WHAT NEW IN

GENERAL RELATIVITY

It is NOT generalization of 'special' relativity.

PRESENCE OF MATTER INFLUENCES SPACE-TIME GEOMETRY

Beginning PRINCIPLE OF EQUIVALENCE

There is no way to locally distinguish gravitational force from acceleration.

Favorite example: 'gently' accelerate space ship

* Einstein published many, many short and often false ideas seeking to incorporate forces (acceleration / gravity) into a fundamental theory. It took +10 years, +100 papers before he figured it out......
The idea that worked

Space-time is not flat

In principle no extra dimension needed to curve this sheet! Look

a) Original deformed from 2 into 3d
b) Image strictly 2d, not flat

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = G T_{\mu\nu} + 2 \Lambda R_{\mu\nu} \]

⇒ Einstein needed to learn about Riemann spaces

\[ g_{\mu\nu} \text{ : metric tensor} \]

Question: What has curved space-time to do with equivalence principle??

Energy-momentum density

grav. const

gravit. 'mass'
Inertial Mass \leftrightarrow \text{ Inertia } \quad F = m_1 \ddot{a}

Gravitational Mass \leftrightarrow \text{ Force } \quad F = mg \ddot{g} \quad (\text{force towards Earth})

\quad = G \frac{m_1 m_2}{r^2} \quad (\text{two grav. masses attract})

Einstein \quad m_1 \ \text{(proportional to)} \ mg

Prior \quad \text{exp. work in particular by Eötvös have shown that the prop. cons. is the same within } 10^{-8} (±1900). \ \text{Today we are still testing, seeking the '15th' force.}
Example

Effect on light by gravity can be deduced from Equivalence Principle

There is no way to locally distinguish gravitational force from acceleration.

Light 'attracted' to a massive body

\[ \text{Inside observer} \]

\[ \text{Curv. of beam due to } \mathbf{a} \]

\[ \text{Outside observer} \]

FIGURE 15.2 Starlight enters a small hole in a spacecraft while the rocket is accelerating. (a) The burst of starlight will hit a spot on the opposite wall at a point lower than where the light came in (greatly exaggerated here).

(b) According to the astronaut inside, the light pulse curved downward and must have been affected by the acceleration.

(c) According to the equivalence principle, the same thing must happen on the Earth because of gravity.
#1. Test of general relativity
Test of gravit bending light path
(Light propagels along 'geodesics')
Equiv. Principle tested for light
Eddington expedition ± 1917 → Einstein famous
Use total eclipse to block sunlight
can be able to see fixed stars
just past the sun

Sun

Eclipse

Moon

Grav. Body

Earth

Today we look for gravitational dark
stuff... technique!!!
Light needs to do work against gravity.

Energy of Light granitates

(Pound-Rebecca)

Experiment tested both gain/loss!

\[ \text{drop} \quad \Delta E = (\hbar \nu)_{\text{obs}} - m'gH \]

\[ \text{gain: } \Delta E = (\hbar \nu)_{\text{obs}} - m'gH \]

\[ \text{loss } \Delta E = (\hbar \nu)_{\text{obs}} + m'gH \]

\[ \Delta E = \text{tiny} \quad h\left(\nu - \nu_{\text{obs}}\right) = \pm \hbar \nu \frac{gH}{c^2} \]

Red/Blue shift

\[ \frac{\Delta \nu}{\nu} = \frac{gH}{c^2} \]

More accurately

\[ \frac{\Delta \nu}{\nu} = -\frac{GM}{c^2} \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \]

~ \(10^{-15}\)

(special 'nuclear' technique required)
#26: Back to time dilation!!

\[ \frac{\Delta \nu}{\nu} = - \frac{\Delta T}{T} = \frac{gH}{c^2} \nu \propto \frac{1}{T} \]

change of frequency means change in period of a clock.

Note, for a flying object at altitude of 10km

\[ \frac{\Delta T}{T} = \frac{(9.8 \text{ m/s}^2)(10000 \text{ m})}{(3 \times 10^8 \text{ m/s})^2} \approx 1.1 \times 10^{-12} \]

Much easier to measure: to get n-s (10^-9s) effect need only 1000s flight time.

B.T.W: Special Relativity

time dilation at \pm 6000km/h is 3 times SMALLER
#2c: Light retardation

(Radar bounces off near-sun planet ↔ equin to clock/v exp)

Measure 'excess' delay of signal! longer geodetic!

Test 3: Perihelion Shift of Mercury.

Somewhat similar to test 1: distortion of geometric path!

Recall: \( F = \alpha \frac{F}{r^2} \)
eliptical (closed) orbit

\( \alpha = \frac{4\pi^2}{G M} \)
Closed Orbits "Rare", 'Open' are most common

General relativity adds distortion of the force-law

$\Rightarrow$ Orbits do not close

'perihelion' shift

\[ F' = \frac{F}{r^2(1 + \frac{\beta}{m^2})} \]

\[ \Rightarrow \text{residual of perihelion shift} \]

---

From Tests to Most Famouse Consequences

- Black Holes
- Grav-Waves

$\downarrow$

Cosmology
'BLACK HOLES'

John Mitchell < 1793 ← earlier
Laplace in 1795

History: p623 'Gravit' by Wheeler

Notes that it is possible for a star to 'attract' light so strongly that it cannot escape

First computation of what we call today Schwarzschild radius

Recall Pound-Rebka Exp

\[ h\nu_{\text{abs}} = h\nu - h\nu \frac{1}{c^2} \frac{GM}{r_2} \left( \frac{1 - \frac{1}{r_2}}{\frac{1}{r_1}} \right) \]

\[ \Rightarrow 0 \text{ at } r_2 = \infty \]

\[ \tau_1 = \frac{2GM}{c^2} \]

\[ \text{all energy entirely gone} \]

General relativity introduces factor 2
If we can put enough mass into a "small" radius, light cannot escape.

→ Event Horizon

Every point mass has this property! No matter how small or big:

\[ M = M_{\text{Sun}} \ \& \ r_s \sim 3\text{km} \ (2.953?) \]

\[ M = M_{\text{Earth}} \ \& \ r_s \sim 1\text{cm} \]

So the question is: Can a stellar body collapse to below its \( r_s \)?
Can an elementary particle be heavy enough to have its own event-horizon?

⇒ Planck's Mass

Set $\gamma_s \sim \left(\frac{\hbar}{mc}\right)$

Planck's action $\hbar = \hbar / 2\pi$

$\frac{\hbar}{mc} \sim \frac{Gm_p}{c^2}$

$m_p = \sqrt{\frac{\hbar c}{G}} = \frac{5.5 \times 10^{-8}}{\sqrt{2\pi}}$

$= 2.2 \times 10^{-8}$ kg

Quantum-Gravity required at point-energy scales beyond this point
**Grav-Waves**

Different from your friendly sound or EM-waves

a) It is the space-time that fluctuates!

b) No 'scalar', 'dipole' but 'quadrupole' 

⇒ need very special antennas ← LIGO etc in apr

c) Coupling to source/receiver

⇒ need huge, time dep. mass to generate

d) Principle of detection

Michelson–Morley set

Compare optical paths as function of time

(MM compared orientations)
\[ \frac{d\tau^2}{dt^2} = \left(1 - \frac{2MG}{rC^2}\right) \frac{dt^2}{dt^2} - \frac{1}{1 - \frac{2MG}{rc^2}} \frac{dr^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2} \]

\( A) \) near the surface of Earth \( r \rightarrow R \)

\( A) \) redshift of photons \( \rightarrow \) Pound-Rebka blue shift

\( B) \) far out \( r \rightarrow \infty \) no gravity, no time dilation due to gravity.

\( \text{note} \)

\[ g = g \times \frac{m}{s^2} = \frac{MG}{Re^2} \]

\[ T^c = \frac{2MG}{c^2} = 2Re^2 ge/c^2 = 9 \text{mm} \]

Grav. Time dilation

\[ 1 - \frac{2MG}{rc^2} = 1 - \frac{T^c}{r} \]

On Earth magnitude

\[ 4 \text{ cm} \times 6.4 \times 10^{-9} \text{ cm} \]

\[ 4.5 \times 10^{-9} \]

\[ u = 0.8 \times 0.5 \]

\[ \frac{\sqrt{1 - \frac{T^c}{r} - \frac{u^2}{c^2}}}{c} \]

\( \frac{u}{c} \approx 4 \times 10^{-7} \) for SR in eq. GR