A, B, C, D

Trick: look at \( \psi' \) near 'matching points' and make it continuous at fixes WF

\[\Rightarrow \text{Result is:}\]
Inside $k_n$: Infinite well effective wavelength larger $\Rightarrow$ wave number smaller

$$\Rightarrow E_n = \left(\frac{\hbar k}{2m}\right)^2$$ smaller than $E_n^{\text{infinite}}$

A way to remember this (generally): Less repulsive potentials allow smaller energies

Also: Wavefunction $\to 0$ at $x \to \pm \infty$
fe $V_0 \to \infty \Rightarrow \psi = 0$ at $x = 0 \pm L$
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