Expected and Actual Forecast Errors by a Non-normal Model of El Niño

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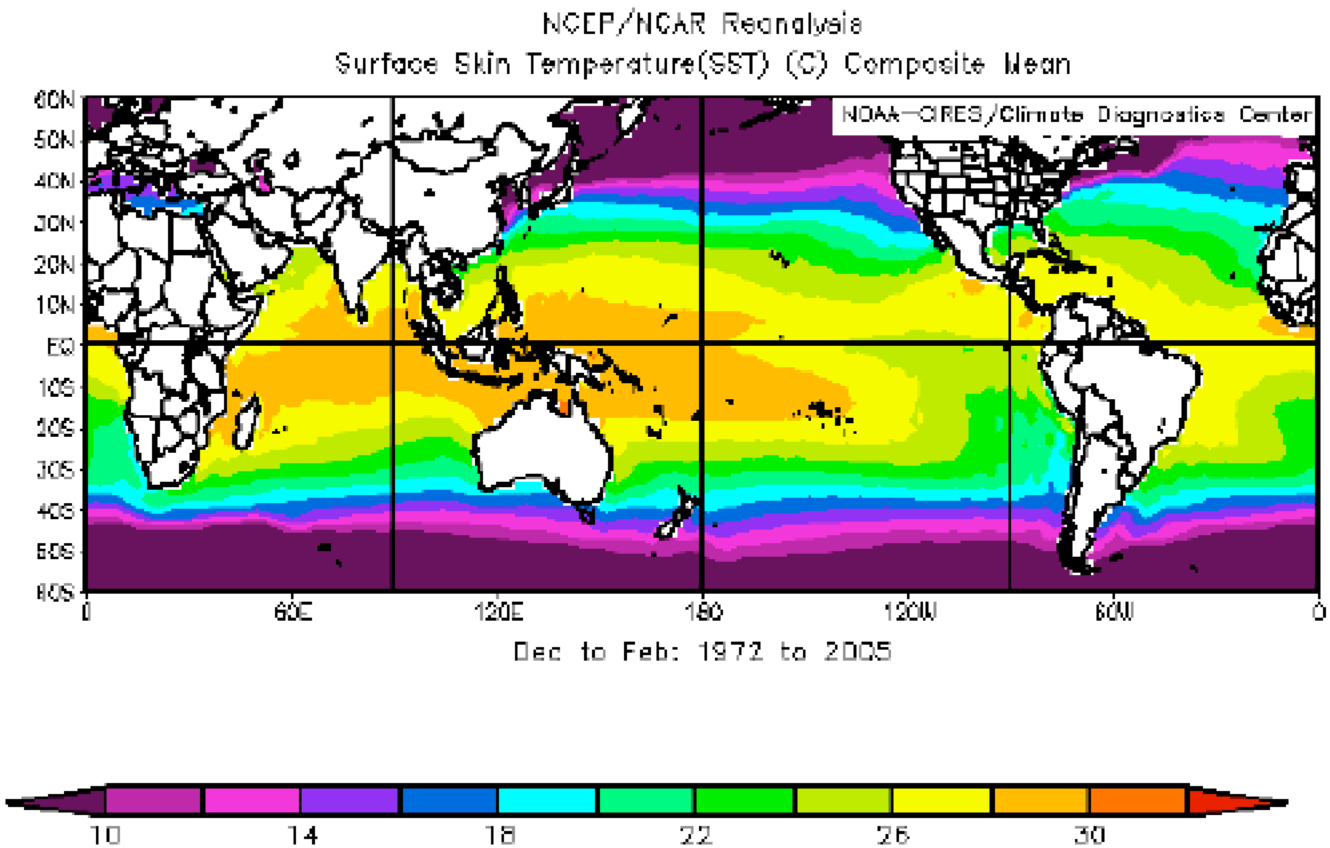
NOAA/ESRL/Physical Sciences Division

Outline

- Some phenomenology: What is El Niño
  - The Annual Cycle
  - Deviations from it, especially El Niño
- An empirical-dynamical model
- Uncertainty and errors


NCEP/NCAR Reanalysis
Surface Skin Temperature(SST) (C) Composite Mean
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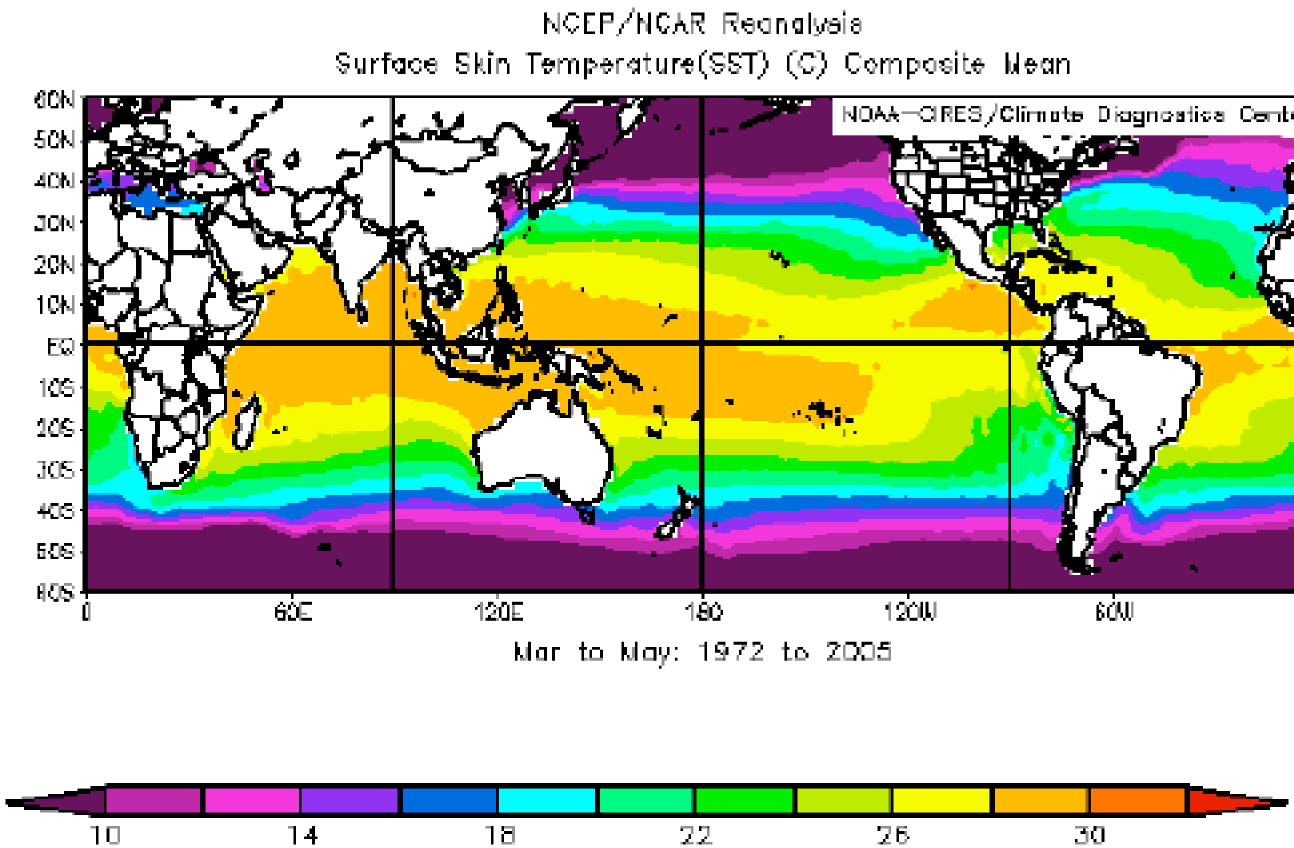
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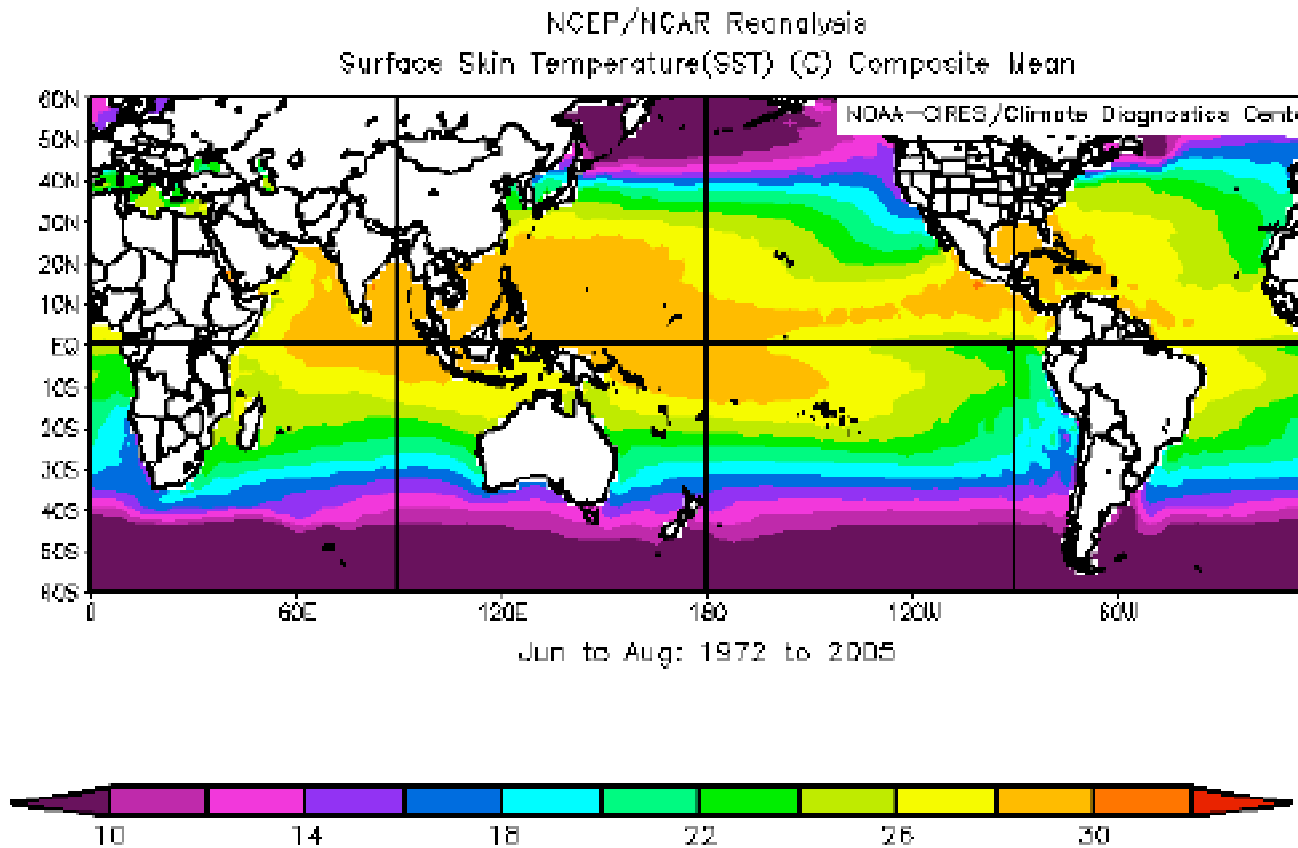
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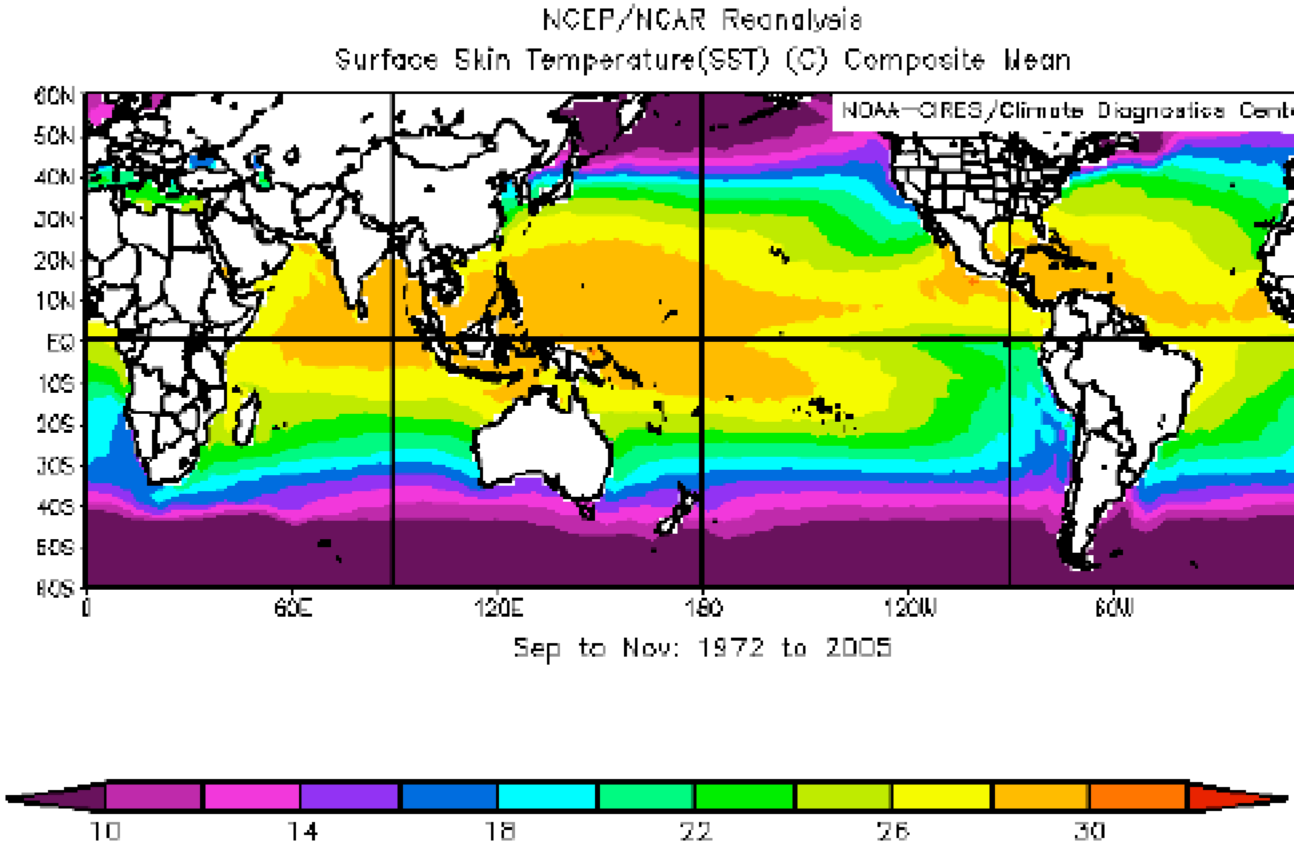
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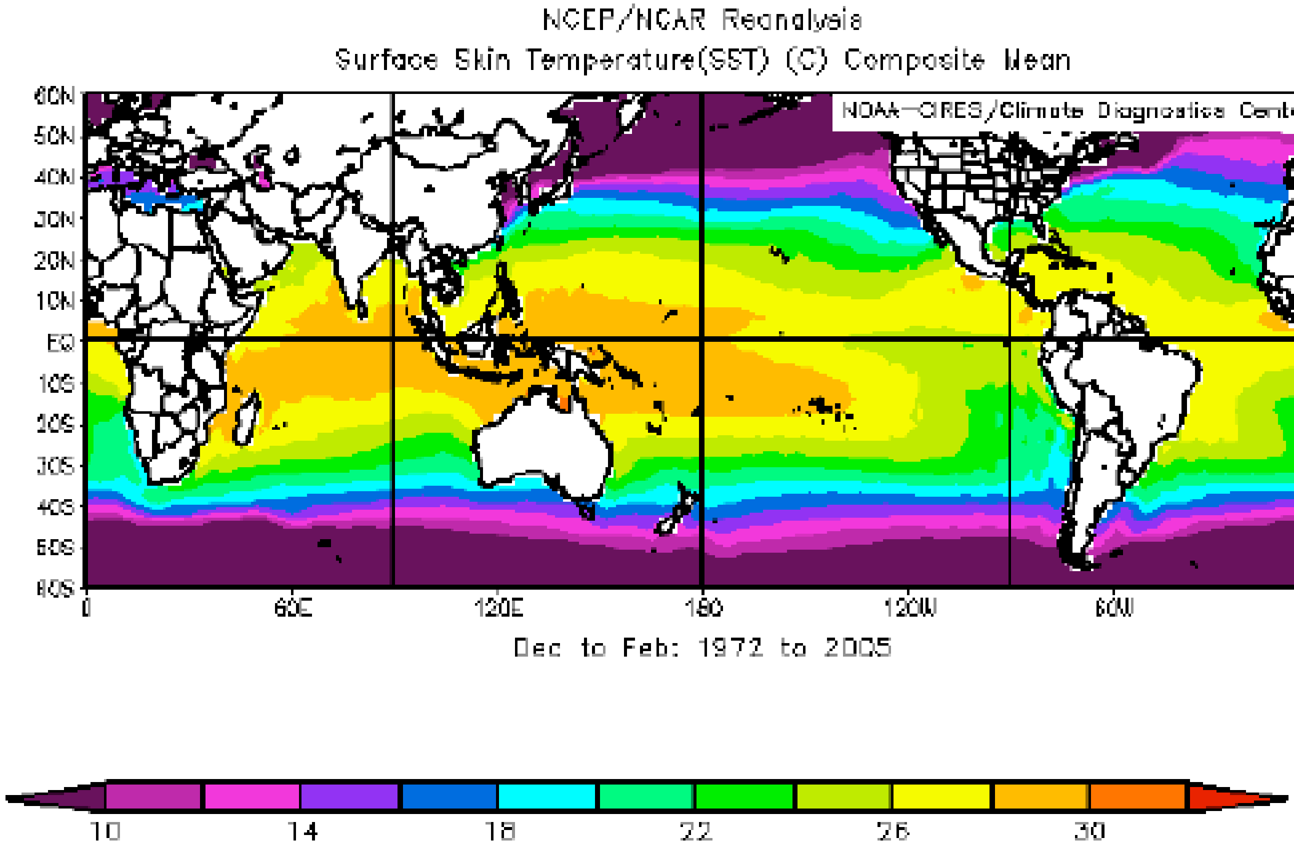
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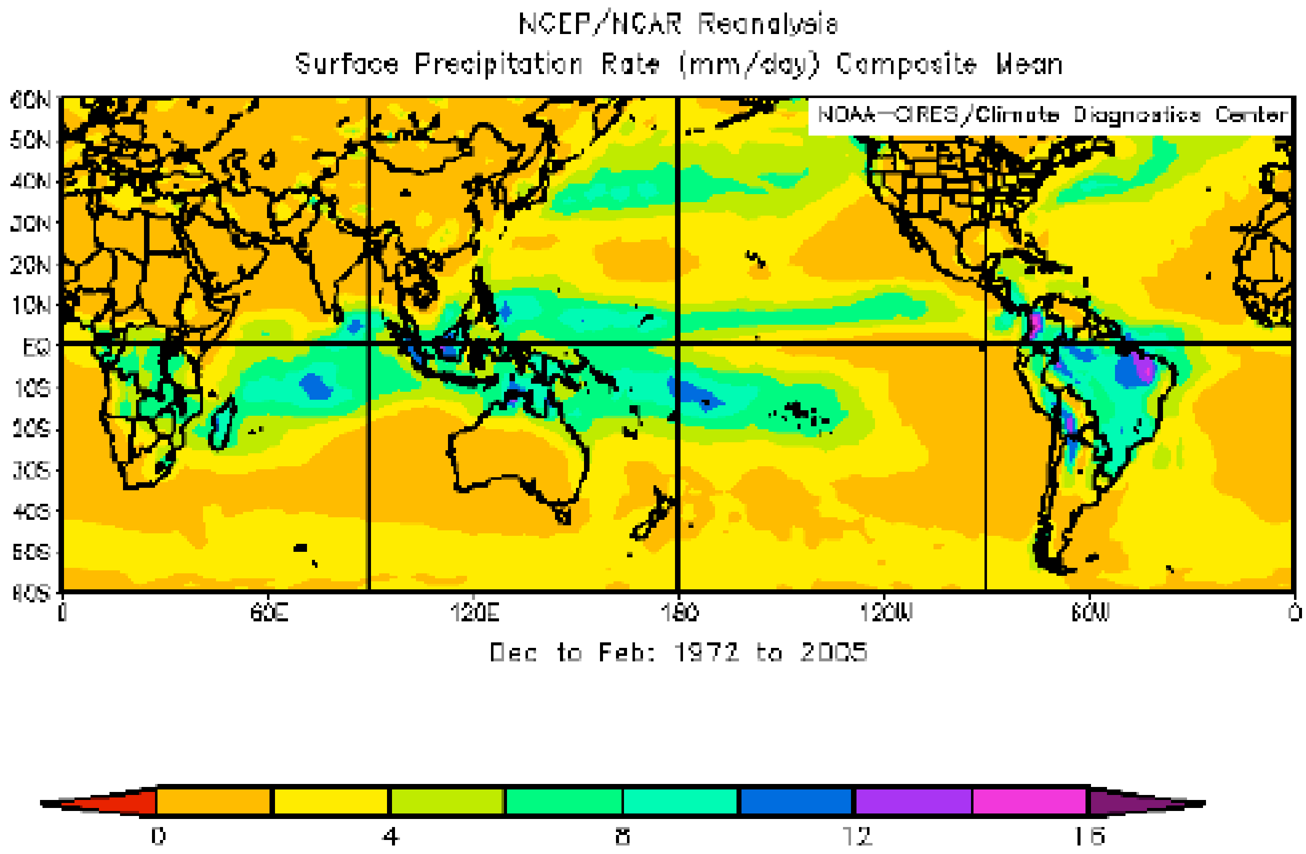
NCEP/NCAR Reanalysis
Surface Precipitation Rate (mm/day) Composite Mean
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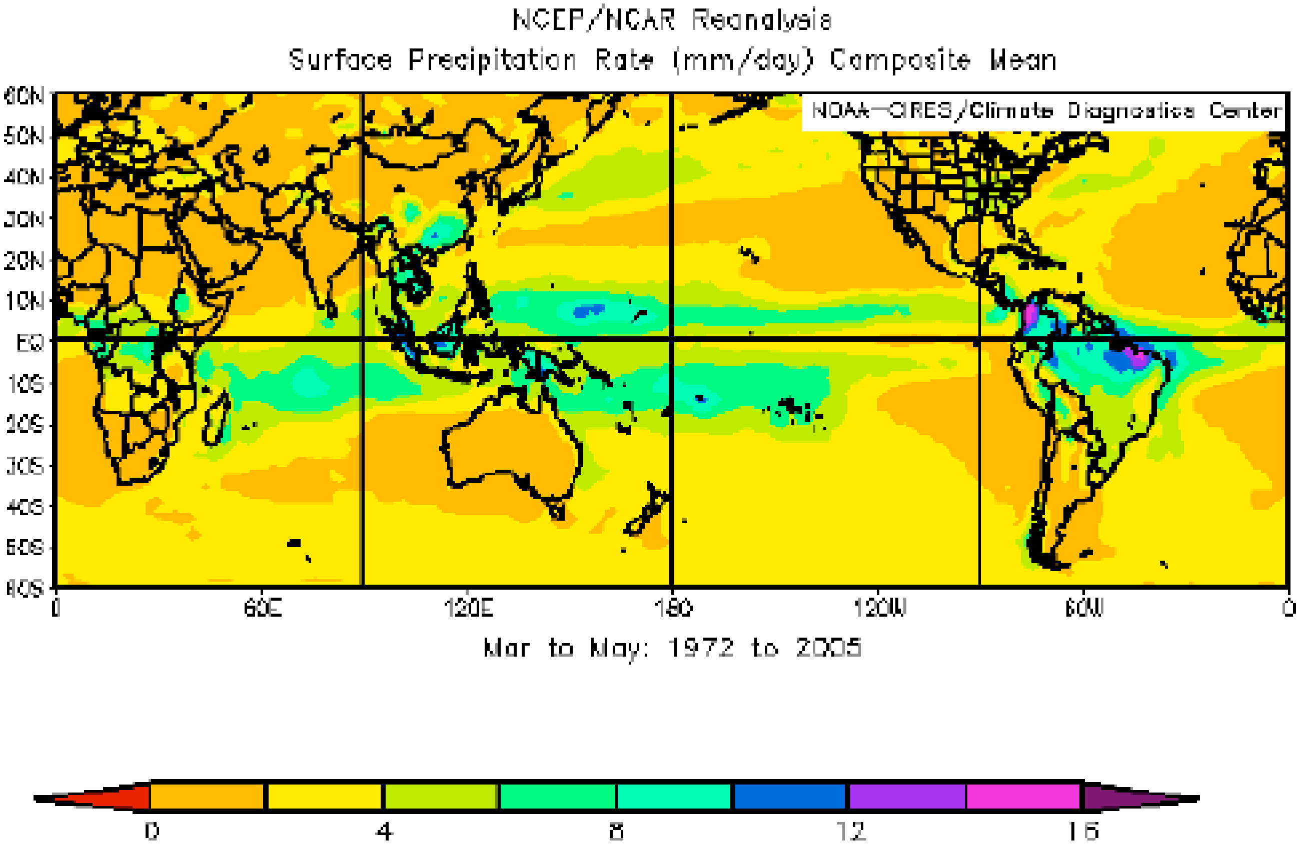
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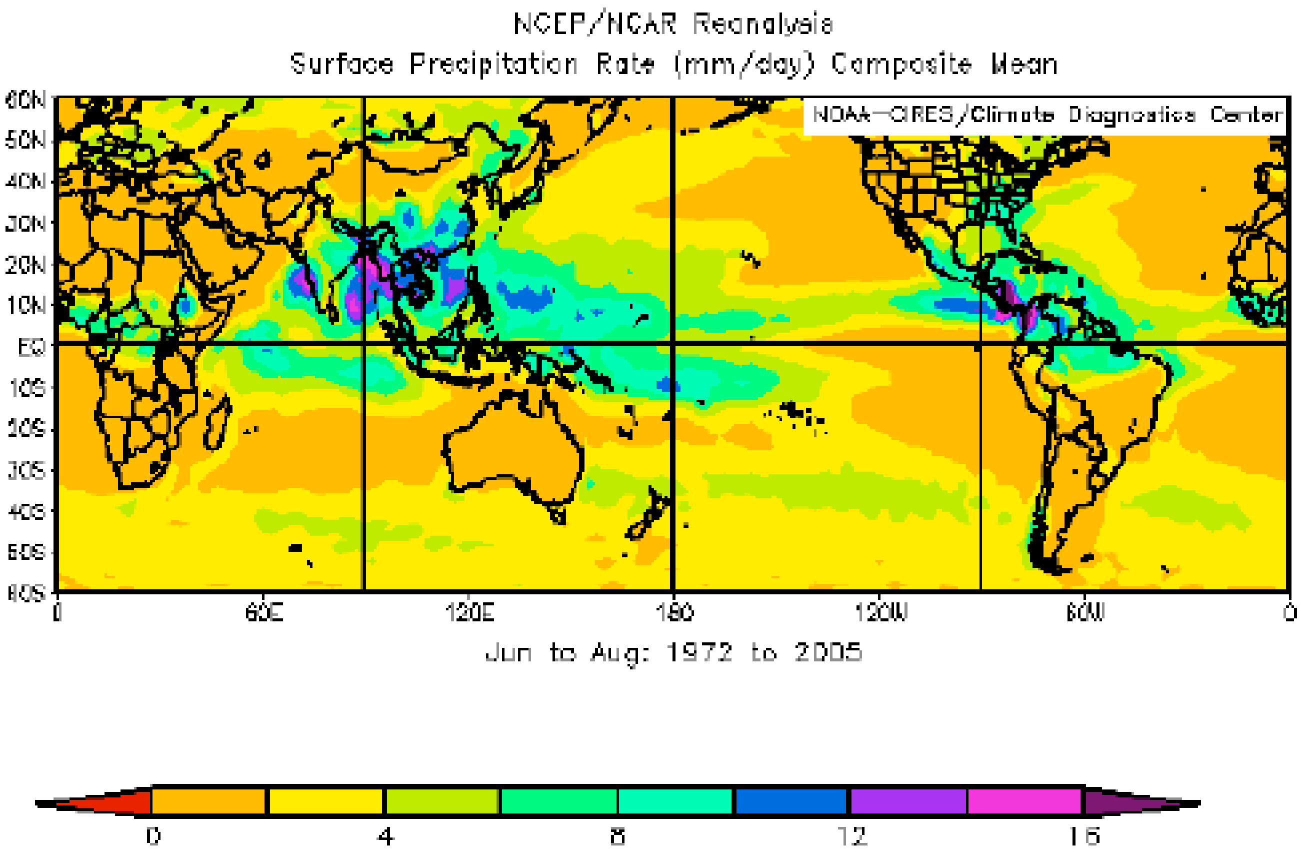
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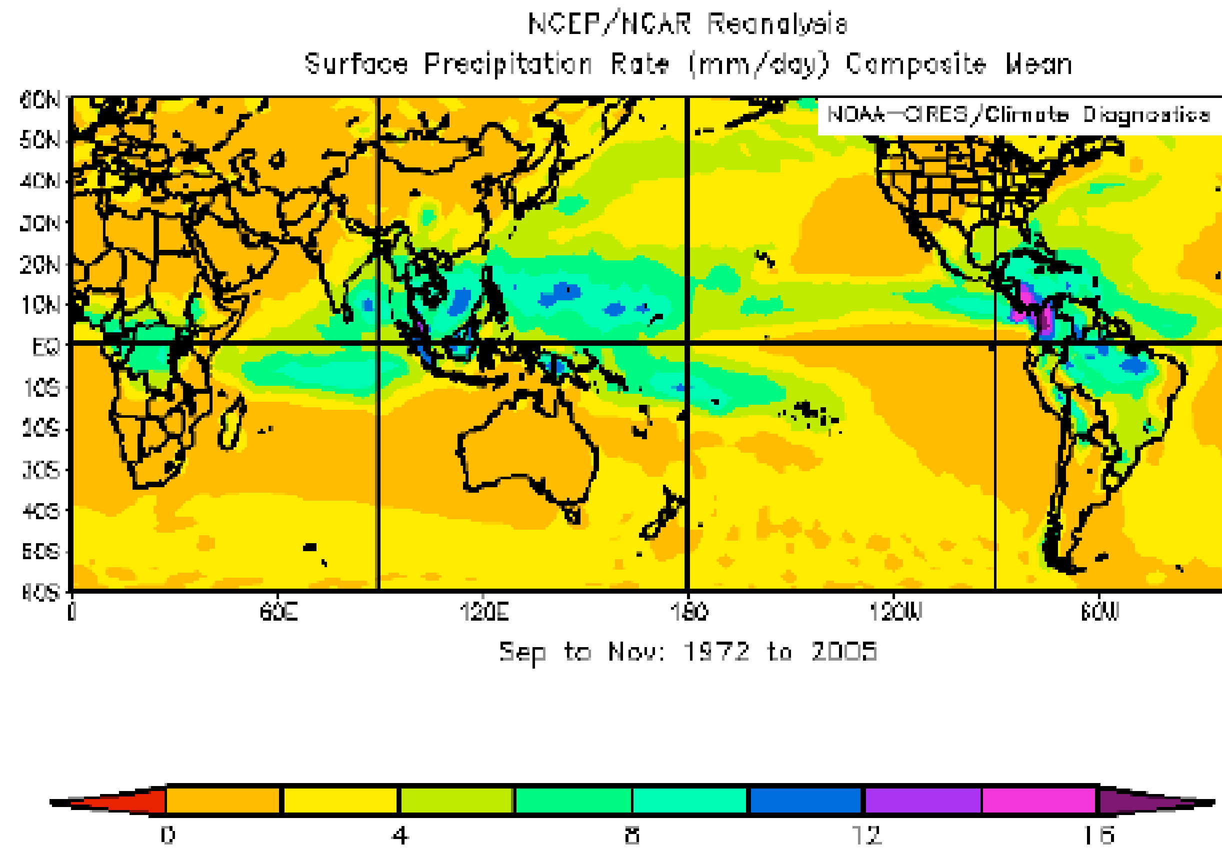

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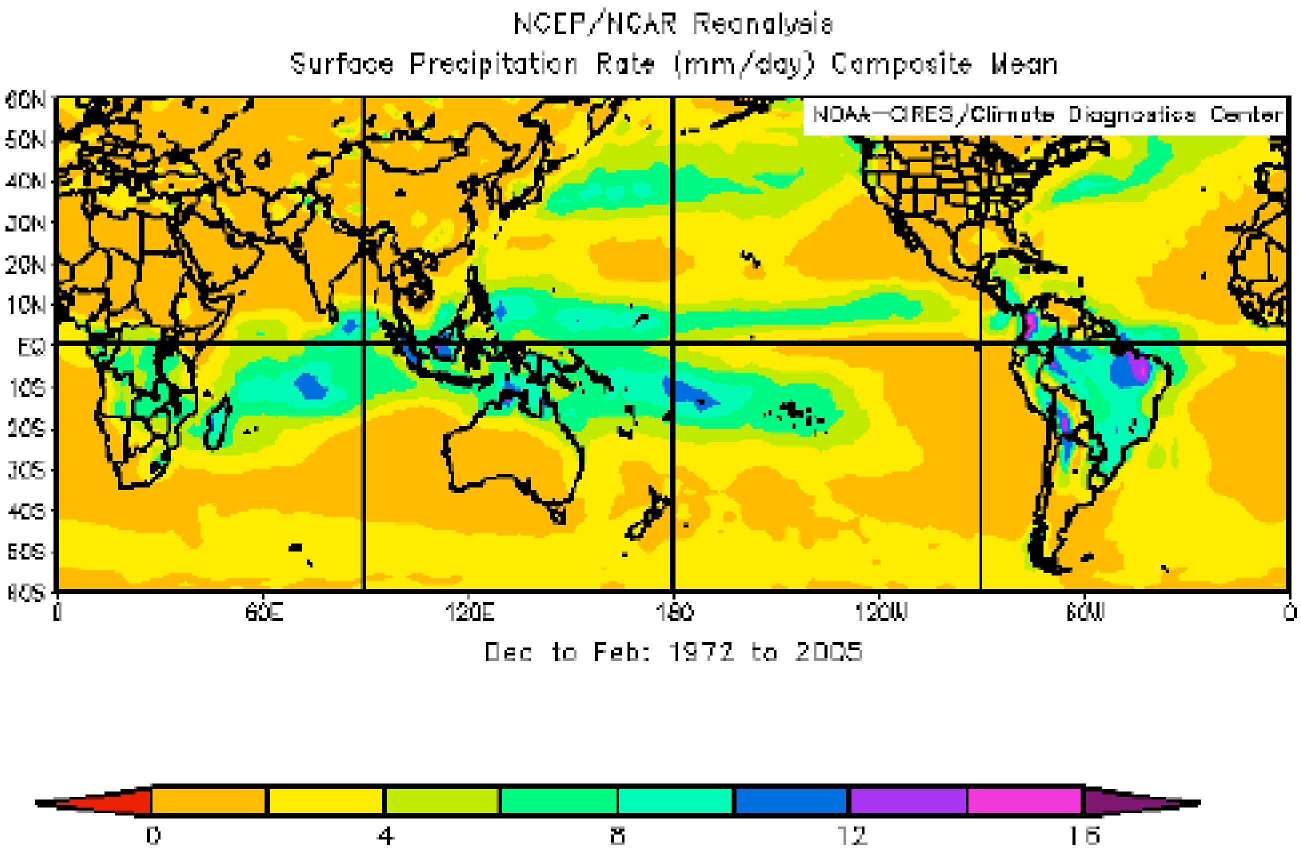
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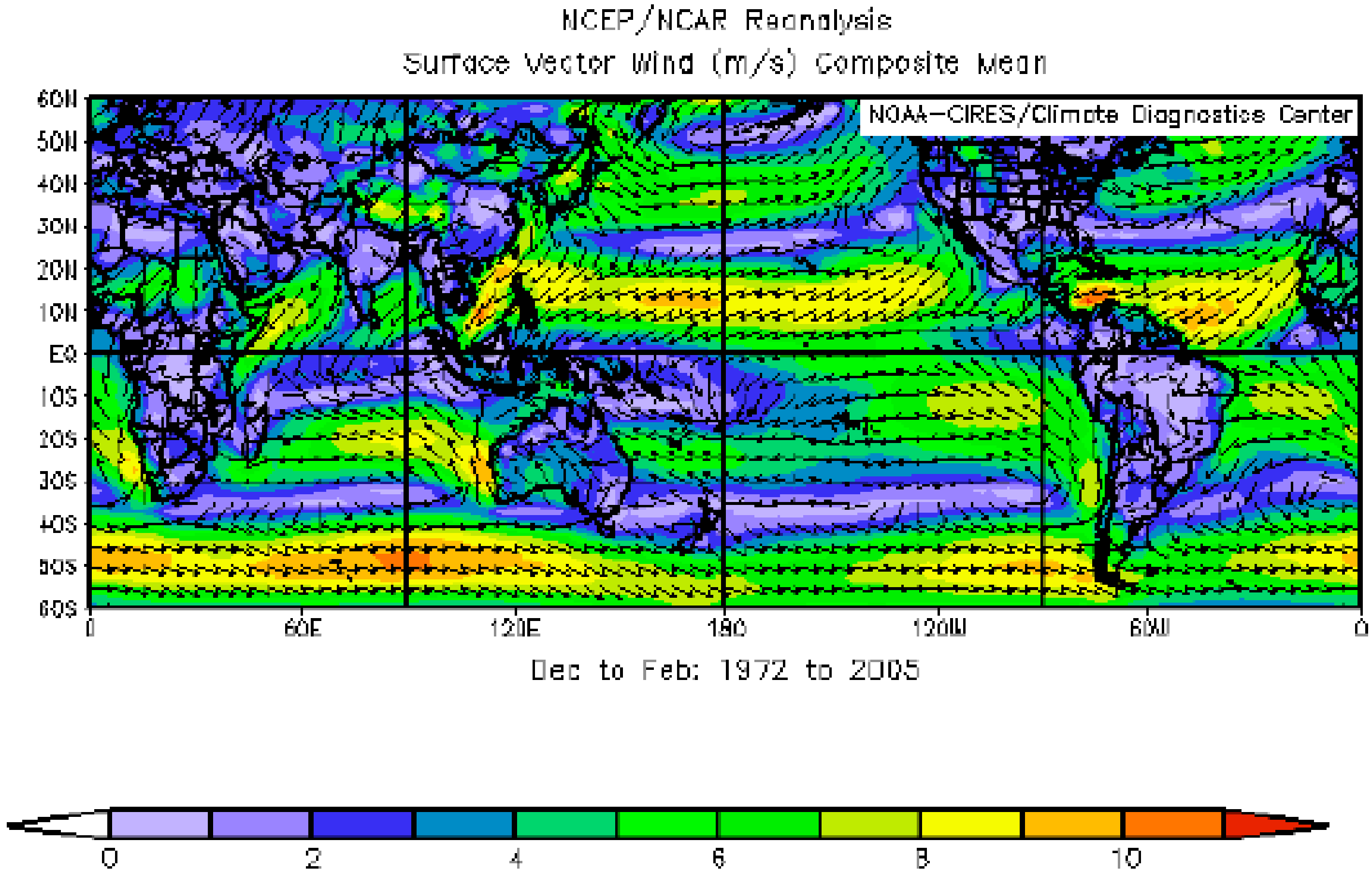

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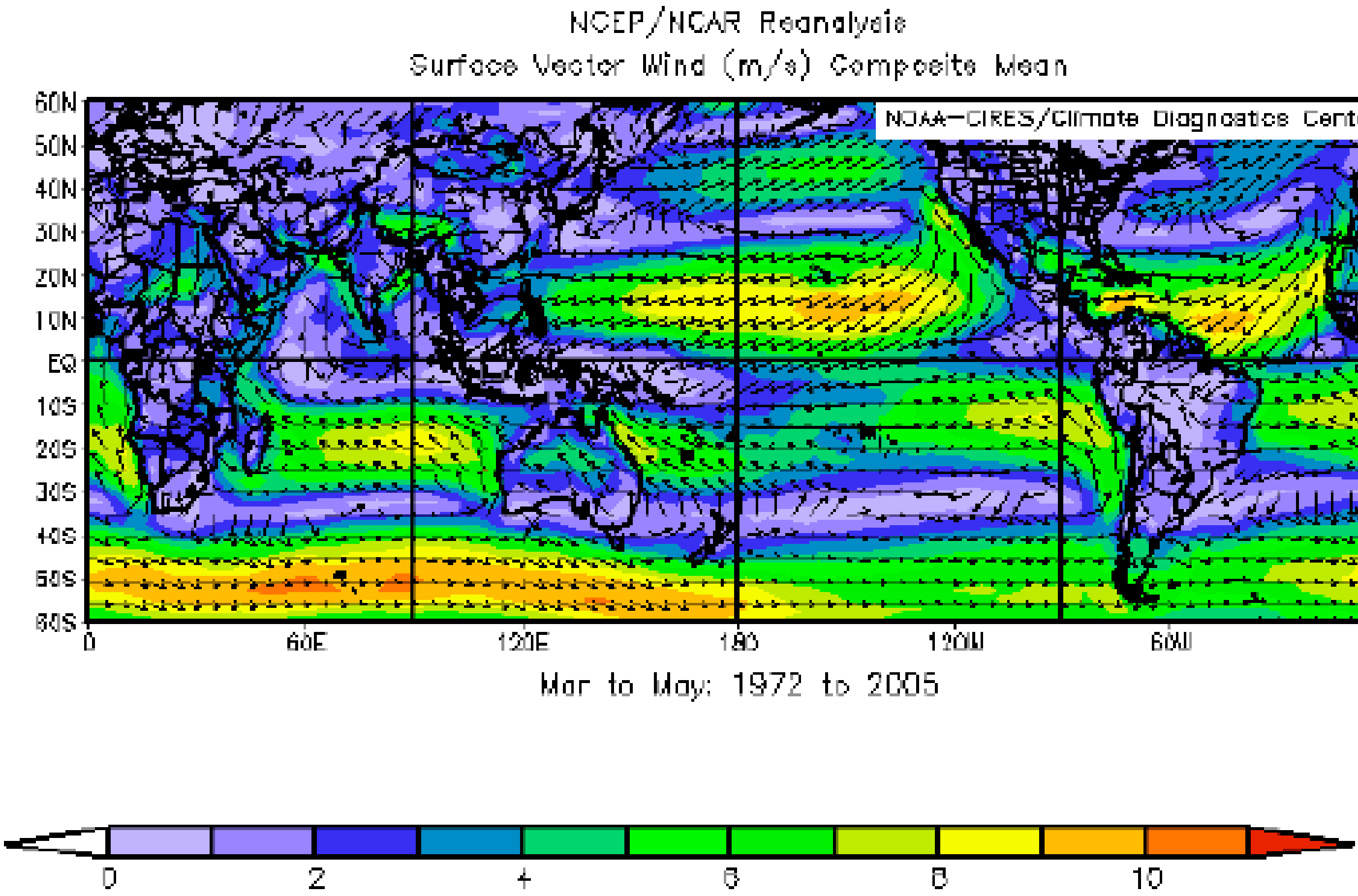
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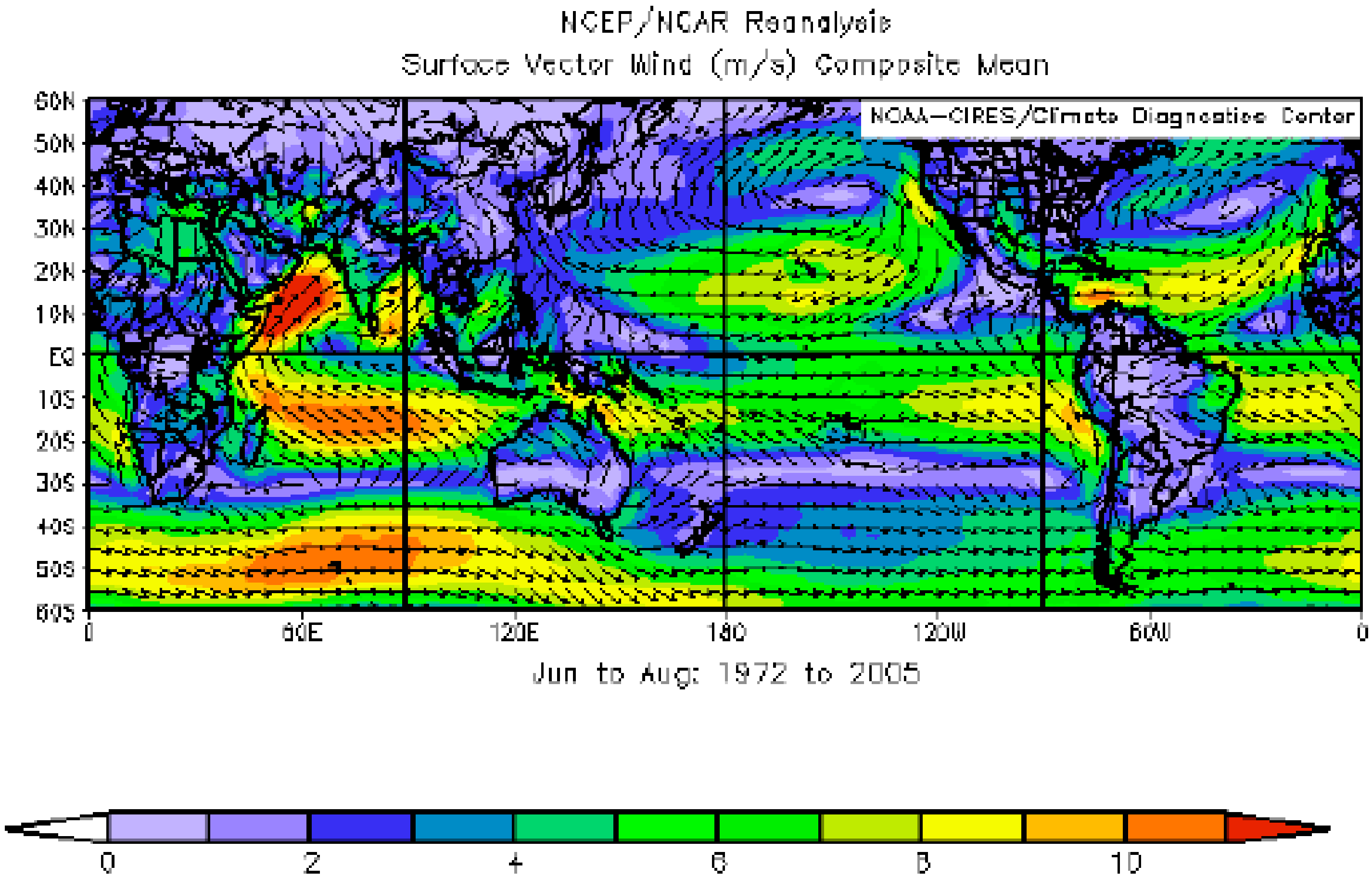
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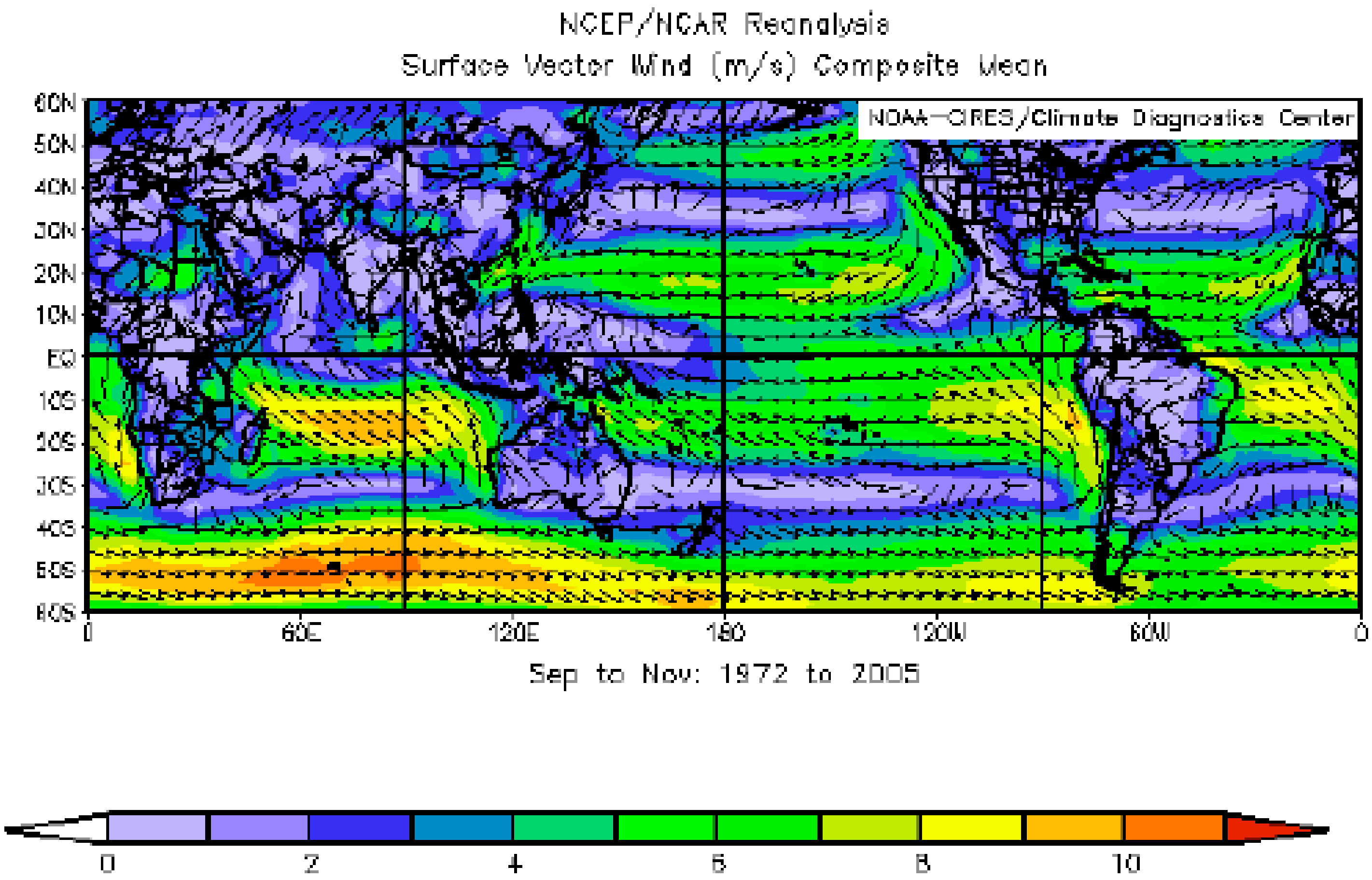
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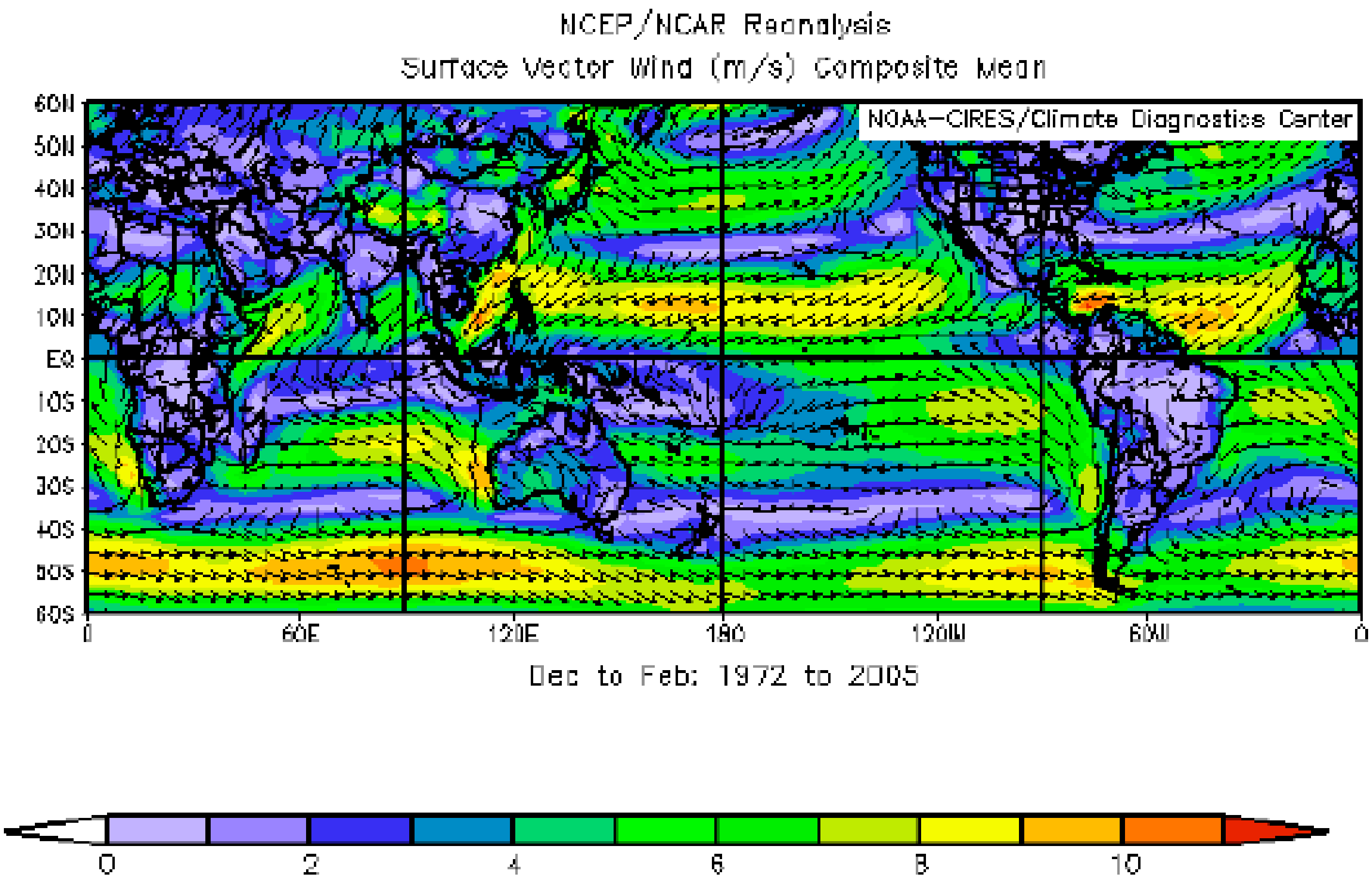
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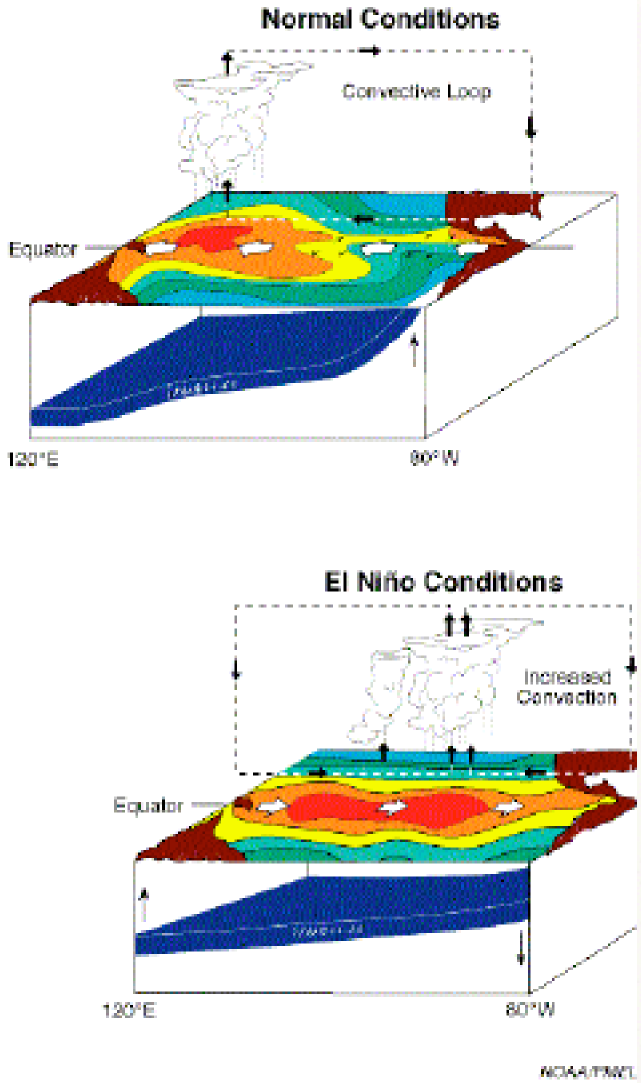
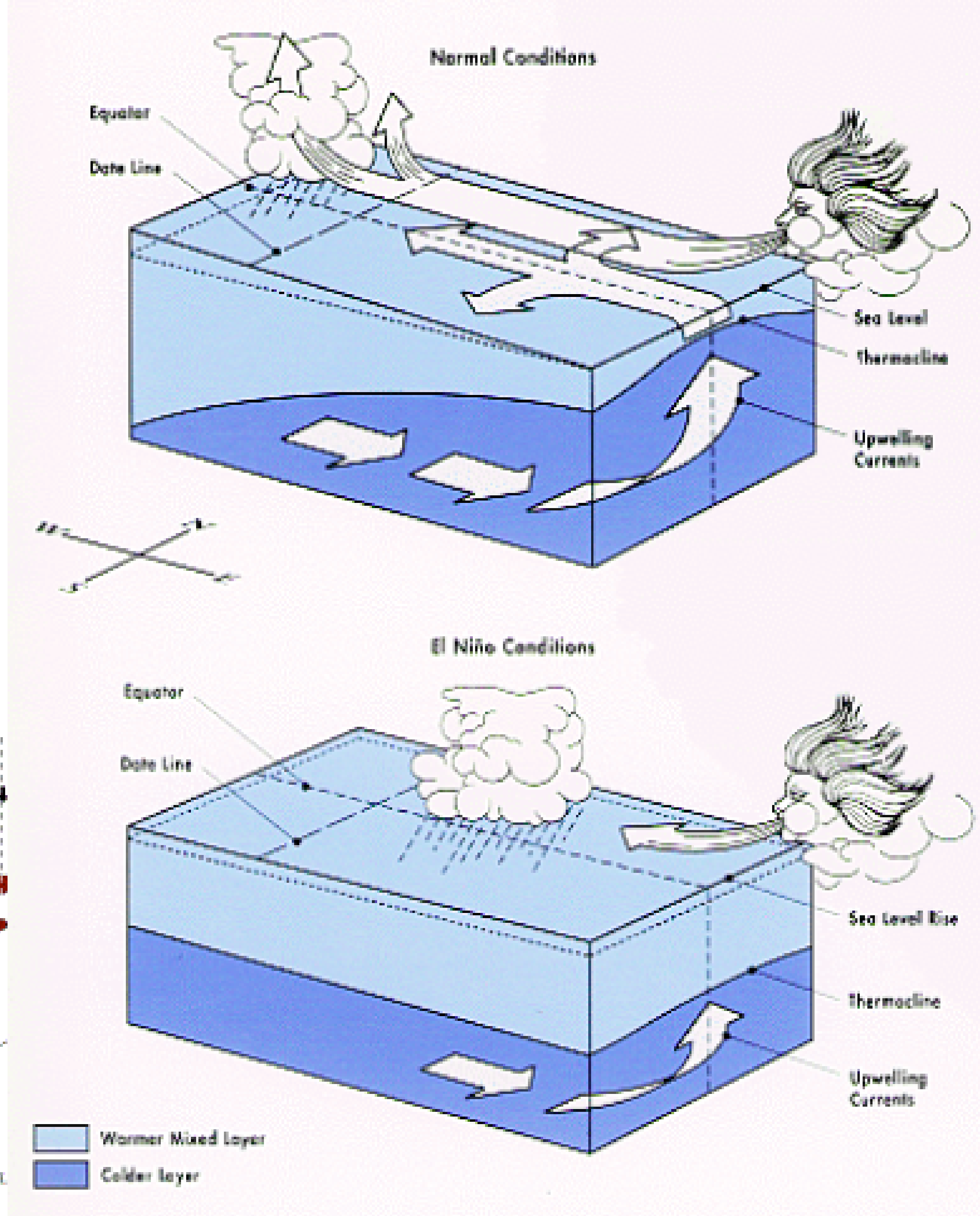
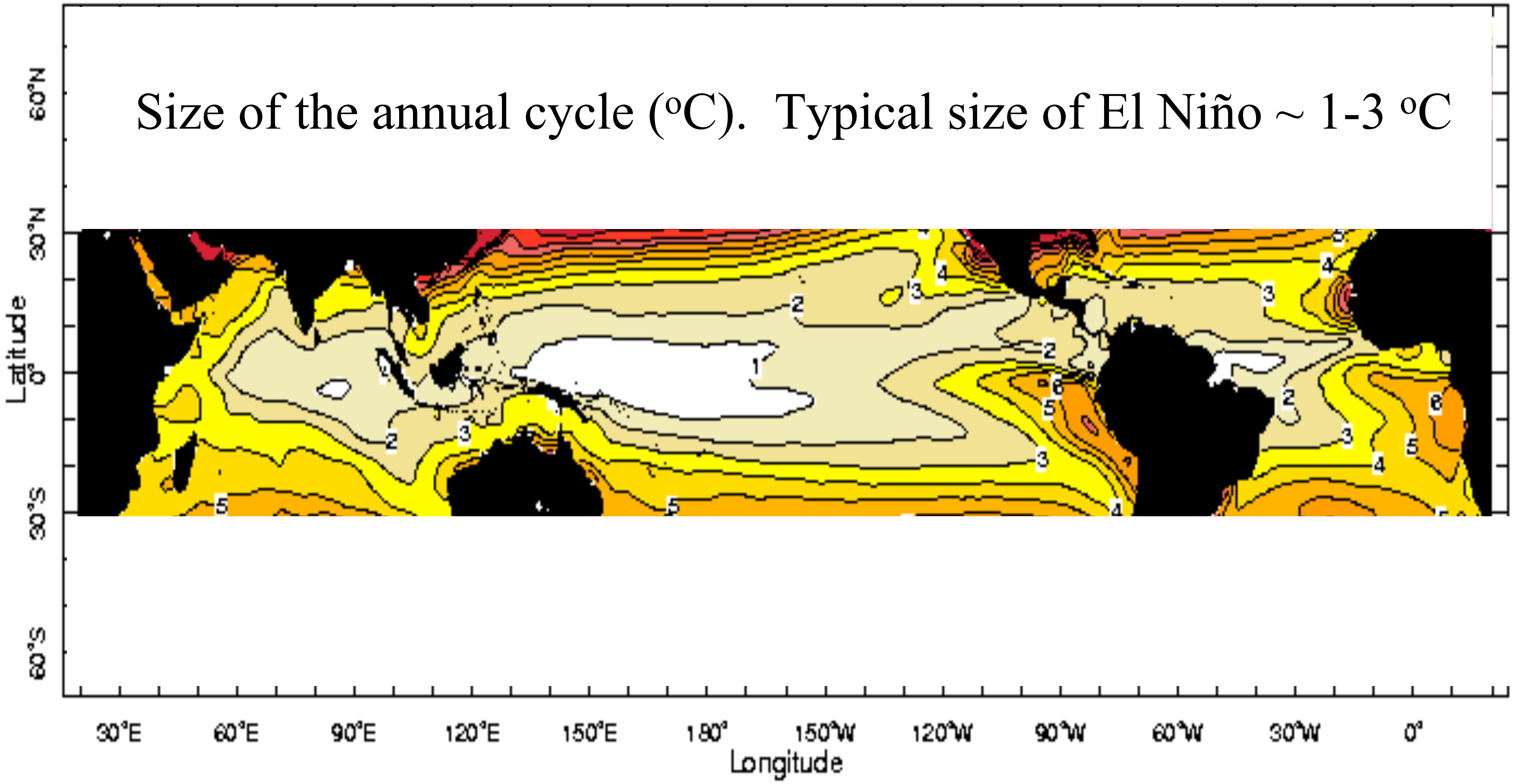

Normal Conditions
Convective Loop
Equator
120°E
80°W
El Niño Conditions
Increased Convection
Equator
120°E
80°W


Normal Conditions
Equator
Date Line
Sea Level
Thermocline
Upwelling Currents
El Niño Conditions
Equator
Date Line
Sea Level Rise
Thermocline
Upwelling Currents
Warmer Mixed Layer
Colder Layer

Size of the annual cycle (^oC). Typical size of El Niño ~ 1-3 ^oC



Linear Inverse Modeling

Assume linear dynamics (dropping the primes):

$$d\boldsymbol{T}/dt = \mathbf{B}\boldsymbol{T} + \xi, \text{ with } <\xi(t+\tau)\,\xi^{T}(t)> = \mathbf{Q}(t)\,\delta(\tau)$$

For now, we'll assume additive noise, although that assumption is false. $\mathbf{Q}(t)$ is periodic.

Corresponding FPE:

$$\frac{\partial p(\boldsymbol{T},t)}{\partial t} = -\sum_{ij}\frac{\partial}{\partial T_i}\left[B_{ij}T_j\,p(\boldsymbol{T},t)\right] + \frac{1}{2}\sum_{ij}\frac{\partial^2}{\partial T_i \partial T_j}\left[Q_{ij}(t)\,p(\boldsymbol{T},t)\right]$$

From the FPE.

$p(\boldsymbol{T}, t+\tau | \boldsymbol{T}_o, t)$ is Gaussian, centered on $\mathsf{G}(\tau)\, \boldsymbol{T}_o$

where $\mathsf{G}(\tau) = \exp(\mathsf{B}\tau) = <\boldsymbol{T}(t+\tau)\boldsymbol{T}^{\mathrm{T}}(t)><\boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t)>^{-1}$.

The covariance matrix of the predictions:

$$\boldsymbol{\Sigma}(t,\tau) = <\boldsymbol{T}(t+\tau)\boldsymbol{T}^{\mathrm{T}}(t+\tau)> - \mathsf{G}(\tau) <\boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t)> \mathsf{G}^{\mathrm{T}}(\tau) .$$

Further,

$$\frac{\partial}{\partial t} <\boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t)> = \mathsf{B} <\boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t)> + <\boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t)> \mathsf{B}^{\mathrm{T}} + \mathsf{Q}(t)$$

Digression : *The disturbing assumption of additive noise*.

Instead of $d\boldsymbol{T}/dt = \mathbf{B}\boldsymbol{T} + \xi$, the system is actually of the form

$$d\boldsymbol{T}/dt = \mathbf{B}\boldsymbol{T} + (\mathbf{A}\boldsymbol{T} + \mathbf{C})\xi_1 + \mathbf{D}\xi_2.$$

All of the LIM formalism follows through, with the identification

$$\mathbf{B} \rightarrow \mathbf{B} + \mathbf{A}^2/2; \quad \mathbf{Q} \rightarrow \langle(\mathbf{A}\boldsymbol{T} + \mathbf{C})(\mathbf{A}\boldsymbol{T} + \mathbf{C})^T\rangle + \mathbf{D}\mathbf{D}^T$$

$$\mathbf{G}(\tau) \rightarrow exp\{(\mathbf{B} + \mathbf{A}^2/2)\tau\}$$

Note: $p(\boldsymbol{T}, t + \tau | \boldsymbol{T}_o, t)$ is no longer Gaussian, but $\mathbf{G}(\tau)\ \boldsymbol{T}_o$ is still the best prediction in the mean square sense.

Eigenvectors of $\mathbf{G}(\tau)$ are the ***"normal" modes*** $\{\mathbf{u}_i\}$.

Eigenvectors of $\mathbf{G}^{\mathrm{T}}(\tau)$ are the ***"adjoints"*** $\{\mathbf{v}_i\}$,

(Recall: $\mathbf{G}(\tau) = \langle \boldsymbol{T}(t+\tau)\boldsymbol{T}^{\mathrm{T}}(t) \rangle \langle \boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t) \rangle^{-1}$)

and $\mathbf{u}\mathbf{v}^{\mathrm{T}} = \mathbf{u}^{\mathrm{T}}\mathbf{v} = 1$.

Most probable prediction: $\boldsymbol{T}(t+\tau) = \mathbf{G}(\tau)\, \boldsymbol{T}_o(t)$

The neat thing: $\mathbf{G}(\tau) = \{\mathbf{G}(\tau_o)\}^{\tau/\tau_o}$.

If LIM's assumptions are valid, the prediction error $\boldsymbol{\varepsilon} = \boldsymbol{T}(t+\tau) - \mathbf{G}(\tau)\ \boldsymbol{T}_o$ does not depend on the lag at which the covariance matrices are evaluated. This is true for El Niño; it is *not* true for the chaotic Lorenz system.

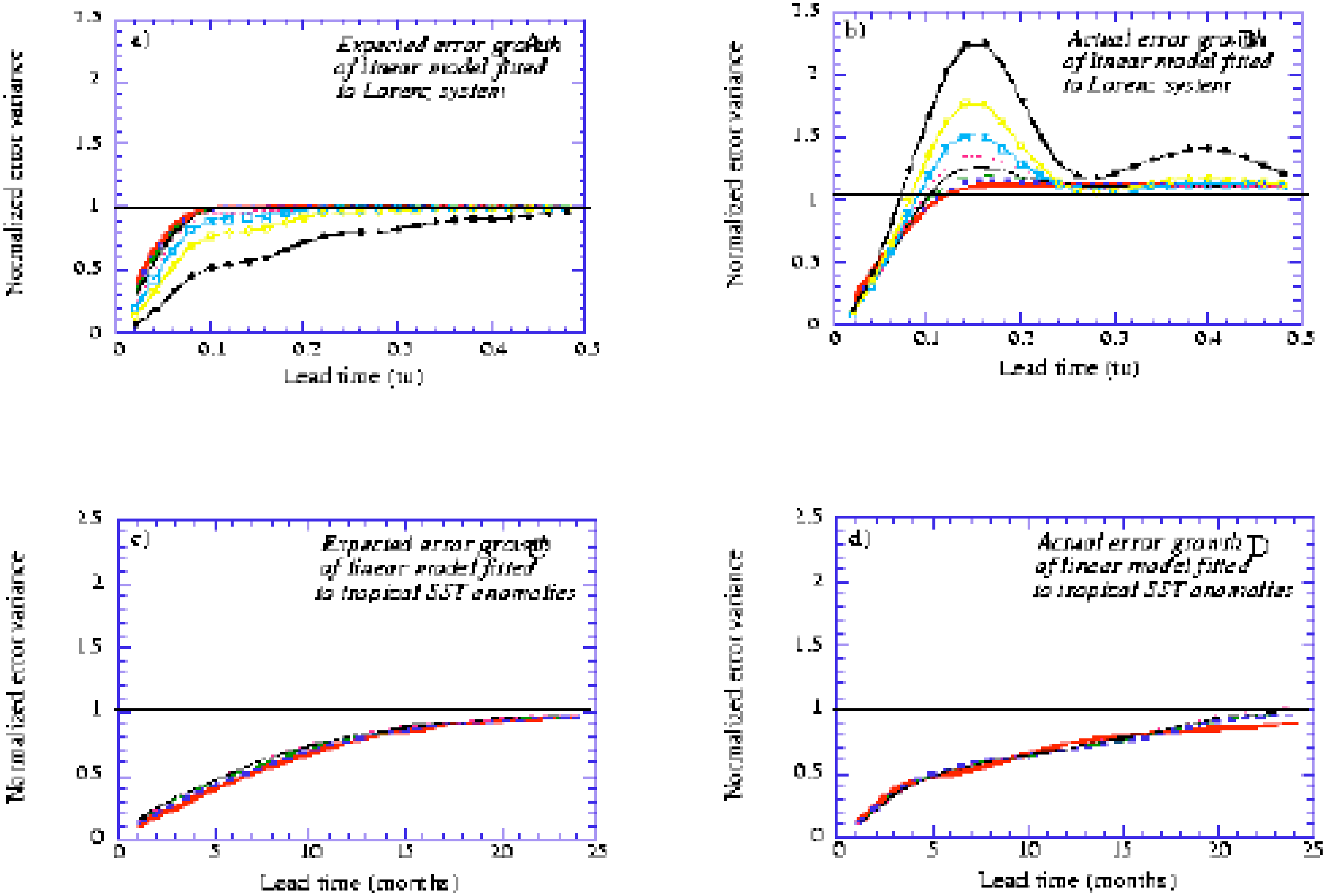
SST Data used:

- COADS (1950-2000) SSTs in 30E-70W, 30N – 30S, or in the global tropical strip, consolidated onto a 4x10-degree grid.

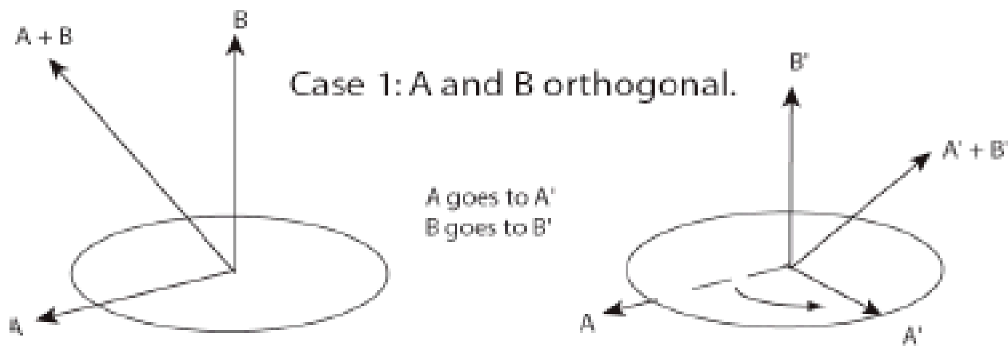
- Subjected to 3-month running mean.

- Projected data onto an orthogonal basis to reduce dimensionality.

Below, different colors correspond to different lags used to identify the parameters. What is plotted: *Tr*(ε) *vs* lead.



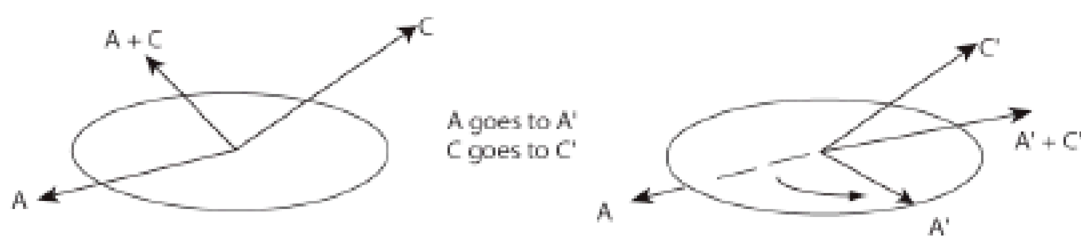
When a system is linear, we can completely identify it from data (***Linear Inverse Modeling***). When it is multi-dimensional, its size can temporarily *grow*, even though all of its components are *decaying*. This happens if the components are almost never orthogonal.



(A' + B') is shorter than (A + B)

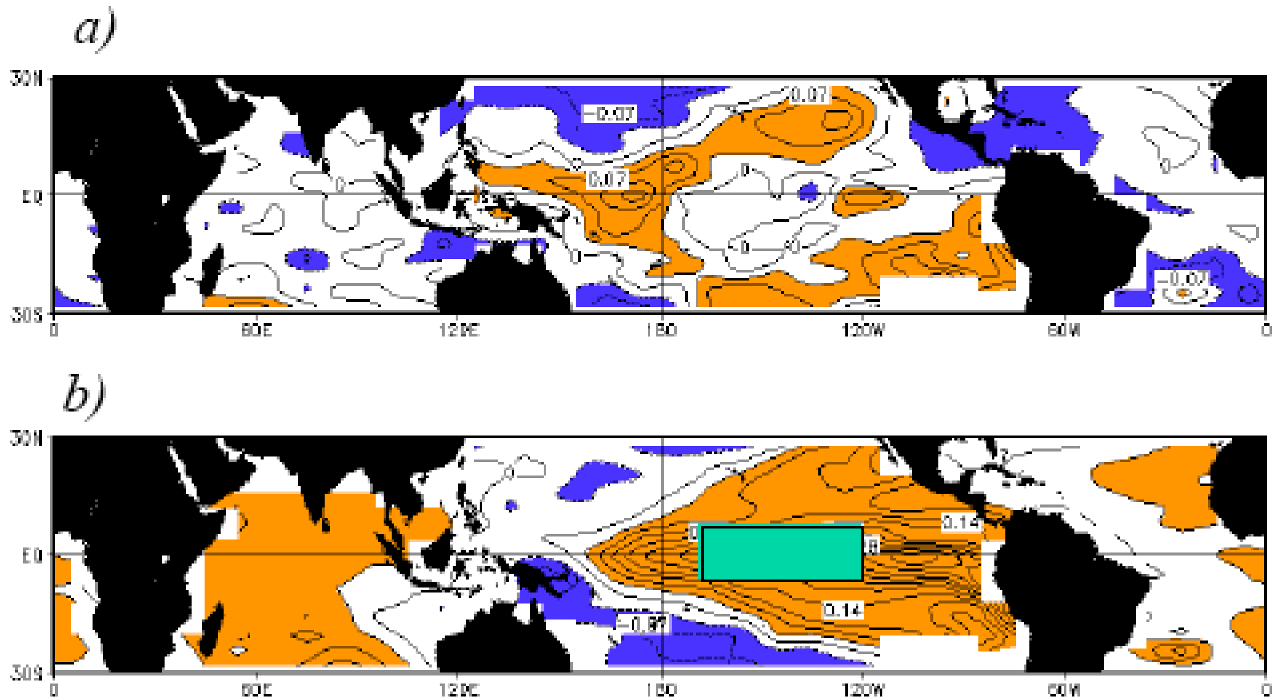
B and C are the same length.
B' and C' are the same length.

Case 2: A and C non-orthogonal.

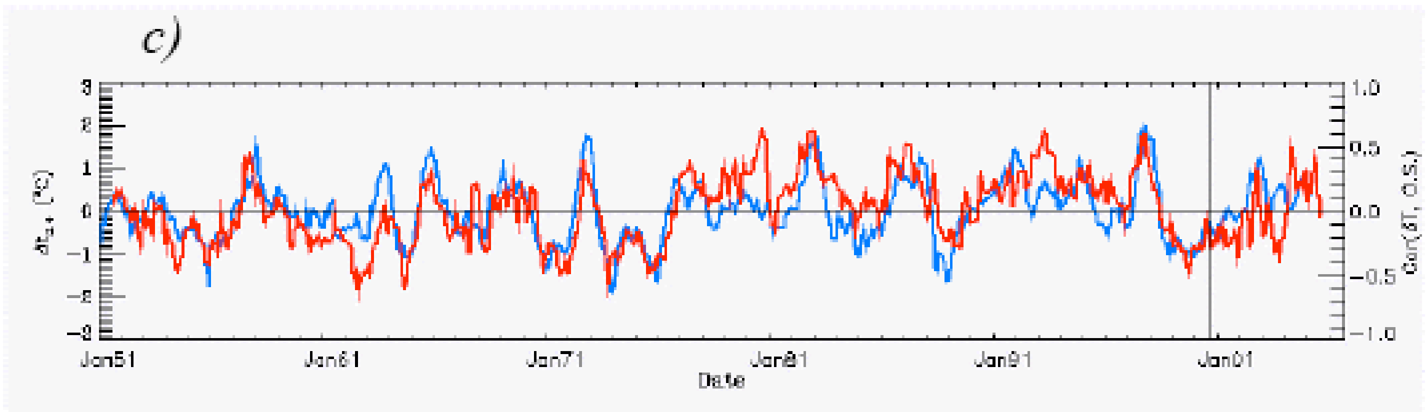


(A' + C') is longer than (A + C)

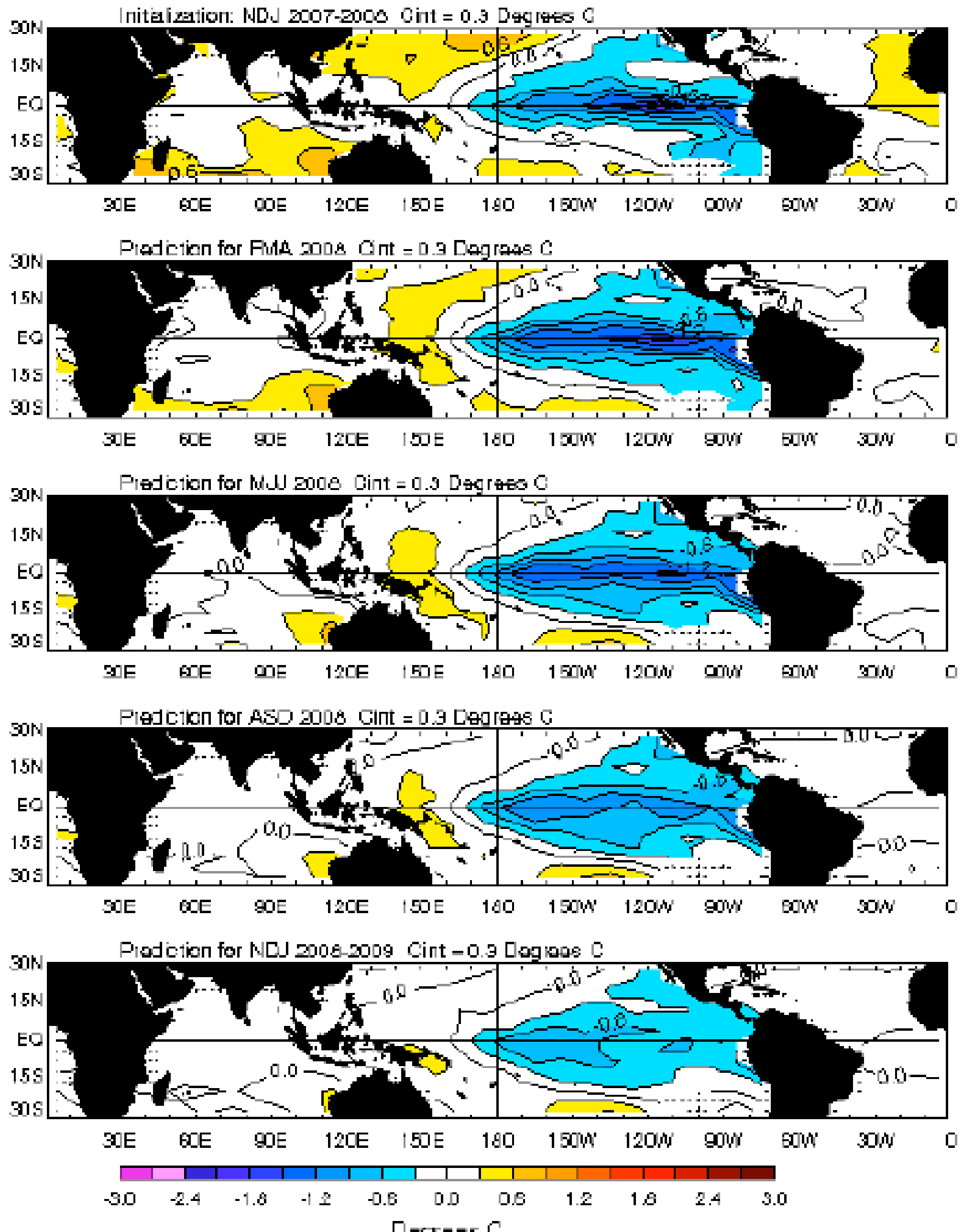
The transient growth possible in a multidimensional linear system occurs when an El Niño develops. ***LIM*** predicts that an *optimal pattern* (*a*) precedes a mature El Niño pattern (*b*) by about 8 months



Temp. Anom. in Niño 3.4 region (6°N-6°S, 170°W-120°W): $\delta T_{3.4}$



Does it? Judge for yourself! The red line is the time series of pattern correlations between pattern (*a*) and the sea surface temperature pattern 8 months earlier. The blue line is a time series index of how strong pattern (*b*) is at the date shown; the blue line is an index of El Niño when it is positive and of La Niña when it is negative.



NOAA/ESRL PSD and CIRES/CDC Experimental Forecast

Several sources of expected error and uncertainty:

- Stochastic forcing:

$$\boldsymbol{\Sigma}(t,\tau) = <\boldsymbol{T}(t+\tau)\boldsymbol{T}^{\mathrm{T}}(t+\tau)> - \mathbf{G}(\tau) < \boldsymbol{T}(t)\boldsymbol{T}^{\mathrm{T}}(t) > \mathbf{G}^{\mathrm{T}}(\tau)$$

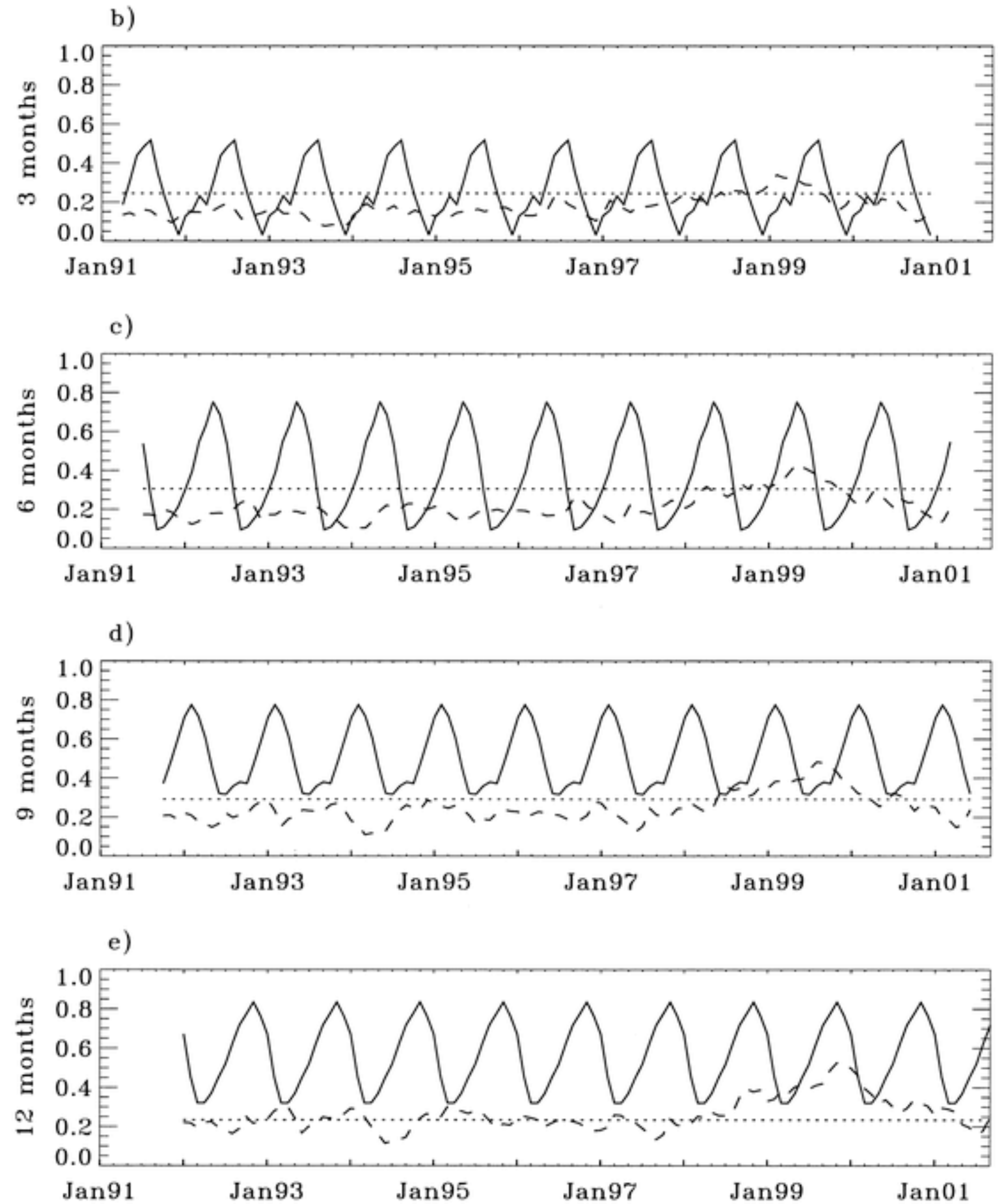
- Uncertain initial conditions:

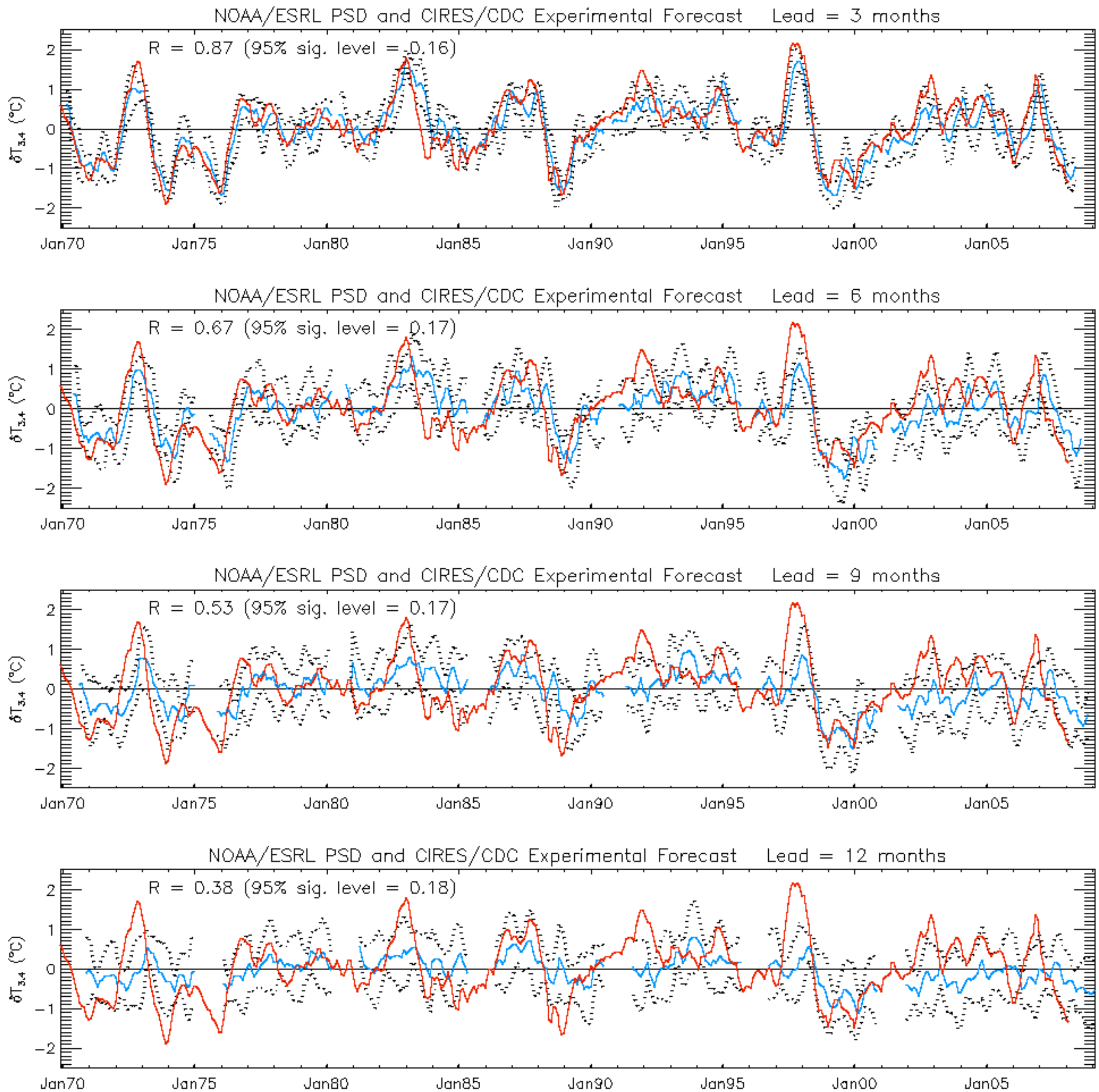
$$<\delta\boldsymbol{T}(t+\tau)\delta\boldsymbol{T}^{\mathrm{T}}(t+\tau)>_{i.c.} = \mathbf{G}(\tau) <\delta\boldsymbol{T}(t)\delta\boldsymbol{T}^{\mathrm{T}}(t)> \mathbf{G}^{\mathrm{T}}(\tau)$$

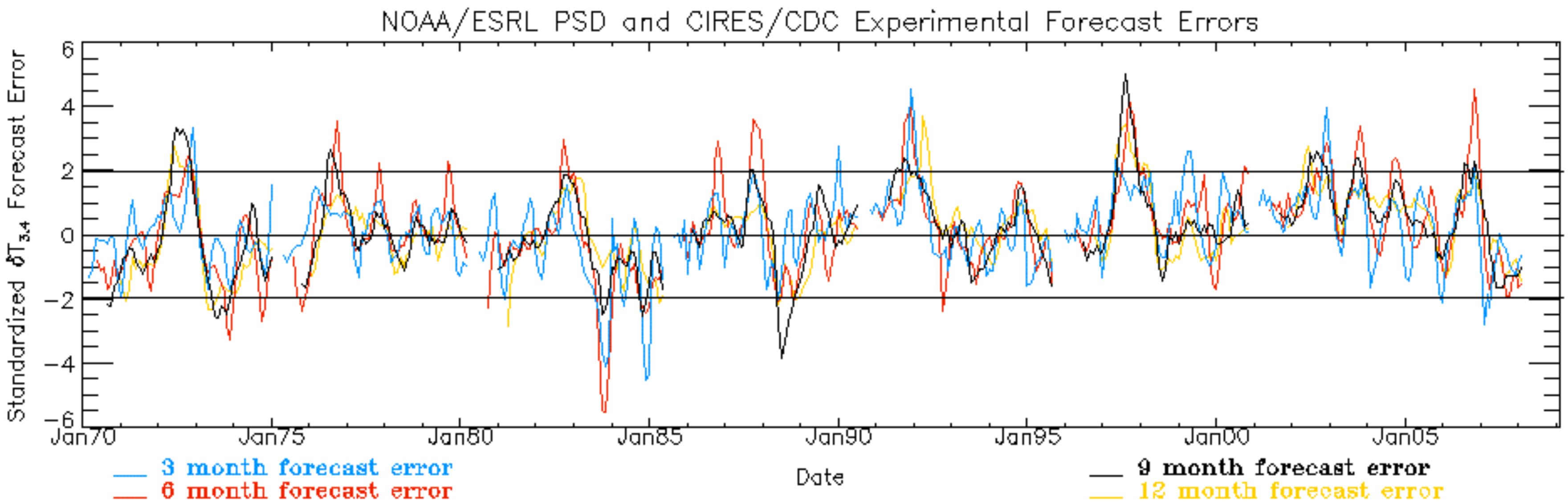
- Sampling errors when estimating $\mathbf{G}(\tau)$:

$$<\delta\boldsymbol{T}(t+\tau)\delta\boldsymbol{T}^{\mathrm{T}}(t+\tau)|\boldsymbol{T}(t)>_{ij,Samp} = \sum_{km} < \delta G_{ik}\delta G_{jm} > T_k(t)T_m(t)$$

Expected error in Niño 3.4 anomaly forecast




NOAA/ESRL PSD and CIRES/CDC Experimental Forecast Lead = 3 months
R = 0.87 (95% sig. level = 0.16)
NOAA/ESRL PSD and CIRES/CDC Experimental Forecast Lead = 6 months
R = 0.67 (95% sig. level = 0.17)
NOAA/ESRL PSD and CIRES/CDC Experimental Forecast Lead = 9 months
R = 0.53 (95% sig. level = 0.17)
NOAA/ESRL PSD and CIRES/CDC Experimental Forecast Lead = 12 months
R = 0.38 (95% sig. level = 0.18)
δT3.4 (°C)
2
1
0
−1
−2
Jan70
Jan75
Jan80
Jan85
Jan90
Jan95
Jan00
Jan05


NOAA/ESRL PSD and CIRES/CDC Experimental Forecast Errors
Standardized $\delta T_{3.4}$ Forecast Error
6
4
2
0
−2
−4
−6
Jan70
Jan75
Jan80
Jan85
Jan90
Jan95
Jan00
Jan05
Date
3 month forecast error
6 month forecast error
9 month forecast error
12 month forecast error

Conclusions

- Expected and actual errors can be a useful diagnostic tool
- El Niño is mainly a linear process maintained by additive and multiplicative cyclostationary stochastic forcing
- Initial condition errors grow and then decay