1. Introducing RELATIVITY

or why \( v \leq c \)

\[ E_0 = mc^2 \quad \text{energy not bd} \]

\[ x = t \sqrt{1 - \frac{v^2}{c^2}} \quad \text{'twin paradox'} \]

\[ l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \quad \text{contract} \]

A difficult task!!

a) you are conditioned by press, science fiction, poor science, high school

b) have own opinions but not always based on exp. fact
If you like, you are welcome to offer on the student info page an answer to a simple question:

You heard about transformation ≠ Lorentz contraction

What is this?
Specifically, is this
a) contraction of space? or
b) contraction of a material body?
c) Is it real or apparent only?
Imagine that like sound, the velocity of light is a property of the medium.

\[ \Rightarrow \]

Velocity of light property of medium, \text{ not } \text{ vacuum } \\
and thus a universal constant, does not depend on conditions of the source, such as velocity.

\[ u = u_{\text{source}} + v \]

\[ \text{ Anti-Compare: } \]

\[ \text{ Stone throw: } \]
**PRINCIPLE OF RELATIVITY**

*The laws of physics are the same in all inertial reference systems* (this is 'Local Flat Universe' postulate)

**Note:** Reaching out to 'global' properties of the Universe, there are preferred inertial systems, absolute motion and all that.

**Special Relativity**

[Deals with 'local' invariance etc]

**Constancy of Velocity of Light**: Restatement of above for a special case

What we **loose**

**Simultaneity**

- Galilean
  \[ t = t' \]
- Einstein
  \[ t \neq t' \]
- Lorentz
  \[ t \neq t' \]
1. all observers in inertial systems measure the same constant c in vacuum.

2. laws of physics (e.g. Maxwell's eqs.) are the same in all inertial systems; no preferred ("ether") inert. system exists locally.

... must know!
1.1. Light-Clock

\[ v \ll c \implies c \approx 3 \times 10^5 \text{ km/s} \]

\[ t_0 = \frac{2l_0}{c} \]

Light pulse takes time

Remember, 1 ns ≈ 30 cm

for \( l_0 = 15 \text{ cm} \), \( t_0 = 2 \text{ ns} \)

to come back after bounce from
the other mirror

Increase \( v \)!

\[ t' = \frac{y}{v} = \frac{\sqrt{y^2 + l_0^2}}{c} \]

\[ y = v \cdot \frac{t_0}{2} \]

\[ \frac{y}{v} = \frac{v}{2} \]

and light has traveled \( \frac{1}{\sqrt{1 + \frac{y^2}{l_0^2}}} \)
TIME OF Travel of mirror = travel of light

\[ \cos \theta = \frac{2}{\sqrt{3} + 1} \]

\[ \frac{Y}{\nu} = \frac{\nu}{c} = \frac{\sqrt{\nu^2 + Y^2}}{c} \]

\[ Y = \frac{\nu}{c} \sqrt{1 - \left(\frac{\nu}{c}\right)^2} \]

\[ t_{0}' = \frac{2Y}{\nu} = \frac{2\nu}{c} \frac{1}{\sqrt{1 - \left(\frac{\nu}{c}\right)^2}} \]

to at rest

to at rest: time period to

to < to'

moving clock: time period to'

fewer clock ticks in moving clock, time passes slow

or
But so far our clocks cannot yet be oriented arbitrarily. Example was 1 to 1.

To accomplish this we need Lorentz contraction.

All material bodies are contracted by factor $\gamma$ along direction of motion.

Light Clock II:

$$t_1 = \frac{l_1'}{c} = \frac{l_1' + x_1}{c} \quad \Rightarrow \quad x_1 = l_1' \frac{u}{c \sqrt{1 - u^2}}$$

$$t_1 = l_1' \frac{1}{c - u}$$
\[ t_2 = \frac{x_2}{v} = \frac{l_2}{c} \quad \Rightarrow \quad x_2 = \frac{l_2}{1 + \frac{v}{c}} \]

\[ t_2 = l_2 \frac{1}{c + v} \]

\[ t_1 + t_2 = t_0' = \frac{c}{c-v} + \frac{l_2'}{c+v} = \frac{2c_0}{c} \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]

Lorentz:
\[ l_1' = l_0 \gamma \quad l_2' = l_0 \gamma \quad l_0 : \text{proper mirror dist} \]
\[ t_0' = l_0 \sqrt{1 - (\frac{v}{c})^2} \left[ \frac{1}{c-v} + \frac{1}{c+v} \right] \]

\[ = \frac{2l_0}{c} \frac{\sqrt{1 - (\frac{v}{c})^2}}{1 - (\frac{v}{c})^2} = t_0 \sqrt{1 - (\frac{v}{c})^2} \quad \text{QED} \]

Verify:
\[ t_0 = \frac{c}{c-v} \left( \frac{c+v}{(c-v)(c+v)} + \frac{c-v}{(c+v)(c-v)} \right) = \frac{2c}{c^2 - v^2} \]

QED
proper 'tick' to < to 'tick in maria clock

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \geq 1 \]

TIME DILATION  Must Know

IMPORTANT FURTHER 'READING'

**Note**

There is a 'exceptional' and observable reference frame

CBR: Cosmic Background Radiation rest frame.

Let us imagine that we always measure time dilation with respect to CBR frame.

⇒ Our planet has \( v = 300 \frac{km}{sec} \pm 38 \text{ km/s} \) with respect to CBR!
Note: Lorentz contraction a must be to have a light clock run right (both 1 & 11) and for c being determined by vacuum, time clock implies time dilation.

**Experimental Verification**

That time clock functions properly: Michelson-Morley Experiment

Mirrors, prisms etc fixed to solid table - all distances contract.
Observation of TIME DILATION

Remember: 'proper' time of a moving body 'ticks' slower

EXPERIMENT:
1) COMPARE CLOCKS - Procedure
   Synchronise and send one away when it comes back, compare again: the traveler is younger

2) Create an unstable particle: $\mu^-$, $N = N_0 e^{-t/\tau}$, $\tau = 2.2 \mu s$
   muon = 'heavy' electron
   \[ \text{proper decay time} \]

From measurement of velocity and averaged traveled distance observed
   \[ \tau' = \frac{\tau}{\sqrt{1 - v^2/c^2}} \]
   moving muon lives longer in our time
IMPORTANT
(and occasionally unclear in book)

Lorentz contraction is an observable effect regarding the property of a physical body.

NOT - Property of space time immediate contradiction

\[ \vec{u} \] "space" contracted by which factor \[ \sqrt{1 - \frac{u^2}{c^2}} \]
Michelson - Morley Exp

in context of 'Relativ. Watch'

- Compare $t''$ with $t_1$
  parallel transverse

relative to motion.

\[ v = 1\% \text{ of } c \]

Earth \[ 30\text{ km/s} \]

Sun \[ 300\text{ km/s} \]

reference

big-bang!
As we rotate the orientation of MM experiment, we are comparing the 'optical' paths $l_1$ and $l_2$.

If no change in interference pattern $\Rightarrow$ "Clock" insensitive to orientation, Lorentz contraction proven to the precision of MM exp.

Historical perspective: search for eather 'wind' $\leftarrow$ earth motion drag $\uparrow$ medium for light $\searrow$ eather. Can be isotropic (compare sound)

Negative result: $\Rightarrow$ no eather wind, i.e. material bodies do not interact with vacuum.

Is there eather? $\leftarrow$ How to observe!!!
The Michelson–Morley Experiments

Consider analogy with material waves (water, sound, etc.)

→ light waves propagate relative to "ether" medium (at rest) with velocity c

→ if light source & ether move relative to each other with vel. v, then effect. Light velocity should depend on direction?!

\[ c - v \leq c_{\text{eff}} \leq c + v \]

, Galilean veloc. addition

\[ \text{Sun} \]
\[ \text{Earth} \]

\[ v = 30 \text{ km/s} \]

\[ \text{300 km/s, rel. to CMBR (Big Bang)} \]

\[ \Rightarrow \text{large v} \ldots \]
--- Locally, light source $L$ in lab.

ether drift

CASE I.

\[ \rightarrow v \]

CASE II.

\[ \uparrow v \]

\[ \text{telescope} \]

\[ \text{interference pattern} \]

\[ 2(l_2 - l_1) = n\lambda \]

\[ \implies \text{time diff. between both paths:} \]

CASE I

\[ t_1 = \frac{l_1}{c+v} + \frac{l_1}{c-v} = \frac{2cl_1}{c^2-v^2} = \frac{2c}{c} \frac{l_1}{1-v^2/c^2} \]

\[ t_2 = \frac{l_2}{\sqrt{c^2-v^2}} + \frac{l_2}{\sqrt{c^2-v^2}} = \frac{2l_2}{c} \frac{1}{\sqrt{1-v^2/c^2}} \]
\[ \Delta t = t_2 - t_1 = \frac{2}{c} \left( \frac{\frac{L}{\sqrt{1-v^2/c^2}}}{\sqrt{1-v^2/c^2}} - \frac{\frac{L}{\sqrt{1-v^2/c^2}}}{\sqrt{1-v^2/c^2}} \right) \]

Similarly, **CASE II**

\[ \Delta t' = t_2' - t_1' = \frac{2}{c} \left( \frac{\frac{L}{1-v^2/c^2}}{\sqrt{1-v^2/c^2}} - \frac{\frac{L}{1-v^2/c^2}}{\sqrt{1-v^2/c^2}} \right) \]

\[ \rightarrow \text{shift of interference pattern, due to...} \]

\[ \Delta t' - \Delta t = \frac{2}{c} \left( \frac{\frac{L}{1+v^2/c^2}}{\sqrt{1-v^2/c^2}} - \frac{\frac{L}{1-v^2/c^2}}{\sqrt{1-v^2/c^2}} \right) \]

\[ \approx \frac{2}{c} \left( \frac{L}{1+v^2/c^2} \right) \left( \frac{1}{1} + \frac{v^2}{2c^2} - ... \right) \]

Taylor expansion, \( v/c \ll 1 \)

\[ \approx \frac{v^2 (L + L_2)}{c^3} \]

**time diff.**

\[ v = 30 \text{ km/s}, \ L_{\text{eff}} \approx \frac{L}{1+v^2/c^2} \approx 1.2 \text{ m} \rightarrow 8 \times 10^{-15} \text{ s} \]
$8 \times 10^{-15}$ corresponds to
0.04 of period $T = \frac{\Lambda}{c}$ for
visible light with $\lambda = 6 \times 10^{-7}$ m
$\Rightarrow$ i.e. 0.04 of (smallest) path diff. between two path, interfere, maxima
$\Rightarrow$ 0.005 still detectable!

$\Rightarrow$ NOTHING FOUND.
(MICHELSON & MORLEY 1887)

$\Rightarrow$ WHY?
- Seasonal dep. of $v$
- Ether dragged by earth
  $\Rightarrow$ Stellar aberration

$\Rightarrow$ CONCLUSION: NO EX. ETHER.

To put it differently:
Maxwell's eqs. determine value of $c$ in vacuum and there is no ether medium for light propagation.