(d) Do the same for light rays coming up to the interface through the glass.

45. ▲ A small light fixture on the bottom of a swimming pool is 1.00 m below the surface. The light emerging from the still water forms a circle on the water surface. What is the diameter of this circle?

46. The walls of a prison cell are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in the eastern wall. The light traverses 2.37 m horizontally to shine perpendicularly on the wall opposite the window. A young prisoner observes the patch of light moving across this western wall and for the first time forms his own understanding of the rotation of the Earth. (a) With what speed does the illuminated rectangle move? (b) The prisoner holds a small, square mirror flat against the wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. With what speed does the smaller square of light move across that wall? (c) Seen from a latitude of 40.0° north, the rising Sun moves through the sky along a line making a 50.0° angle with the southeastern horizon. In what direction does the rectangular patch of light on the western wall of the prisoner's cell move? (d) In what direction does the smaller square of light on the eastern wall move?

47. A hiker stands on an isolated mountain peak near sunset and observes a rainbow caused by water droplets in the air at a distance of 8.00 km along her line of sight. The valley is 2.00 km below the mountain peak and entirely flat. What fraction of the complete circular arc of the rainbow is visible to the hiker? (See Fig. 35.24.)

48. Figure P35.48 shows a top view of a square enclosure. The inner surfaces are plane mirrors. A ray of light enters a small hole in the center of one mirror. (a) At what angle θ must the ray enter if it exits through the hole after being reflected once by each of the other three mirrors? (b) **What If?** Are there other values of θ for which the ray can exit after multiple reflections? If so, sketch one of the ray's paths.

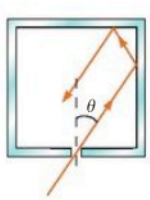

Figure P35.48

49. ▲ A laser beam strikes one end of a slab of material as shown in Figure P35.49. The index of refraction of the slab is 1.48. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.

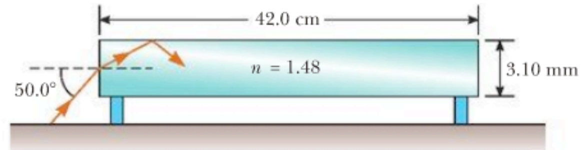

Figure P35.49

50. A 4.00-m-long pole stands vertically in a lake having a depth of 2.00 m. The Sun is 40.0° above the horizontal. Determine the length of the pole's shadow on the bottom of the lake. Take the index of refraction for water to be 1.33.

51. The light beam in Figure P35.51 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

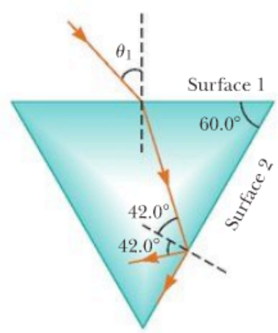

Figure P35.51

52. Builders use a leveling instrument in which the beam from a fixed helium–neon laser reflects in a horizontal plane from a small, flat mirror mounted on a vertical rotating shaft. The light is sufficiently bright and the rotation rate is sufficiently high that the reflected light appears as a horizontal line, wherever it falls on a wall. (a) Assume the mirror is at the center of a circular grain elevator of radius 3.00 m. The mirror spins with constant angular velocity 35.0 rad/s. Find the speed of the spot of laser light on the curved wall. (b) Now assume the spinning mirror is at a perpendicular distance of 3.00 m from point O on a long, flat, vertical wall. When the spot of laser light on the wall is at distance x from point O, what is its speed? (c) What is the minimum value for the speed? What value of x corresponds to it? How does the minimum speed compare with the speed you found in part (a)? (d) What is the maximum speed of the spot on the flat wall? (e) In what time interval does the spot change from its minimum to its maximum speed?

53. ▲ ● A light ray of wavelength 589 nm is incident at an angle θ on the top surface of a block of polystyrene as shown in Figure P35.53. (a) Find the maximum value of θ for which the refracted ray undergoes total internal reflection at the left vertical face of the block. **What If?** Repeat the calculation for the case in which the polystyrene block is immersed in (b) water and (c) carbon disulfide. You will need to explain your answers.

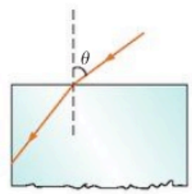

Figure P35.53

54. ● As sunlight enters the Earth's atmosphere, it changes direction due to the small difference between the speeds of light in vacuum and in air. The duration of an *optical* day is defined as the time interval between the instant when the top of the rising Sun is just visible above the horizon and the instant when the top of the Sun just disappears below the horizontal plane. The duration of the *geometric* day is defined as the time interval between the instant when a mathematically straight line between an observer and the top of the Sun just clears the horizon and the instant at which this line just dips below the horizon. (a) Explain which is longer, an optical day or a geometric day. (b) Find the difference between these two time intervals. Model the Earth's atmosphere as uniform, with index of refraction 1.000 293, a sharply defined upper surface, and depth 8 614 m. Assume the observer is at the Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon.

55. A shallow glass dish is 4.00 cm wide at the bottom as shown in Figure P35.55. When an observer's eye is located as shown, the observer sees the edge of the bottom of the empty dish. When this dish is filled with water, the observer sees the center of the bottom of the dish. Find the height of the dish.

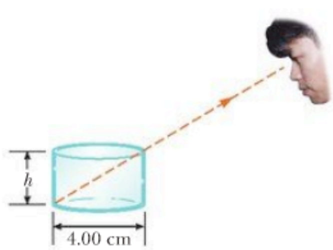

Figure P35.55

56. A ray of light passes from air into water. For its deviation angle $\delta = |\theta_1 - \theta_2|$ to be 10.0°, what must its angle of incidence be?

57. A material having an index of refraction n is surrounded by a vacuum and is in the shape of a quarter circle of radius R (Fig. P35.57). A light ray parallel to the base of the material is incident from the left at a distance L above the base and emerges from the material at the angle θ. Determine an expression for θ.

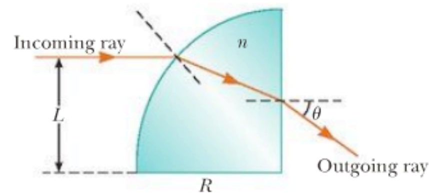

Figure P35.57

58. *Fermat's principle.* Pierre de Fermat (1601–1665) showed that whenever light travels from one point to another, its actual path is the path that requires the smallest time interval. The simplest example is for light propagating in a homogeneous medium. It moves in a straight line because a straight line is the shortest distance between two points. Derive Snell's law of refraction from Fermat's principle. Proceed as follows. In Figure P35.58, a light ray travels from point P in medium 1 to point Q in medium 2. The two points are respectively at perpendicular distances a and b from the interface. The displacement from P to Q has the component d parallel to the interface, and we let x represent the coordinate of the point where the ray enters the second medium. Let $t = 0$ be the instant at which the light starts from P. (a) Show that the time at which the light arrives at Q is

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2 + x^2}}{c} + \frac{n_2\sqrt{b^2 + (d - x)^2}}{c}$$

(b) To obtain the value of x for which t has its minimum value, differentiate t with respect to x and set the derivative equal to zero. Show that the result implies

$$\frac{n_1 x}{\sqrt{a^2 + x^2}} = \frac{n_2(d - x)}{\sqrt{b^2 + (d - x)^2}}$$

(c) Show that this expression in turn gives Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

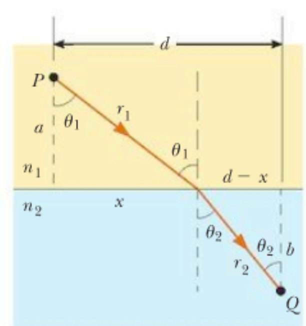

Figure P35.58