Uncertainty in Models of the Ocean and Atmosphere

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Quantifying Uncertainty

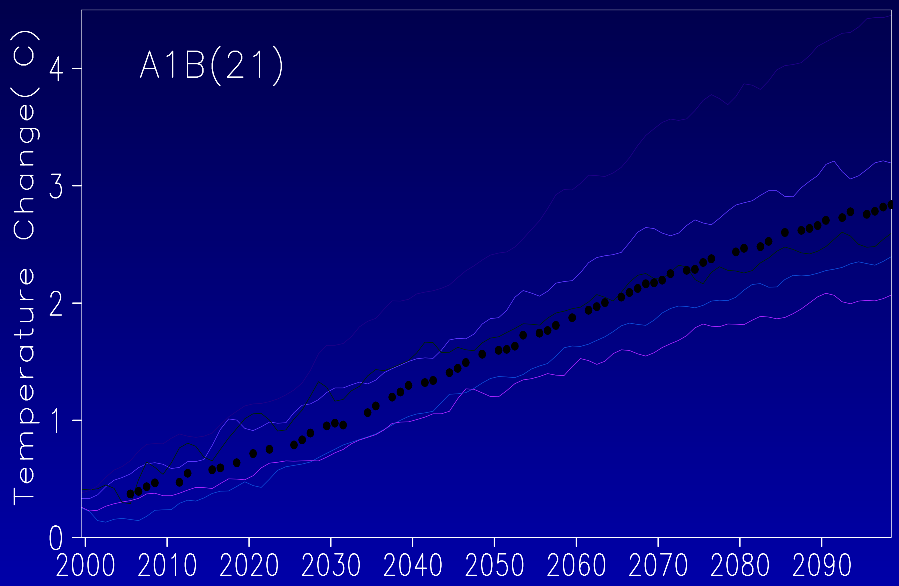
What, exactly are we talking about?

- We want to estimate specific properties of complex natural systems. Today: the ocean and atmosphere.
- We want to have some quantitative measure of the extent we can trust these estimates
- We want to make these estimates of uncertainty available along with the estimates of nature
- We want some quantitative evaluation of the reliability of our uncertainty estimates

$\cdots$ *a tall order*

What We Have in Mind

Estimates of uncertainty are often based on ensemble calculations. From a multi-model ensemble:



Globally averaged surface warming for one CO_2 scenario.Heavy dots depict ensemble average. Redrawn from figure 10.5, IPCC AR4

Quantifying Uncertainty

Prediction is very difficult, especially about the future

–variously attributed:

Niels Bohr

Sam Goldwyn

Yogi Berra

$\cdots$ and others

The Dimensions of Uncertainty

- These models can *potentially* contain $O(10^6)$ independent degrees of freedom
- These models cannot contain faithful representations of all relevant physics due to inevitable resource limitations
- These models contain dozens of parameters, many of which are, at best, empirically determined
- These models contain significant nonlinearities
- This leads to distinct and closely interrelated problems in design of ensembles for evaluation of uncertainty

Contributing Factors to Uncertainty

- Uncertainty estimates must take errors into account, but not all uncertainty stems from error:

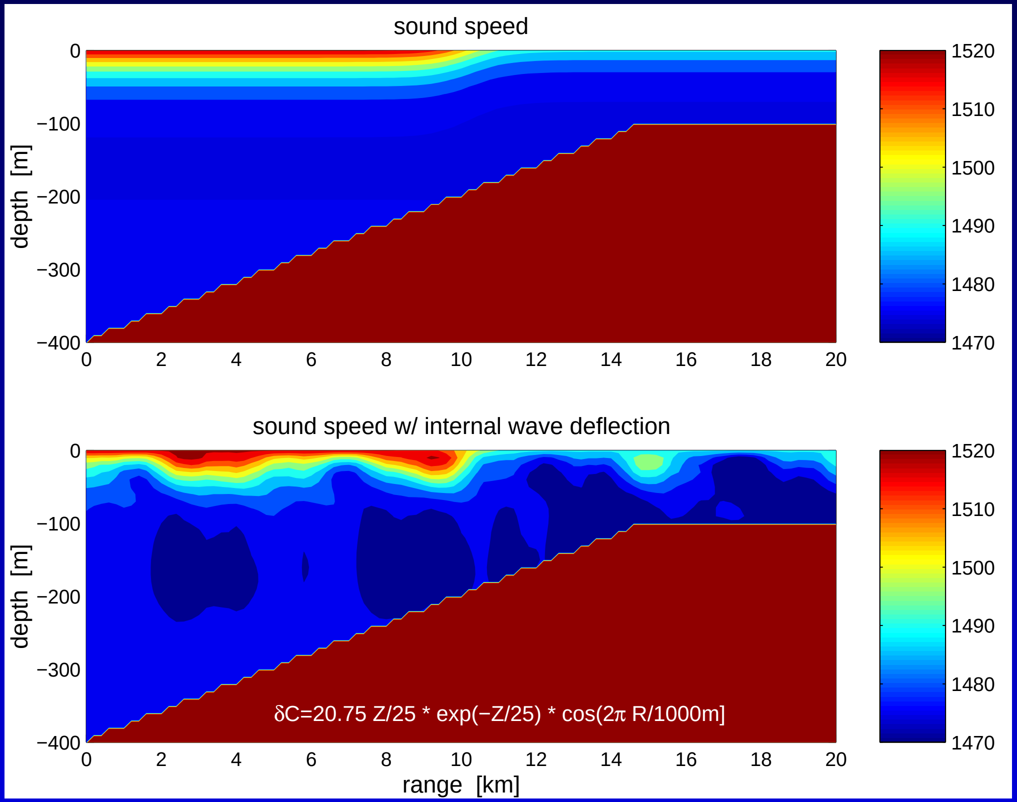


Figure 1: Mean and perturbed sound speed fields.

Contributions to Uncertainty: Error

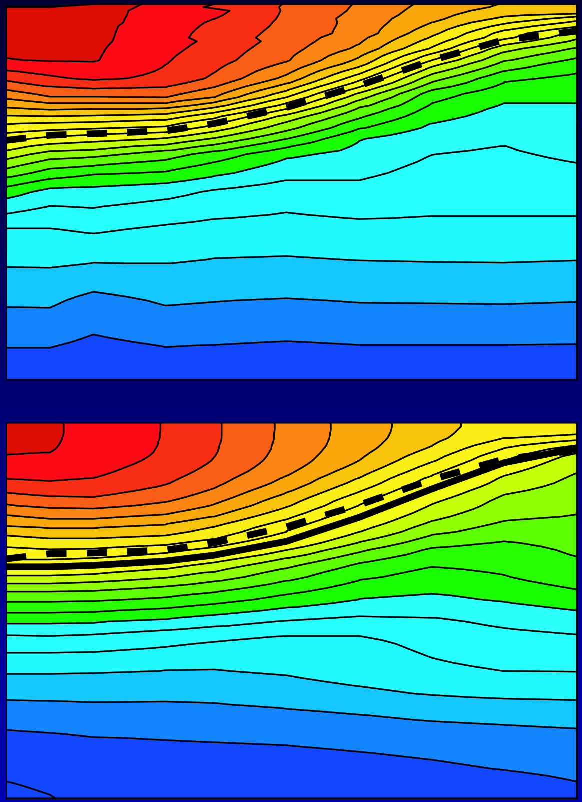


Figure 2: Five-year mean temperature along the equator, observed (top) and modeled (bottom). From R. C. Perez, 2006.

Sampling Variability

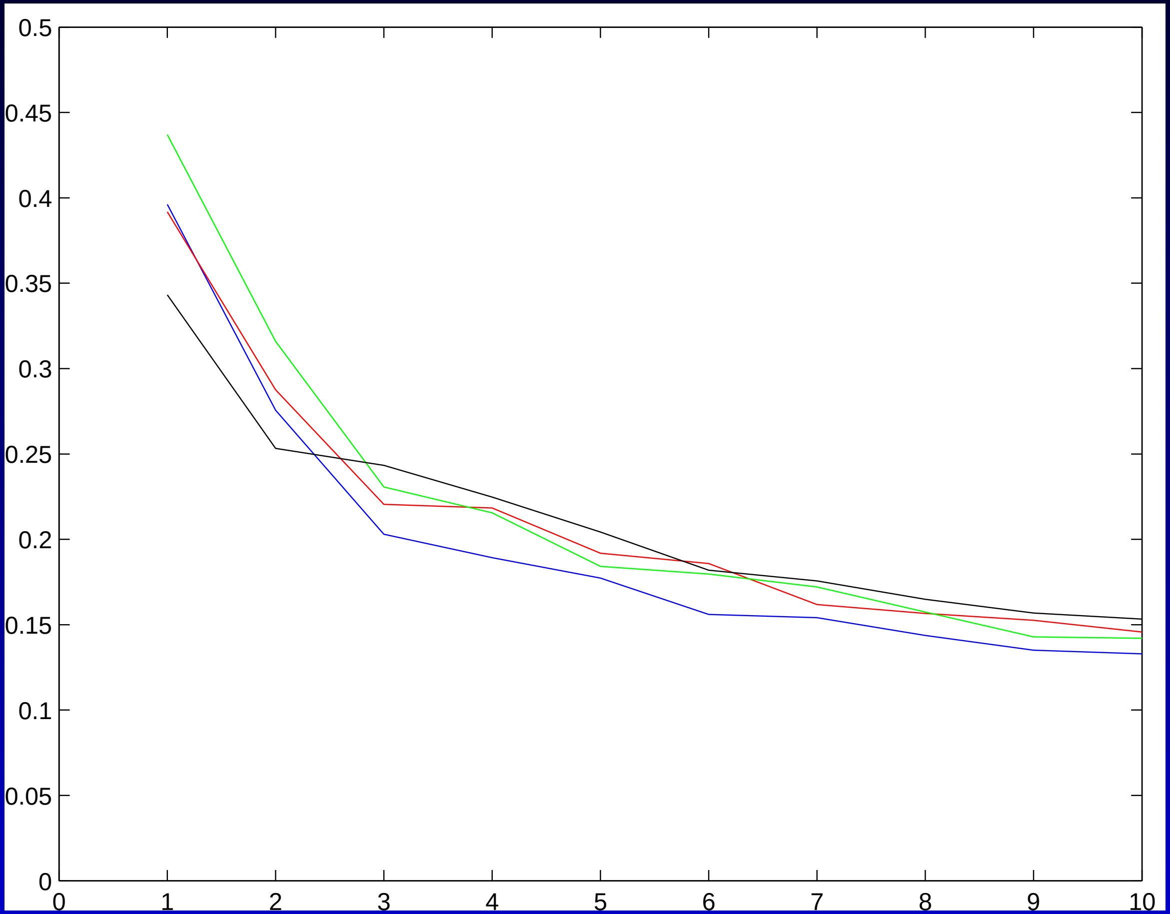


Figure 3: Eigenvalues of SST anomaly covariance, variously sampled

Data Assimilation, Very Briefly

Data assimilation methods are usually derived:
The model system:

$$\mathbf{u}^f_{j+1} = \mathbf{f}(\mathbf{u}^a_j) + \mathbf{F}_j$$

is chosen to mimic the true system:

$$\mathbf{u}^t_{j+1} = \mathbf{f}(\mathbf{u}^t_j) + \mathbf{F}_j + \epsilon_j$$

ϵ_j is a random process with covariance $< \epsilon_j \epsilon_k^T >= Q\delta_{jk}$.

Data Assimilation, Continued

One way to do it:

- We have observations $\mathbf{z} = H\mathbf{u}^t + \epsilon^o$
- ϵ^o is the obs error with covariance R
- Data assimilation makes use of data misfits, aka *innovations*: $\mathbf{z} - H\mathbf{u}^{(f)}$
- The forecast error covariance $\mathbf{P}_j^f$ evolves according to:

$$\mathbf{P}_{j+1}^f = \mathbf{L}\mathbf{P}_j^a\mathbf{L}^T + \mathbf{Q}$$

where $\mathbf{P}_j^a$ is the *analysis error covariance*

Data Assimilation, Continued

- The analysis :

$$\mathbf{u}_j^a = \mathbf{u}_j^f + \mathbf{K}(\mathbf{z} - \mathbf{H}\mathbf{u}_j^f)$$

- $\mathbf{K} = \mathbf{P}\mathbf{H}^T(\mathbf{H}\mathbf{P}\mathbf{H}^T + R)^{-1}$ is the *Kalman Gain Matrix*
- The analysis covariance:

$$\mathbf{P}_j^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_j^f$$

- This is the *Kalman filter*

Posterior Tests

In a Kalman filter operating properly:

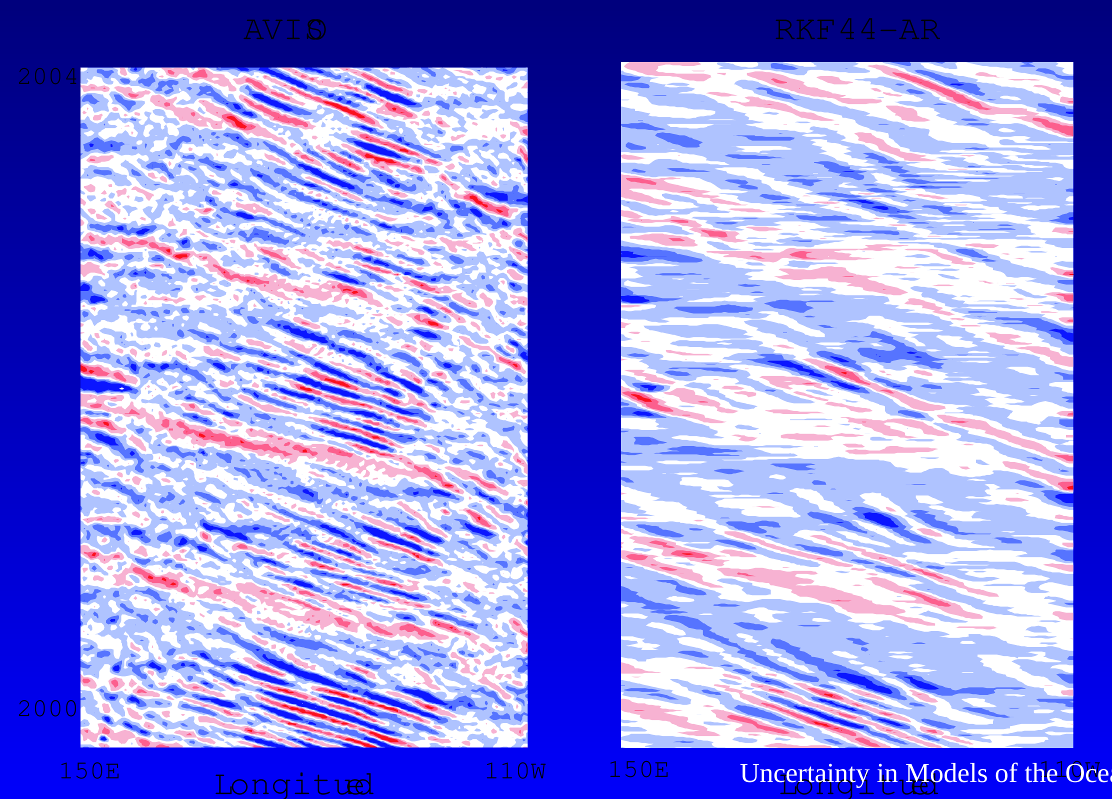
- The innovation sequence will be white
- The quantity:

$$(\mathbf{z} - \mathbf{H}\mathbf{u}_j^f)^T (\mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R})^{-1} (\mathbf{z} - \mathbf{H}\mathbf{u}_j^f)$$

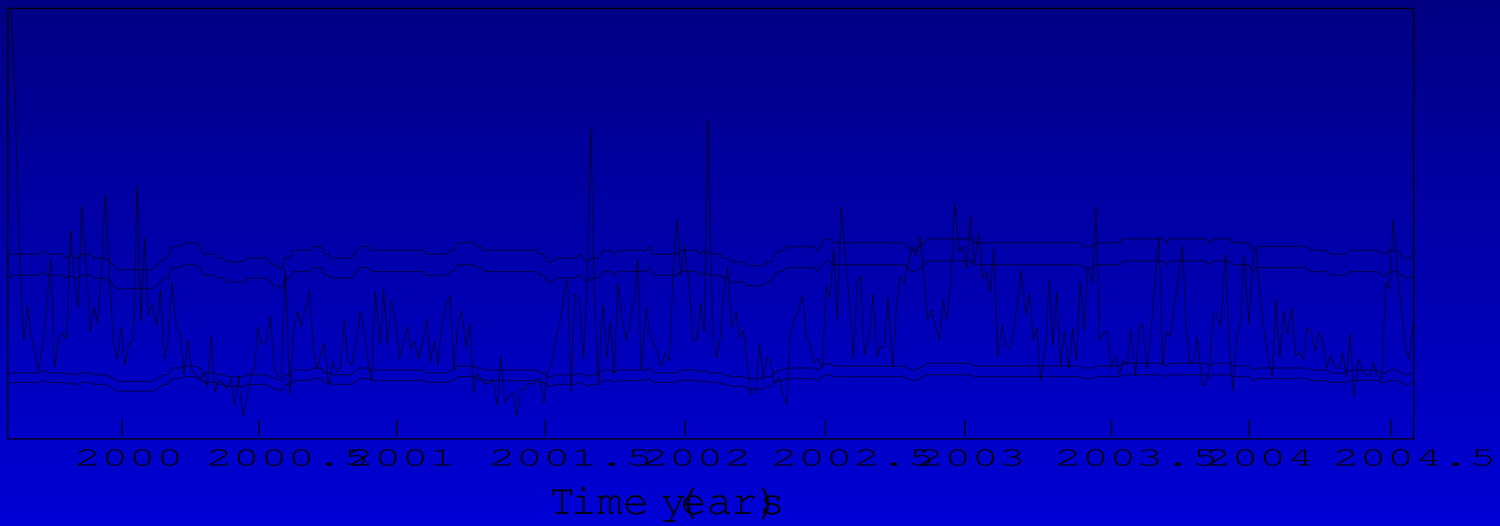
will be a random variable with χ^2 distribution on a number of degrees of freedom equal to the number of observations

Assimilation of Dynamic Height Data

- Model: The GCM of Gent and Cane (1989) applied to the tropical Pacific
- Reduced state space Kalman filter
- Dynamic height data from the TAO array



The χ^2 Test



(R. C. Perez, 2006)

What, Exactly, are we Modeling?

Data assimilation methods are usually derived:
The model system:

$$\mathbf{u}^f_{j+1} = \mathbf{f}(\mathbf{u}^a_j) + \mathbf{F}_j$$

is chosen to mimic the true system:

$$\mathbf{u}^t_{j+1} = \mathbf{f}(\mathbf{u}^t_j) + \mathbf{F}_j + \epsilon_j$$

ϵ_j is a random process.
But what do we mean by *true*?

In Search of the *True* State

- The ocean measured by instruments doesn't know about physical approximations, coarse resolution or their consequences
- It is not subject to the limitations in computing power that restrict models to coarse resolution
- Measurements are not subject to the same requirements for approximate physical parameterizations

So ask: What quantity in nature is the "true" value of the model state? Does it even exist in a meaningful way?
No specific answers today (but see, e.g., L. Smith, 2000); Rather a suggestion for what to do while we are waiting.

Representation Error

- Data assimilation makes use of data misfits, aka *innovations*: $\mathbf{z} - H\mathbf{u}^{(f)}$
- $\mathbf{u}^{(f)}$ is the forecast state
- Let $\tilde{\mathbf{u}}^t$ be the “true” ocean, as the instruments measure it.

Representation Error

Write the innovation:

$$
\begin{aligned}
\mathbf{z} - H\mathbf{u}^{(f)} &= \mathbf{z} - \mathbf{z}^{(t)} + \mathbf{z}^{(t)} - H\mathbf{u}^{(f)} \\
&= \epsilon^0 + \mathbf{H}(\tilde{\mathbf{u}}^{(t)} - \mathbf{u}^{(t)}) + \mathbf{H}(\mathbf{u}^{(t)} - \mathbf{u}^{(f)})
\end{aligned}
$$

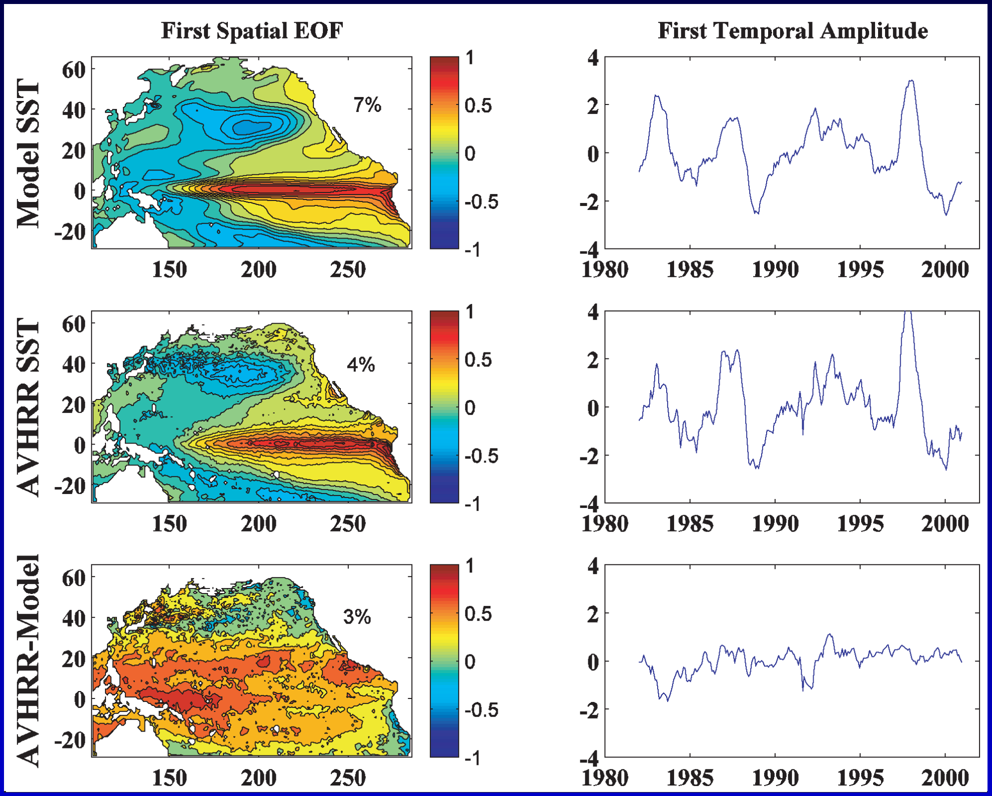
- $\epsilon^0 = \mathbf{z} - \mathbf{z}^{(t)}$,the *instrument error*
- $\mathbf{H}(\tilde{\mathbf{u}}^{(t)} - \mathbf{u}^{(t)})$ is *representation error*
- Estimates of its statistics must appear in the terms reserved for instrument error
- $\mathbf{u}^{(t)} - \mathbf{u}^{(f)}$ is the *forecast error*

Estimating Representation Error

Our method for estimating the representation error for SST:

1. Generate a long model run
2. Calculate EOFs of the model run, considered as a matrix whose (i, j) element is the value of state element j at time i
3. Determine the number of meaningful degrees of freedom
4. Project the innovations on the meaningful singular vectors
5. Subtract the result from the innovations.
6. The difference is an estimate of the representation error

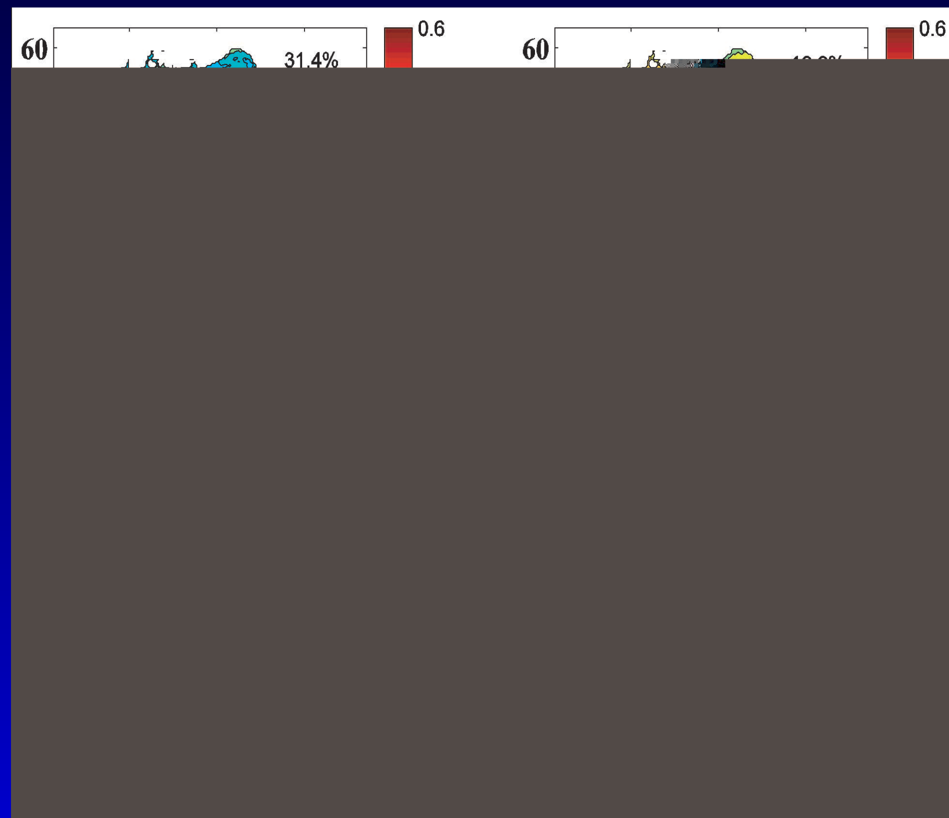
Model and AVHRR Seasonal Anomalies: First EOF



Representation Error

- Project model-data misfits on multivariate EOFs. This is the portion of the data that is compatible with the model.
- Subtract the result from the model-data misfits. This is an estimate of the error of representation.
- Calculate EOFs of error of representation

EOFs of SST Representation Error



Representation Error in Climate Forecast Models

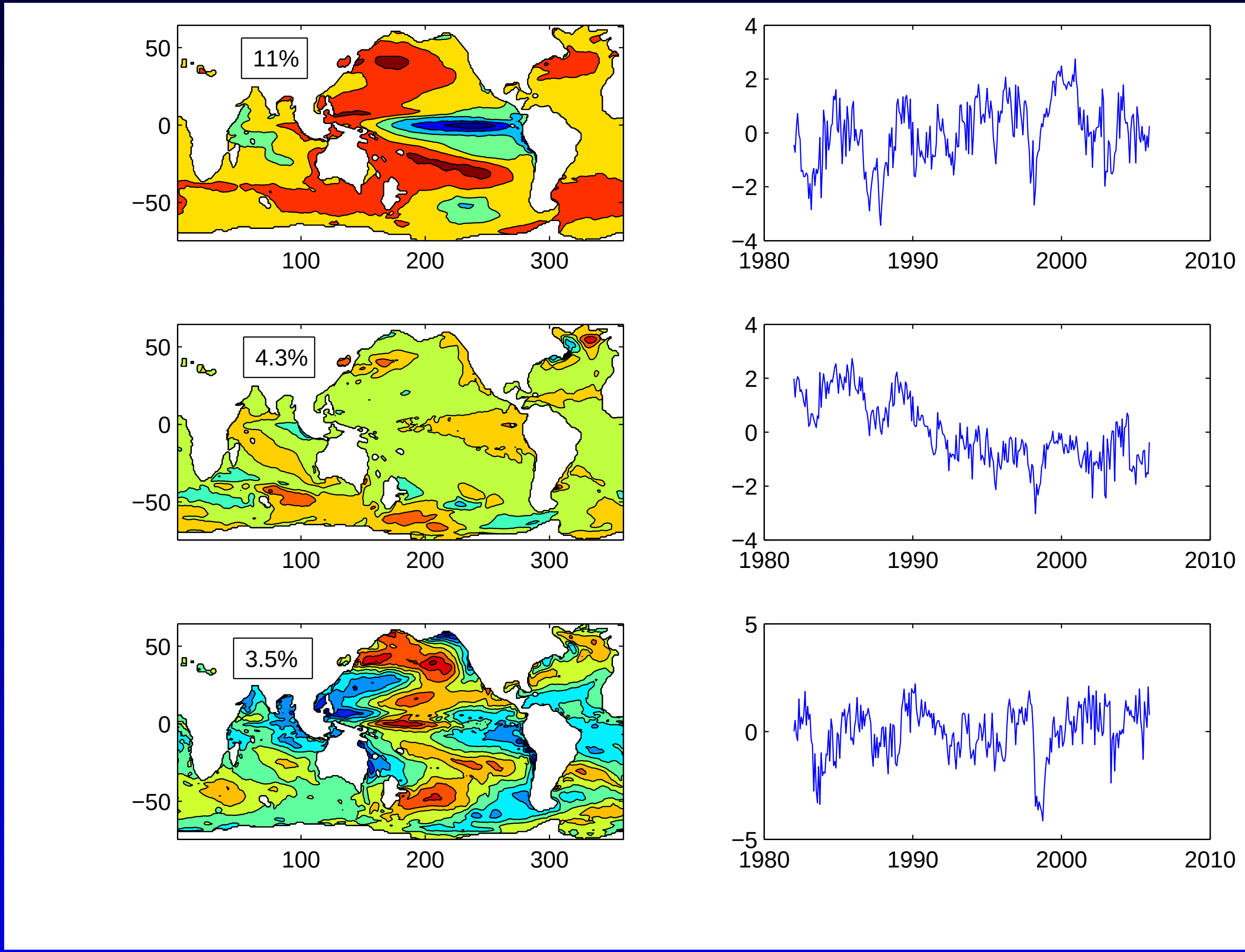
- Climate models are often run past their forecast horizons
- Climate models can only produce forecasts consistent with their internal physics
- Representation errors could take on crucial importance

Ocean Component, NCEP CFS

- Basically global MOM/POP
- Resolution 1^o over most of the ocean, tapering to 0.33^o from 30^o N/S to 8.5^o
- 24 years (1982-2005), 9-month forecasts, from the restart files

Spectral Analysis of the CFS Restart Records

Temperature EOFs and Time Series of Amplitudes



Amplitude of the Lead Model SST EOF

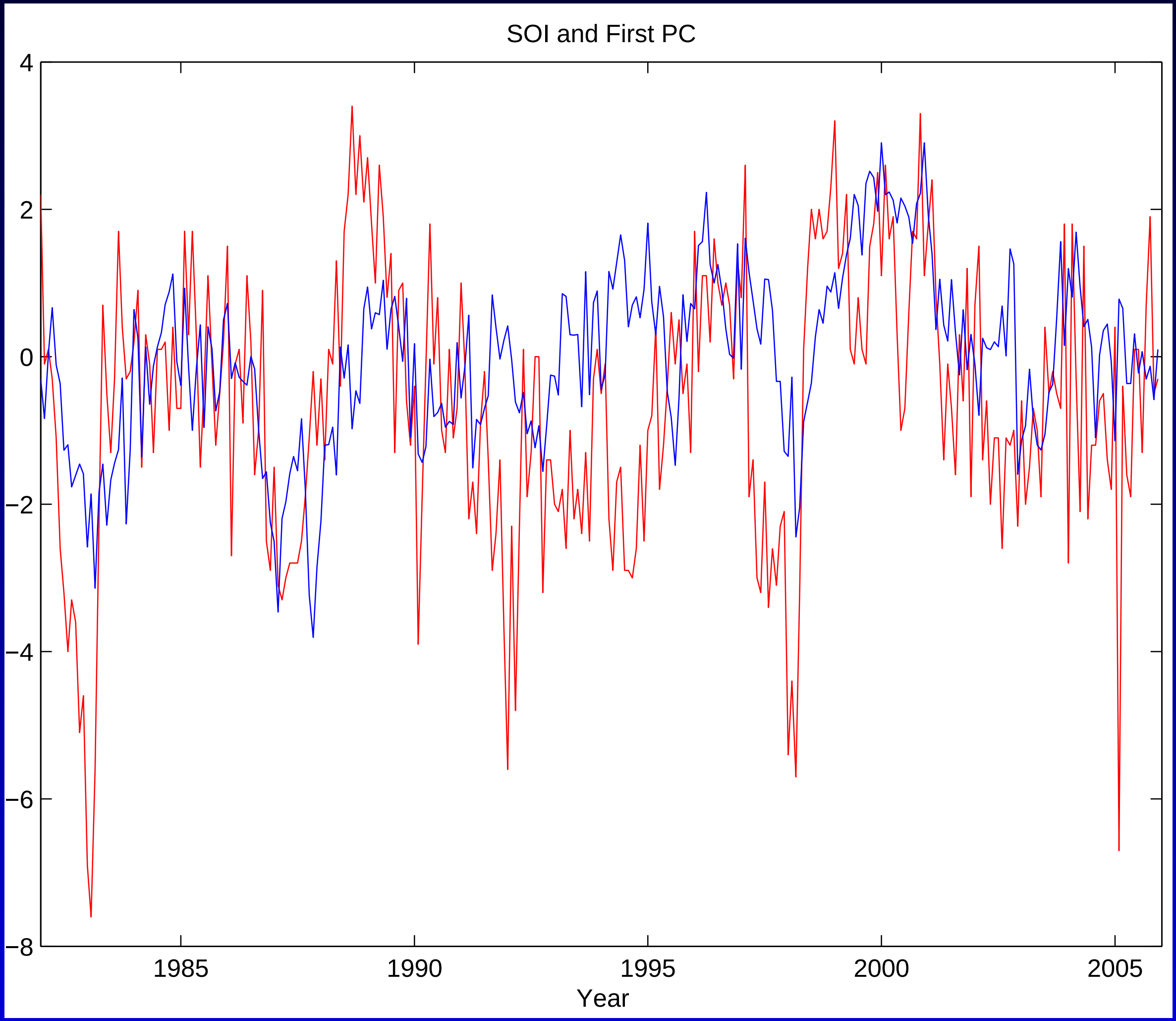


Figure 4: Blue=lead PC; Red=SOI. Correlation ≈ 0.4

Amplitude of Lead EOF of AVHRR SSTA

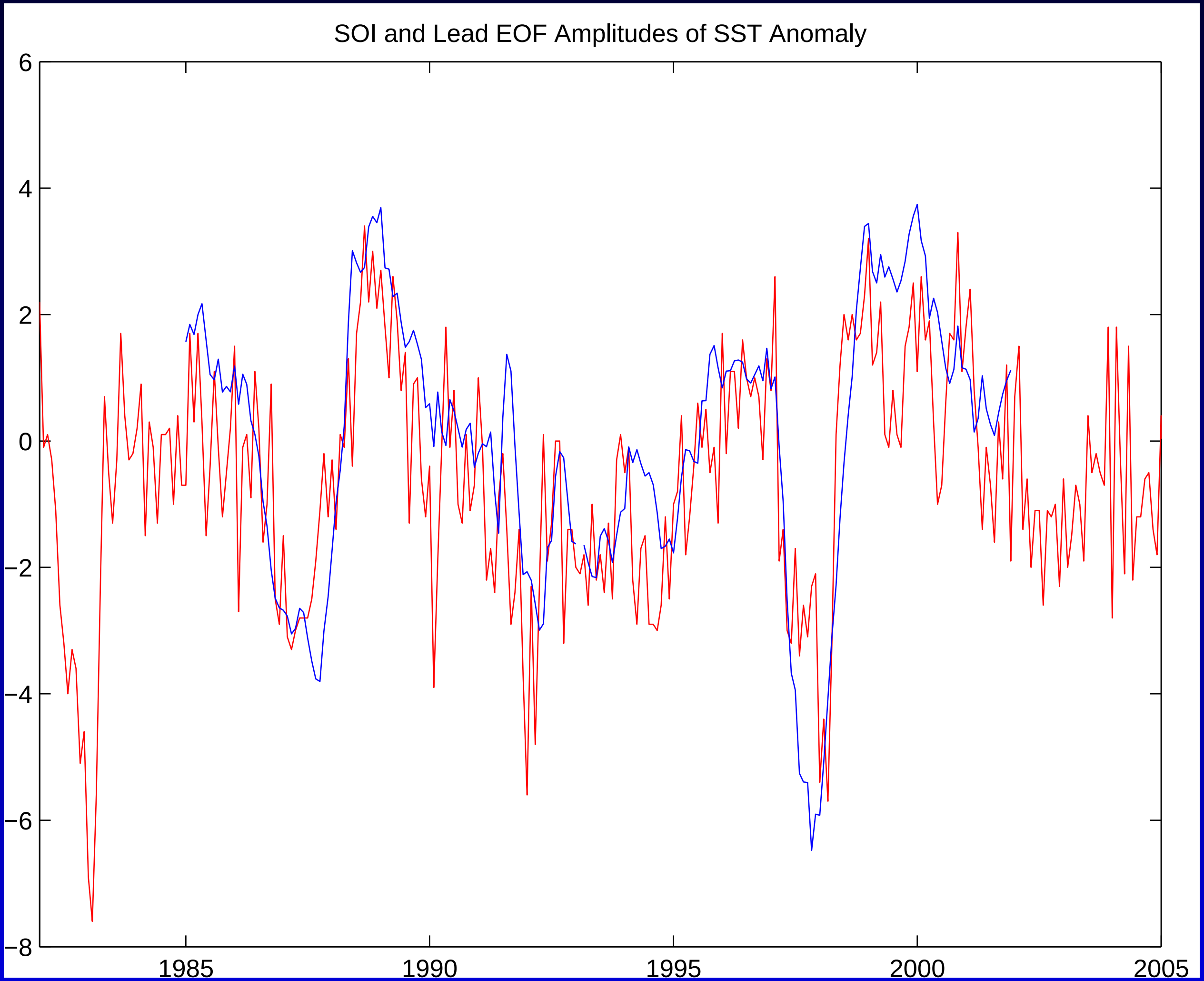
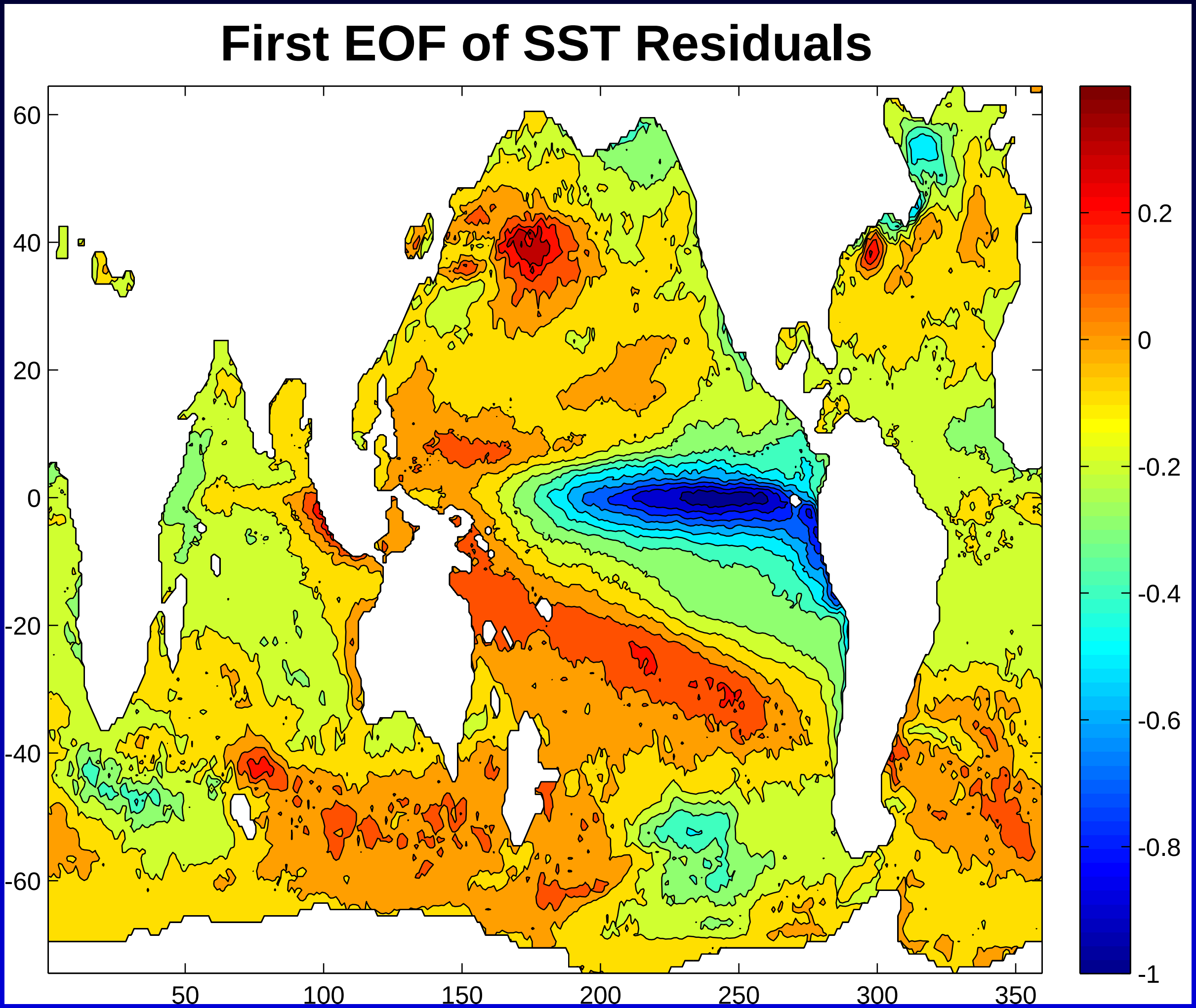
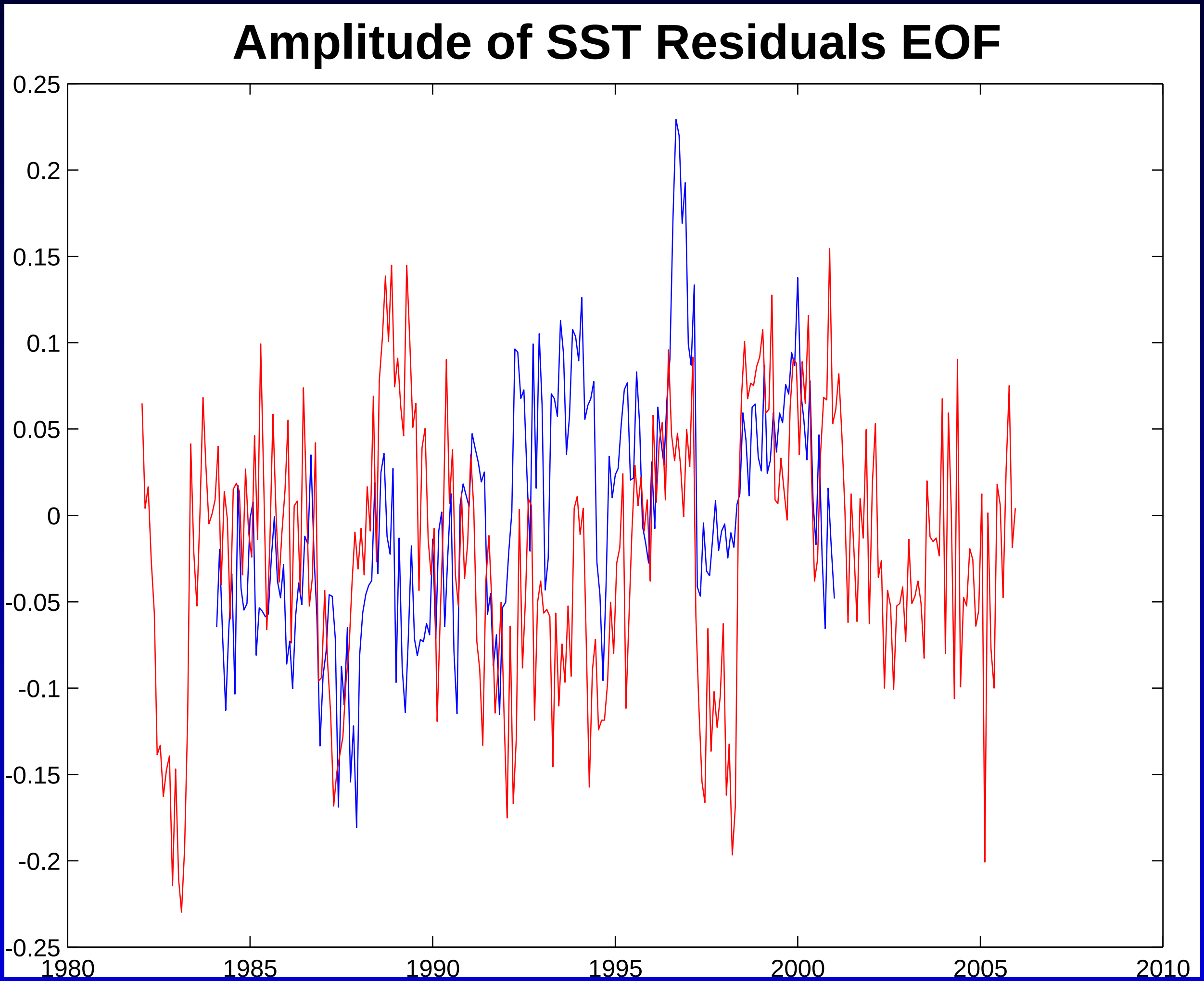


Figure 5: Blue=lead PC; Red=SOI.

Lead EOF of Misfits



Amplitude of Lead EOF of Mis-fits



· · · This is going to be harder than we thought.

Final Thoughts

There's more:

- Nonlinearity
- Dealing with high dimensionality: Construction of small ensembles for high-dimensional systems:
  - Bred vectors, Singular vectors
  - Single model/Multi-model ensembles
- Non-parametric tests for representativeness of ensembles: Talagrand's test and generalizations.

Summary and Conclusions

- We wish to estimate uncertainty in complex high dimensional nonlinear systems.
- We will use complex models to attempt to predict hypthetical outcomes for the future
- Our uncertainty estimates will be based on ensemble behavior. These ensembles will necessarily be small compared to the potential number of degrees of freedom in the problem

Summary and Conclusions, cont'd

- Factors contributing to uncertainty:
  - Model error, including representation error
  - Random influences, e.g. ambient noise, as in the acoustic example
  - Sampling variability
  - Strong nonlinearity
- We should develop and apply objective methods for evaluating ensemble performance

Prediction is very difficult, especially about the future