Unparticle Physics

Yi Liao 廖益
Nankai Univ 南开大学

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1. *Introduction and mini-review*
2. *Bounds on unparticle couplings to electron*
3. *Fermionic unparticle*
4. *Thermodynamics of unparticles*
5. *First attempt at gauging unparticles*

Talk at KITPC on Sep 25th, 2008: 'New Phys Beyond Standard Model'

1. Introduction and mini-review

1.1 Original idea due to Georgi

hep-ph/0703260, 0704.2457[hep-ph]

- **Some very high energy theory contains SM fields and the fields from a scale invariant (SI) sector**

- **The two interact via heavy particles of mass $M_{\mathcal{U}}$**

- **Effective interactions below $M_{\mathcal{U}}$:** $\sim$ (SM operators) $\times$ (SI operators)

- **Dimensional transmutation occurs at $\Lambda_{\mathcal{U}}$ in SI sector:**
(SI operators) $\rightarrow \mathcal{U}$: **unparticle fields as new degrees of freedom**

- **Effective int. below $\Lambda_{\mathcal{U}}$:** $\sim$ (SM operators) $\times \mathcal{U}$

1.2 Basics on unparticles

- **No dispersion relation for unparticle, $\mathcal{U}$**
- **Kinematical behavior largely fixed by scaling dim., d, of $\mathcal{U}$ field**

(1) scale invar. $\rightarrow$ state density of bosonic unparticle of momen. p:
$\propto \theta(p_0)\theta(p^2)(p^2)^{d-2}$

Reminiscent of state density of a system of d massless particles, suggesting

$$\text{normalization factor } A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d+\frac{1}{2})}{\Gamma(d-1)\Gamma(2d)}$$

No known restrictions on d but from analysis of unitary rep in conformal theory; e.g., $d \geq 1$ for scalar $\mathcal{U}$

(2) Via unitarity, state density implies the propagator:

$$\frac{A_d}{2\sin(\pi d)} \frac{i}{(-p^2 - i\epsilon)^{2-d}}$$

- **Comments:**
  - **Extendable to fields with spin, both tensors and spinors**
  - **Theoretical considerations prefer $1 < d < 2$ for scalar $\mathcal{U}$**
  - **Enough info for pheno. analysis in effective field theory**

1.3 Status

195 papers as of Sep 24, 08 quoted Georgi's 1st paper in Mar 07

- **Collider phys**
- **Low energy effects (FCNC, CP, neutrinos, precision tests, etc)**
- **Astrophys and cosmology**
- **Theoretically oriented approaches including various topics, e.g.:**

understanding $\mathcal{U}$ via continuum mass, deconstruction, relation to hidden sectors, CFT, breaking of scale invar at low energies, etc

- **Interplay with electroweak SSB and Higgs**
- **5th force**
- **Supersymmetric $\mathcal{U}$**
- **Thermodynamics**
- **Fermionic $\mathcal{U}$**
- **Gauged $\mathcal{U}$**

This talk is based on:

Liao: 0705.0837; Liao, Liu: 0706.1284; Liao: 0708.3327; Chen, He, Hu, Liao: 0710.5129; Liao: 0804.0752; Liao: 0804.4033

2. Bounds on unparticle couplings to electron

2.1 Why unparticle-electron interactions?

Want to say something certain about relevance of unparticle phys

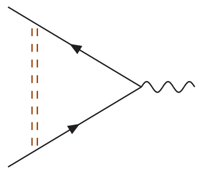
Theoretically, leptons as the cleanest fundamental particles connect directly to theory
Experimentally, electron systems are most precisely tested

$\Rightarrow$ Aims: (1) precision QED; (2) long-ranged forces

Language: effective field theory with interactions:

$$\begin{aligned}
\mathcal{L}_{\textbf{int}} &= C_S\overline{\psi}\psi\mathcal{U}_S + C_P\overline{\psi}i\gamma_5\psi\mathcal{U}_P + C_V\overline{\psi}\gamma_\mu\psi\mathcal{U}_V^\mu + C_A\overline{\psi}\gamma_\mu\gamma_5\psi\mathcal{U}_A^\mu \\
\psi &: \text{electron field} \\
\mathcal{U}_{S,P,V,P} &: \text{unparticle fields of scaling dim } d \\
C_{S,P,V,A} &= \pm\Lambda_{S,P,V,A}^{1-d} \text{ couplings and effective energy scales}
\end{aligned}$$

2.2 Electron $g-2$



$\mathcal{U}_{S,P,V,A}$ **contribute to** $(g-2) = 2a$ **as follows (**$m =$ **electron mass):**

$$a_S = -\frac{A_d}{2\sin(\pi d)}\frac{(C_S m^{d-1})^2}{8\pi^2}\frac{3\Gamma(2d-1)\Gamma(2-d)}{\Gamma(2+d)}$$

also in 0704.3532

$$a_P = +\frac{A_d}{2\sin(\pi d)}\frac{(C_P m^{d-1})^2}{8\pi^2}\frac{\Gamma(2-d)\Gamma(2d)}{\Gamma(2+d)}$$

$$a_V = -\frac{A_d}{2\sin(\pi d)}\frac{(C_V m^{d-1})^2}{4\pi^2}\frac{\Gamma(3-d)\Gamma(2d-1)}{\Gamma(d+2)}$$

also in 0704.2588

$$a_A = +\frac{A_d}{2\sin(\pi d)}\frac{(C_A m^{d-1})^2}{\pi^2}\frac{\Gamma(2d-2)\Gamma(3-d)}{\Gamma(2+d)}$$

deviation between SM and expt $|\delta a| < 15 \times 10^{-12}$ PRL97

$$\Rightarrow \quad \Lambda_S > 110 \text{ TeV}, \quad \Lambda_P > 73 \text{ TeV}, \quad \Lambda_V > 37 \text{ TeV}, \quad \Lambda_A > 146 \text{ TeV} \quad \text{for } d = 1.5$$

Comments

- **Bounds are weakened as d increases.**

- **Partial cancellation occurs but not more can be gained by treating $\mathcal{U}_{S,P,V,A}$ together since d's are generally different.**

2.3 Exotic positronium decays

Positronium: EM bound state of e^-e^+, with orbital ℓ and spin s, have $C = (-1)^{\ell+s}, \ P = (-1)^{\ell+1}$

Spin-1: ortho-positronium (o-Ps); $C = P = -1$ **for ground-state**

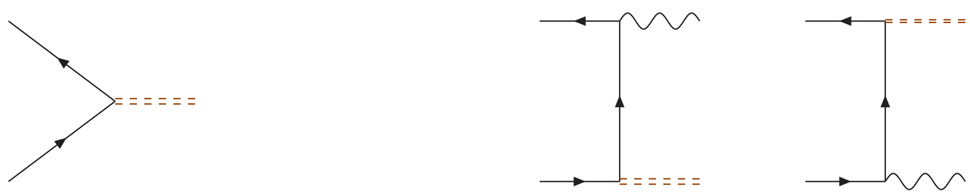
Spin-0: para-positronium (p-Ps); $-C = P = -1$ **for ground-state**

Dominant decays by C and P conservation:

$$\text{o-Ps} \quad \rightarrow \quad \gamma\gamma\gamma \ \text{–good place to discover new phys}$$
$$\text{p-Ps} \quad \rightarrow \quad \gamma\gamma$$

$\mathcal{L}_{\rm int}$ also conserves C and P, thus dominant new modes are

$$\text{invisible decays} \quad : \quad \text{o-Ps} \rightarrow \mathcal{U}_V;$$
$$\text{p-Ps} \rightarrow \mathcal{U}_P$$
$$\text{exotic decays} \quad : \quad \text{o-Ps} \rightarrow \gamma\mathcal{U}_S, \ \gamma\mathcal{U}_P, \ \gamma\mathcal{U}_A;$$
$$\text{p-Ps} \rightarrow \gamma\mathcal{U}_V \ \text{–too small}$$



Results:

$$\mathbf{Br}(\mathbf{o\text{-}Ps} \to \mathcal{U}_V) = \frac{3 \cdot 2^{2d-6}}{(\pi^2 - 9)\alpha^3} A_d \left(\frac{C_V}{m^{1-d}} \right)^2$$

$$\mathbf{Br}(\mathbf{p\text{-}Ps} \to \mathcal{U}_P) = \frac{2^{2d-5}}{\pi\alpha^2} A_d \left(\frac{C_P}{m^{1-d}} \right)^2$$

$$\mathbf{Br}\left(\mathbf{o\text{-}Ps} \to \gamma\mathcal{U}_{S,P,A}\right) = \frac{\Gamma\left(d+\frac{1}{2}\right)}{\Gamma(2d)\Gamma(d+1)} \frac{3}{4(\pi^2-9)\alpha^2} \pi^{\frac{3}{2}-2d} \left(\frac{C_{S,P,A}}{m^{1-d}} \right)^2$$

Data:

$$\mathbf{Br}(\mathbf{o\text{-}Ps} \to \mathbf{invisible}) \leq 4.2 \times 10^{-7}\ (90\%\mathbf{C.L}) \quad \text{PRD75}$$

$$\mathbf{Br}(\mathbf{p\text{-}Ps} \to \mathbf{invisible}) \leq 4.3 \times 10^{-7}\ (90\%\mathbf{C.L}) \quad \text{PRD75}$$

$$\mathbf{Br}(\mathbf{o\text{-}Ps} \to \gamma X^0) \leq 1.1 \times 10^{-6}\ (90\%\mathbf{C.L}) \quad \text{PRL66}$$

Bounds at $d = 1.5$:

$$\mathbf{invisible} : \Lambda_V \geq 4.3 \times 10^5\ \mathbf{TeV}, \quad \Lambda_P \geq 5.6 \times 10^2\ \mathbf{TeV}$$

$$\mathbf{exotic} : \Lambda_{S,P,A} \geq 5.1 \times 10^2\ \mathbf{TeV}$$

Comments

• $1 \to 1$ **transition impossible between particles of different masses due to kinematics**
But 1 particle $\to$ 1 unparticle possible, and no resonance expected

$\Rightarrow$ **very strong bound from invisible $\mathrm{o\text{-}Ps}$ decays**

• **Continuous spectrum of (instead of a monochromatic) photon in exotic 2-body decays.**

• **Bounds more stringent than from $g - 2$**

2.4 Long-range forces between electrons

2.4.1 Motivations

• **Known long-range (macroscopic) forces: EM and gravity**

Interpretation in quantum theory:

- force quanta are massless, i.e., $p^2 = 0$(particles) $\leftrightarrow$ propagator $\propto 1/p^2$ (fields)
- physical space is 3-dim

$$\left.\begin{array}{l} \text{force quanta are massless, i.e., } p^2 = 0 \text{ (particles)} \leftrightarrow \text{propagator} \propto 1/p^2 \text{ (fields)} \\ \text{physical space is 3-dim} \end{array}\right\} \Rightarrow V \propto \frac{1}{r}$$

Due to its fundamental importance, search for extra (5th) long-range forces has been a great tradition in phys

- **No further surprise for forces mediated by particles:** **spin-indept or spin-dept, multiple exchange of quanta, etc** $\Rightarrow$ **Always** $V \propto r^{-n}$ **for integral** n **at macroscopic** r

- **Unparticles can be different because of strange-looking propagator or lack of dispersion relation.** $\Rightarrow$ **Different** V **expected at macroscopic** r**.**

- **Microscopic particle-unparticle interactions must be feeble enough**

to evade particle phys detection.
Interactions due to unparticles can be amplified between macroscopic bodies if they are long-ranged like gravity.

$\Rightarrow$ Stronger constraints on particle-unparticle interaction may result!

- **Why spin-spin interactions of electron?**

If $(e, \mathcal{U})$ interact, very likely also $(N, \mathcal{U})$ do. The latter is perhaps more important for macroscopic bodies because they contain more N than e.

But $(e, \mathcal{U})$ is cleaner and connected more directly to theory.

$\Rightarrow$ Remove contamination from N!

The only data not involving N are for spin-spin interactions of e.

2.4.2 e^-e^- interaction potential due to $\mathcal{U}$

t-channel exchange of $\mathcal{U}$ yields

$$
\begin{aligned}
U_t^{--}(\mathbf{r}) &= U_{\rm spin}^{--}(\mathbf{r}) + U_{\rm non}^{--}(\mathbf{r}) \\
U_{\rm spin}^{--}(\mathbf{r}) &= \frac{A_d}{4\pi^2}\frac{1}{r^{2d-1}}\left\{-C_A^2\Sigma_s\Gamma(2(d-1)) - C_A^2\Sigma_s\frac{\Gamma(2d)}{4m^2r^2}(2-d)\right. \\
&\quad -C_P^2\frac{1}{4m^2r^2}\frac{\Gamma(2d)}{2(d-1)}\left[\Sigma_s-(2d+1)\Sigma_a\right] \\
&\quad \left.+(C_A^2-C_V^2)\frac{1}{4m^2r^2}\frac{\Gamma(2d)}{2(d-1)}\left[(1-2d)\Sigma_s+(2d+1)\Sigma_a\right]\right\} \\
U_{\rm non}^{--}(\mathbf{r}) &= \frac{A_d}{4\pi^2}\frac{1}{r^{2d-1}}\left\{(C_V^2-C_S^2)\Gamma(2(d-1)) + \left[(2-d)C_V^2-(3-d)C_S^2\right]\frac{\Gamma(2d)}{4m^2r^2}\right\}
\end{aligned}
$$

$$\Sigma_s = \sigma_1\cdot\sigma_2, \quad \Sigma_a = \sigma_1\cdot\hat{\mathbf{r}}\sigma_2\cdot\hat{\mathbf{r}}, \quad \hat{\mathbf{r}} = \mathbf{r}/r, \quad m = \textbf{electron mass}$$

Comments

- **Even higher terms not important for $mr \gg 1$.**

- **Standard results for exchange of a massless particle are recovered as $d \to 1$ up to contact terms.**

- **Dominant spin-spin interaction:** $\propto C_A^2 \sigma_1 \cdot \sigma_2 \, r^{1-2d}$ **generally non-integral power law!**

2.4.3 Bounds from macroscopic spin-spin interaction of e^-

- **4 precise yet reliable experiments:**

$*$ **2 with torsion pendulum**

$*$ **2 by induced para-magnetization**

Best data: PRL 71 (1993), Physica B194-196 (1994)

- **Results:**

Largest term $\sim r^{1-2d} C_A^2 \Sigma_s$

$$\left(\frac{\Lambda_A}{\mathrm{TeV}}\right)^{2(d-1)} \geq 3.17 \times 10^4 \, \frac{\Gamma(d-\frac{1}{2})}{(2\pi)^{2d}\Gamma(d)} \left(\frac{10^{16}}{0.1973} \frac{r_0}{\mathrm{cm}}\right)^{2(2-d)}$$

r_0 : **typical distance of two samples**

- **For $1.5 < d < 2$, Λ_A is very stringently bounded.**

- **For $1 < d < 1.5$, practically $C_A \sim 0$: should consider next terms.**

d	$\log_{10}(\Lambda_A/\mathrm{TeV})$	$\log_{10}(\Lambda_P/\mathrm{TeV})$	$\log_{10}(\Lambda_V/\mathrm{TeV})$
1.2	×	×	×
1.3	×	6.44	5.77
1.4	×	0.126	−0.307
1.5	20.3	–	–
1.6	13.7	–	–
1.7	9.04	–	–
1.8	5.53	–	–

Table 1: Bounds on $\Lambda_{A,P,V}$ as a function of d for $r_0 = 25$ cm.
×: scales far in excess of the Planck scale;
–: scales too low to be useful.

Next terms $\sim r^{-1-2d} C_{P,V}^2 m^2 \Sigma_s$ **for** $1 < d < 1.5$

$$\left(\frac{\Lambda_P}{\text{TeV}}\right)^{2(d-1)} \geq 6.07 \times 10^{16}\, \frac{\Gamma(d+\frac{1}{2})}{(2\pi)^{2d}\Gamma(d)} \left(\frac{0.1973}{10^{16}}\frac{\text{cm}}{r_0}\right)^{2(d-1)}$$

$$\left(\frac{\Lambda_V}{\text{TeV}}\right)^{2(d-1)} \geq 1.52 \times 10^{16}\, \frac{\Gamma(d+\frac{1}{2})}{(2\pi)^{2d}\Gamma(d)}(2d-1) \left(\frac{0.1973}{10^{16}}\frac{\text{cm}}{r_0}\right)^{2(d-1)}$$

2.4.4 Bounds from hyperfine splitting of positronium

- **hfs: level splitting between o-Ps and p-Ps. For ground-states:**

$$\textbf{QED}: E(1^3S_1) - E(1^1S_0) = +203.391\,69\ (41\ \textbf{or}\ 16)\ \text{GHz} \quad \textbf{PRL 85(2000), PRL 86 (2001)}$$

$$\textbf{expt}: E(1^3S_1) - E(1^1S_0) = +203.387\,5(16)\ \text{GHz} \quad \textbf{PRL 34 (1975), PR A27 (1983)}$$

- **Potential for e^-e^+, $U^{-+}(\mathbf{r})$. 2 contributions**
  - $t-$**channel exchange of $\mathcal{U}$: obtained from $U_t^{--}(\mathbf{r})$ by $C_V^2 \to -C_V^2$**

- $s-$**channel annihilation to $\mathcal{U}$:**

$$U_s^{-+}(\mathbf{r}) = \frac{A_d}{4\sin(\pi d)} \frac{1}{(-4m^2c^2 - i\epsilon)^{2-d}} \delta^3(\mathbf{r}) \times \left[(3C_V^2 + C_P^2 + C_A^2) + (C_V^2 - C_P^2 - C_A^2)\sigma_- \cdot \sigma_+\right]$$

Higher order in α than $U_t^{-+}(\mathbf{r})$ for $1 < d < 2$, ignored below.

- **Results:**

$$E(1^3S_1) - E(1^1S_0) = -m\alpha^{2d-1}\left(\frac{C_A}{m^{1-d}}\right)^2 \frac{A_d}{2\pi^2}\Gamma(2(d-1))\Gamma(2(2-d))$$

$$\Longrightarrow \quad \Lambda_A \geq 21 \text{ TeV } \textbf{at } d = 1.5$$

Summary of this section

• 1 particle to 1 unparticle transition possible $\Longrightarrow$ invisible decays
Continuous spectrum of 2-body decays containing an unparticle
Strange-looking propagator $\Longrightarrow$ new macroscopic forces

• Global pattern of constraints on $e\, e\, \mathcal{U}$ interactions

• Long-ranged spin-spin forces:
For $1 < d < 1.5$, $\mathcal{U}_A$ cannot couple to ee.
For $1.5 < d < 2$, $\Lambda_A > 10^5$ TeV.

• Invisible positronium decays:
$\Lambda_V > 4.3 \times 10^5$ TeV, $\Lambda_P > 5.6 \times 10^2$ TeV

Exotic positronium decays: $\Lambda_S > 5.1 \times 10^2$ TeV

• $g - 2$ and positronium hfs provide milder bounds

3. Fermionic unparticle

3.1 Motivations

Why bosonic $\mathcal{U}$ first?

- **It is easy to couple them as a SM singlet to particles**
- **Their propagators were known at the very start**

No fundamental reason against fermionic $\mathcal{U}$:

- **QCD analogue: mesons and baryons from quarks and gluons**
- **If we imagine that fermionic $\mathcal{U}$ are charged under SM, they can couple to particles as easily as bosonic ones.**
- **Not more difficult to construct a propagator for them from scale and Lorentz symmetry**

3.2 Basic considerations

- $\mathcal{U}$ **is free when its interactions with particles and itself are ignored**

$\Rightarrow$ **field and propagator is determined up to**

- **unknown d ($\leftarrow$ given by fundamental theory at a high scale)**
- **arbitrary normalization ($\leftarrow$ not affecting phys)**

compared to canonical QFT:

- **mass as a given physical parameter**
- **normalization arbitrary though canonical one popular**

Actually the only thing available for constructing $\mathcal{U}$ and its propagator that is also ***sufficient and necessary*** **is scale and Lorentz symmetry.**

- **It is highly desired that $\mathcal{U}$ and its propagator have a massless fermionic particle limit as $d \to \frac{3}{2}$.**

3.3 Fermionic $\mathcal{U}$ field

Can be built as in the canonical case:

$$\begin{aligned}\mathcal{U}(x) &= \int \frac{d^4p}{(2\pi)^4}\Xi_d(p)\sum_s \left(a_p^s u^s(p)e^{-ip\cdot x} + b_p^{s\dagger} v^s(p)e^{ip\cdot x}\right)\\ \Xi_d(p) &= F(d)\theta(p^0)\theta(p^2)(p^2)^{d-\frac{5}{2}}\end{aligned}$$

- **Scale symmetry incorporated:** $\mathcal{U}(\lambda x) = \lambda^{-d}\mathcal{U}(x)$
dimensions: $[a] = [b] = \frac{1}{2} - d,\ [u] = [v] = \frac{1}{2}$

- $a_p^s,\ b_p^s$ **assumed to satisfy the anti-commutation relations, e.g.,**

$$\Xi_d(p)\Xi_d(q)\{a_p^r, a_q^{s\dagger}\} = \Xi_d(p)(2\pi)^4\delta^4(p-q)\delta^{rs}$$

with normalized single-unparticle states, e.g.,

$$|p, s\rangle = \Xi_d(p)a_p^{s\dagger}|0\rangle$$

$$\langle p,r|q,s\rangle = \Xi_d(p)(2\pi)^4\delta^4(p-q)\delta^{rs}$$

• **The above $\mathcal{U}$ is completely similar to a complex scalar $\mathcal{U}$ in the absence of $u,\ v$, with the correspondence:**
$(d-\frac{1}{2})$ **(fermionic)** $\leftrightarrow d$ **(bosonic)**

suggesting the normalization: $F(d)=A_{d-\frac{1}{2}}\equiv B(d-\frac{1}{2})$
$\Rightarrow$ **Correct canonical limit of all formulas guaranteed with** $F(d)$

Comment: absolute normalization immaterial

3.3.1 Construction of $u(p),\ v(p)$

Point: Although dispersion relation is missing for $\mathcal{U}$, the standard procedure still works by the definition of its Lorentz properties.

• **Physical unparticle has $p^2>0,\ p_0>0$: go to its rest frame**
$\sqrt{p^2}$ **plays the same role of mass as for a canonical fermion**

- **Make a boost to a general p. The wavefunction then satisfies**

$$(\not{p} - \sqrt{p^2})u(p) = 0, \qquad \sum_{\textbf{spin}} u(p)\bar{u}(p) = \not{p} + \sqrt{p^2}, \quad \textbf{etc}$$

K-G eqn is a trivial identity due to the lack of dispersion relation.

3.4 Propagator

Start from definition:

$$\tilde{S}^{\mathcal{U}}_{F\alpha\beta}(x-y) = \langle 0|T\mathcal{U}_\alpha(x)\overline{\mathcal{U}}_\beta(y)|0\rangle$$

where, e.g.,

$$\langle 0|\mathcal{U}_\alpha(x)\overline{\mathcal{U}}_\beta(y)|0\rangle = +i(\not{\partial}^x)_{\alpha\beta}\int\frac{d^4p}{(2\pi)^4}\Xi_d(p)e^{-ip\cdot(x-y)} + \delta_{\alpha\beta}\int\frac{d^4p}{(2\pi)^4}\Xi_d(p)(p^2)^{\frac{1}{2}}e^{-ip\cdot(x-y)}$$

Final Answer after some manipulation

$$\tilde{S}_F^{\mathcal{U}}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} S_F^{\mathcal{U}}(p),$$

$$S_F^{\mathcal{U}}(p) = \frac{iF(d)}{2\sin(d\pi)} \left[\frac{1}{(-p^2-i\epsilon)^{2-d}} - \tan(d\pi) \frac{\not{p}}{(-p^2-i\epsilon)^{\frac{5}{2}-d}} \right]$$

non$-\gamma$ term $\qquad$ γ term

$S_F^{\mathcal{U}}(p)$ **has correct discontinuity across the cut** $p^2 > 0$**:**

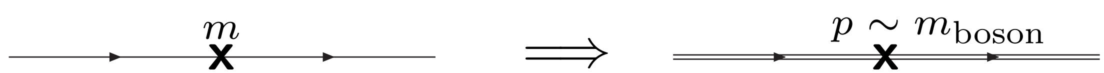
$$F(d)(p^2)^{d-\frac{5}{2}}(\not{p} + \sqrt{p^2})$$

Features

- **Relative phase $\frac{\pi}{2}$ between non-γ and γ terms for $p^2 > 0$**
$\Rightarrow$ **New interference possible beyond the one for bosonic $\mathcal{U}$**

- **Ordinary mass term replaced by p-dependent non-γ term**

⇒ **Can enhance chirality-flipped transitions when $\mathcal{U}$ appears in loop together with a heavy bosonic particle:**



We demonstrate this feature by an example.

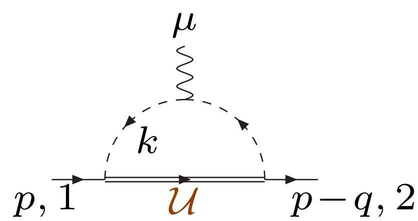
3.5 Chirality-flipped EM transitions due to fermionic $\mathcal{U}$

Consider the effective interaction:

$$\mathcal{L}_{\rm int}^{\mathcal{U}} = \Lambda_{\mathcal{U}}^{\frac{3}{2}-d}\overline{\mathcal{U}}(a_j + b_j\gamma_5)\psi_j\varphi + \Lambda_{\mathcal{U}}^{\frac{3}{2}-d}\bar{\psi}_j(a_j^* - b_j^*\gamma_5)\mathcal{U}\varphi^\dagger \quad (1)$$

- φ, ψ_j: **ordinary particle fields of mass** m_φ **and** m_j; $j = 1, 2$
- $\Lambda_{\mathcal{U}}$ **unparticle scale;** $a_j,\ b_j$ **unknown pure numbers**
- $\mathcal{U}$ **chosen to be neutral:** $Q(\varphi) = -Q(\psi_j) \equiv -Q_j$ **in units of** $e > 0$

3.5.1 Amplitudes for $\psi_1 \to \psi_2\gamma$



On-shell amplitude for $m_\varphi \gg m_{1,2}$:

$$
\begin{aligned}
\mathcal{A}_\mu &= -\frac{1}{(4\pi)^2}\frac{Q_1 e \Lambda_{\mathcal{U}}^{3-2d} F(d)}{2\sin(d\pi)} m_\varphi^{2(d-2)} \mathcal{F}_\mu \\
\mathcal{F}_\mu &= \frac{\tan(d\pi)}{6 m_\varphi}\Gamma(d+1/2)\Gamma(7/2-d) \\
&\quad \times \left[-\bar{m}(a_1 a_2^* + b_1 b_2^*) i\sigma_{\mu\nu}q^\nu + \Delta m (a_2^* b_1 + a_1 b_2^*) i\sigma_{\mu\nu}q^\nu \gamma_5\right] \\
&+ \frac{1}{2}\Gamma(d)\Gamma(3-d)\left[(a_1 a_2^* - b_1 b_2^*) i\sigma_{\mu\nu}q^\nu + (a_2^* b_1 - a_1 b_2^*) i\sigma_{\mu\nu}q^\nu \gamma_5\right]
\end{aligned}
$$

with $\bar{m} = (m_1 + m_2)/2, \;\; \Delta m = (m_1 - m_2)/2$

Non-γ term enhanced by $\frac{m_\varphi}{m_j}$ compared to the ordinary case!

3.5.2 Transition rates

Keeping only the enhanced term and assuming $m_1 \gg m_2$

$$\begin{aligned}
\Gamma(\psi_1 \to \psi_2\gamma) &= 2^{-7} m_1 \alpha \left[f(d) Q_1 \left(\frac{m_\varphi}{\Lambda_{\mathcal{U}}}\right)^{2d-3} \frac{m_1}{m_\varphi}\right]^2 X_{12} \\
X_{ij} &= |a_i a_j^* - b_i b_j^*|^2 + |a_j^* b_i - a_i b_j^*|^2 \\
f(d) &= \frac{1}{2^{2d}\pi^{2d-\frac{3}{2}}\sin(d\pi)} \frac{[\Gamma(d)]^2\Gamma(3-d)}{\Gamma(d-\frac{3}{2})\Gamma(2d-1)}
\end{aligned}$$

3.5.3 EM dipole moments

$$
\begin{aligned}
a_{\psi_1} &= Q_1 \frac{2m_1}{(4\pi)^2} \frac{\Lambda_{\mathcal{U}}^{3-2d} F(d)}{2\sin(d\pi)} m_\varphi^{2(d-2)} \left[\frac{1}{2}\Gamma(d)\Gamma(3-d)(|a_1|^2 - |b_1|^2) \right. \\
&\quad \left. - \tan(d\pi)\frac{m_1}{6m_\varphi}\Gamma\left(d+\frac{1}{2}\right)\Gamma\left(\frac{7}{2}-d\right)(|a_1|^2+|b_1|^2) \right] \\
d_{\psi_1} &= -Q_1 \frac{1}{(4\pi)^2} \frac{e\Lambda_{\mathcal{U}}^{3-2d} F(d)}{2\sin(d\pi)} m_\varphi^{2(d-2)} \Gamma(d)\Gamma(3-d)\Im(a_1^* b_1)
\end{aligned}
$$

Compare to pure particle case assuming $m_\varphi \gg m_\chi, m_\psi$:

$$
\begin{aligned}
& \mathcal{L}_{\rm int}^{\chi} = \bar{\chi}(a+b\gamma_5)\psi\varphi + \bar{\psi}(a^* - b^*\gamma_5)\chi\varphi^\dagger \\
\Rightarrow \quad & a_\psi = -\frac{Q_\psi}{(4\pi)^2}\frac{m_\psi}{m_\varphi^2}\left[m_\chi(|a|^2 - |b|^2) + \frac{1}{3}m_\psi(|a|^2+|b|^2) \right] \\
& d_\psi = \frac{Q_\psi e}{(4\pi)^2}\frac{m_\chi}{m_\varphi^2}\Im(a^* b)
\end{aligned}
$$

3.5.4 Constraints from EM dipole moments

$$
\begin{aligned}
|\delta a_e| &< 15\times 10^{-12}\ [\mathrm{PRL97(2006)}], \quad \delta a_i \equiv a_i^{\mathrm{expt}} - a_i^{\mathrm{SM}} \\
\delta a_\mu &= 22(10)\times 10^{-10}\ [\mathrm{PDG}] \\
d_e &= (0.07\pm 0.07)\times 10^{-26}\ \mathrm{e\ cm}\ [\mathrm{PDG}] \\
d_\mu &= (3.7\pm 3.4)\times 10^{-19}\ \mathrm{e\ cm}\ [\mathrm{PDG}] \\
d_n &< 0.63\times 10^{-25}\ \mathrm{e\ cm}\ [\mathrm{PDG}]
\end{aligned}
$$

d	1.6	1.7	1.8	1.9
$10^3(\lvert a\rvert^2 - \lvert b\rvert^2)_e$	7.3	6.5	6.5	5.4
$10^3(\lvert a\rvert^2 - \lvert b\rvert^2)_\mu$	5.2	4.6	4.6	3.8
$-10^9\Im(a^*b)_e$	8.9	7.8	7.9	6.5
$-\Im(a^*b)_\mu$	4.7	4.1	4.2	3.4
$-10^7\Im(a^*b)_n$	8.0	7.0	7.1	5.9

Table 2: **Bounds for** $\Lambda_{\mathcal{U}} = 1$ TeV, $m_\varphi = 200$ GeV.

3.5.4 Constraints from lepton flavor changing radiative decays

$$\mathbf{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11} \text{ [MEGA]}$$

$$\mathbf{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8} \text{ [Belle]}, \quad \mathbf{Br}(\tau \to e\gamma) < 1.2 \times 10^{-7} \text{ [Belle]}$$

d	1.6	1.7	1.8	1.9
$10^{12} X_{\mu e}$	3.3	2.6	2.7	1.8
$10^{5} X_{\tau e}$	5.3	4.1	4.2	2.9
$10^{5} X_{\tau \mu}$	2.0	1.5	1.6	1.1

Table 3: **Bounds on X_{ij}. Same input parameters as above.**

Comments

- **High quality in $X_{\mu e}$ not only from better precision but also from less power in m_μ.**
- **Bound on X_{bs} from $b \to s\gamma$ less stringent.**

Summary of this section

• **The propagator of fermionic $\mathcal{U}$ is determined by scale and Lorentz symmetry. It has the correct canonical limit.**

• **Two (γ and non-γ) terms appear in propagator.**

• **Relative phase of $\frac{\pi}{2}$ between the two in time-like regime.**
$\Rightarrow$ Expect new interference phenomenon beyond the known one.

• **Non-γ term can effect chirality flip *without* the usual suppression of light fermion mass. Instead, it can enhance the flip by the mass of a virtual boson in loop.**

4. Thermodynamics of unparticles

4.1 Motivations and main results

• **Density of states for $\mathcal{U}$, very different from particles, could result in very different thermodynamics even if no other differences occur.**

• **$\mathcal{U}$ effects have hitherto been studied either at $T = 0$ (few-body systems) or in astrophysical environments with $T \neq 0$.**

Thermodynamics of photons is taken for granted in the latter case.
Is this justified?

We find:
• **Equation of state (EoS) parameter $\omega_{\mathcal{U}} = \frac{1}{2d+1}$ — Some new form of energy between matter ($\omega_{\mathrm{M}} = 0$) and radiation ($\omega_\gamma = \frac{1}{3}$)**

- $\omega_{\mathcal{U}} < \omega_\gamma \Rightarrow \mathcal{U}$, **once produced and decoupled, evolve more slowly than photons in the expansion of universe!**

Possible to obtain significant relic $\mathcal{U}$ in the present even if the universe started with a low $\rho_{\mathcal{U}}$!

4.2 Thermodynamics

Basic idea

Start from bosonic particles of mass μ whose partition function is

$$\ln Z(\mu^2) = -g_s V \int \frac{d^4p}{(2\pi)^4} 2\pi 2p^0 \theta(p^0) \delta(p^2 - \mu^2) \ln(1 - e^{-p^0\beta}), \quad \beta = T^{-1}$$

No such dispersion-relation constraint on $\mathcal{U}$ whose density of states is

$$\propto \frac{d^4p}{(2\pi)^4} 2p^0 \theta(p^0) \theta(p^2) (p^2)^{d-2}$$

It may be interpreted as a continuous collection of particles with the spectral function $\varrho(\mu^2) \propto \theta(\mu^2)(\mu^2)^{d-2}$:

$$\frac{d^4p}{(2\pi)^4} 2\pi 2p^0 \theta(p^0) \delta(p^2 - \mu^2) \varrho(\mu^2) d\mu^2$$

μ^2 serves as a new quantum number to be summed over with weight $\varrho(\mu^2)$!

Normalization of $\varrho(\mu^2)$

$\mathcal{U}$ exists only below $\Lambda_{\mathcal{U}}$:

- **$\varrho(\mu^2)$ must terminate at $\mu^2 = \Lambda_{\mathcal{U}}^2$**
- **Our results apply only to $\beta\Lambda_{\mathcal{U}} > 1$. Beyond certain T, may be resolved – no more suitable degrees of freedom to cope with**

Result: $\varrho(\mu^2) = (d-1)\Lambda_{\mathcal{U}}^{2(1-d)} \theta(\mu^2)(\mu^2)^{d-2}$

Thermodynamics of $\mathcal{U}$

Its partition function is

$$\ln Z = \int_0^{\Lambda_{\mathcal{U}}^2} d\mu^2 \varrho(\mu^2) \ln Z(\mu^2)$$
$$= -\frac{g_s V (d_{\mathcal{U}} - 1)}{4\pi^2 \beta^3 (\beta\Lambda_{\mathcal{U}})^{2(d_{\mathcal{U}}-1)}} \int_0^{(\beta\Lambda_{\mathcal{U}})^2} dy\ y^{d_{\mathcal{U}}-2} \int_y^{\infty} dx\ \sqrt{x-y} \ln(1 - e^{-\sqrt{x}})$$

For $\beta\Lambda_{\mathcal{U}} > 1$,

$$\ln Z = \frac{g_s V}{\beta^3 (\beta\Lambda_{\mathcal{U}})^{2(d-1)}} \frac{\mathcal{C}(d)}{4\pi^2}, \qquad \mathcal{C}(d) = B(3/2, d)\Gamma(2d+2)\zeta(2d+2)$$

Then

$$p_{\mathcal{U}} = \frac{1}{\beta}\frac{\partial}{\partial V} \ln Z = g_s T^4 \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2(d-1)} \frac{\mathcal{C}(d)}{4\pi^2},$$
$$\rho_{\mathcal{U}} = -\frac{1}{V}\frac{\partial}{\partial \beta} \ln Z = (2d+1) g_s T^4 \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2(d-1)} \frac{\mathcal{C}(d)}{4\pi^2}$$
$$\omega_{\mathcal{U}} = \frac{p_{\mathcal{U}}}{\rho_{\mathcal{U}}} = \frac{1}{2d+1}$$

Comments

- **The case of massless particles is recovered as $d \to 1$.**
- **EoS parameter $\omega_{\mathcal{U}}$ lies between $\omega_{\rm M} = 0$ and $\omega_\gamma = \frac{1}{3}$.**
- **Important impacts on cosmology.**

4.3 Cosmological implications

Why $w_{\mathcal{U}}$ important

It determines the evolution of $\rho(T)$ or $\rho(R)$ after decoupling from equilibrium in the expanding universe of Friedmann-Robertson-Walker (FRW),

$$\rho(R) = \rho(R_{\rm D}) \left(\frac{R_{\rm D}}{R}\right)^{3(1+\omega)} \qquad R: \ \textbf{scale factor}, \ _{\rm D}: \ \textbf{decoupling}$$

For photon, $R_{\rm D}/R = T_\gamma/T_{\rm D}$, thus

$$\rho_{\mathcal{U}}(T_\gamma) = \rho_{\mathcal{U}}(T_{\rm D}) \left(\frac{T_\gamma}{T_{\rm D}}\right)^{3(1+\omega_{\mathcal{U}})}, \qquad \rho_\gamma(T_\gamma) = \rho_\gamma(T_{\rm D}) \left(\frac{T_\gamma}{T_{\rm D}}\right)^4$$

Comments:

• If $\mathcal{U}$ always in equilibrium with photon, they share the same T so that

$*$ always $\rho_{\mathcal{U}} < \rho_\gamma$ due to extra power in $T/\Lambda_{\mathcal{U}} < 1$ and

$*$ $\rho_{\mathcal{U}}$ drops faster than ρ_γ as the universe cools down

• If $\mathcal{U}$ decouples at $T_{\rm D}$, it evolves afterwards more slowly than photon — this case more interesting:
We could have a significant $\rho_{\mathcal{U}}(T_\gamma)$ today, albeit $\rho_{\mathcal{U}}(T_{\rm D})$ is very small!

Feasibility reserved for a global fit of cosmological and astrophysical data. We studied a simple scenario with the photon-$\mathcal{U}$ interaction:

$$\mathcal{L} = \frac{\lambda}{\Lambda_{\mathcal{U}}^d} F^{\mu\nu} F_{\mu\nu} \mathcal{U}$$

Summary of results

- **First study on thermodynamics of unparticles and surprising results obtained.**

$\rho_{\mathcal{U}},\ p_{\mathcal{U}}$: **higher powers in T for $\mathcal{U}$ than for photons**
$\omega_{\mathcal{U}} = \frac{1}{2d+1}$ **lies between those of matter and photon: new form of energy in the universe.**

- **Different evolution of $\mathcal{U}$ from photons could result in a significant $\mathcal{U}$ relic today after taking into account strong constraints from star-cooling and BBN, etc.**

- **Actual relevance of $\mathcal{U}$ to cosmology relies more on numerical fitting of data and a survey of interactions.**

5. First attempt at gauging unparticles

Unparticles must interact with particles to be physically relevant.
$\Rightarrow$ It seems natural they are charged under SM gauge group.

Gauging $\mathcal{U}$ was already challenged in Georgi's original paper —
'but I have no idea whether this is possible'

Why non-trivial? Non-integral d or $d > 2 \Longrightarrow$ non-local theory!

Minimal coupling for local fields doesn't work.

First attempt by Cacciapaglia, Marandella, Terning, 0708.0005
with the help of Wilson line and lessons from non-local chiral quark model

Free theory

$$S_0 = \int d^4x\, d^4y\, \mathcal{U}^\dagger(x)\tilde{D}^{-1}(x-y)\mathcal{U}(y),$$

$$\tilde{D}^{-1}: \text{ Fourier transf of } D^{-1}(p) \propto (m^2 - p^2)^{2-d}$$

Gauge theory

$$S = \int d^4x\, d^4y\, \mathcal{U}^\dagger(x)\tilde{D}^{-1}(x-y) P \exp\left[-igT^a \int_x^y A_\mu^a\, dw^\mu\right]\mathcal{U}(y),$$

$$P: \text{ path ordering in both } A_\mu^a \text{ and } T^a$$

Feynman rules

Complicated but systematically manageable Terning 1991, Holdom 1992

Examples:

$$A\mathcal{U}\mathcal{U}^\dagger \qquad ig\Gamma^{a\mu}(-(p+q), p; q) = igT^a(2p+q)^\mu E_1(p;q)$$

$$
\begin{aligned}
\mathcal{AAUU}^\dagger \qquad & ig^2\Gamma^{ab\mu\nu}(-(p+q_1+q_2),p;q_1,q_2) \\
= \quad & ig^2\Big\{\{T^a,T^b\}g^{\mu\nu}E_1(p;q_1+q_2) \\
+ \quad & T^aT^b(2p+q_2)^\nu(2p+2q_2+q_1)^\mu E_2(p;q_2,q_1) \\
+ \quad & T^bT^a(2p+q_1)^\mu(2p+2q_1+q_2)^\nu E_2(p;q_1,q_2)\Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
E_0(p) &= D^{-1}(p), \\
E_1(p;q_1) &= \frac{E_0(p+q_1)-E_0(p)}{(p+q_1)^2-p^2}, \\
E_2(p;q_1,q_2) &= \frac{E_1(p;q_1+q_2)-E_1(p;q_1)}{(p+q_1+q_2)^2-(p+q_1)^2}.
\end{aligned}
$$

Particle case arises as a limit:

$d \to 1 \Rightarrow D(p) \to i(p^2-i\epsilon)^{-1},\ E_1 \to 1,\ E_2 \to 0$

But there are serious problems with this gauge model.

$(2-d)$ rule

I found for 2- and 3-point Green's functions of gauge bosons at one-loop that

$$\begin{aligned}\left[\mathcal{A}^{ab}_{\alpha\beta}\right]_{\rm unparticle} &= (2-d)\left[\mathcal{A}^{ab}_{\alpha\beta}\right]_{\rm particle}\\ \left[\mathcal{A}^{abc}_{\alpha\beta\gamma}\right]_{\rm unparticle} &= (2-d)\left[\mathcal{A}^{abc}_{\alpha\beta\gamma}\right]_{\rm particle}\end{aligned}$$

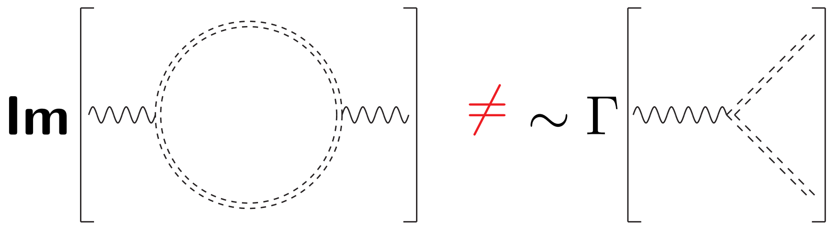
I suspect it holds true generally. The model looks trivial!

Unitarity violation

Unitarity is preserved by non-gauge interactions of $\mathcal{U}$ because
- **propagator has the correct cut structure by construction and**
- **interactions are Hermitian**

But above gauge interactions are non-Hermitian in time-like region!
$\Rightarrow$ **may signal failure of unitarity**

Example:



finite and odd in $(2-d)$ **non-integrable and even in** $(2-d)$

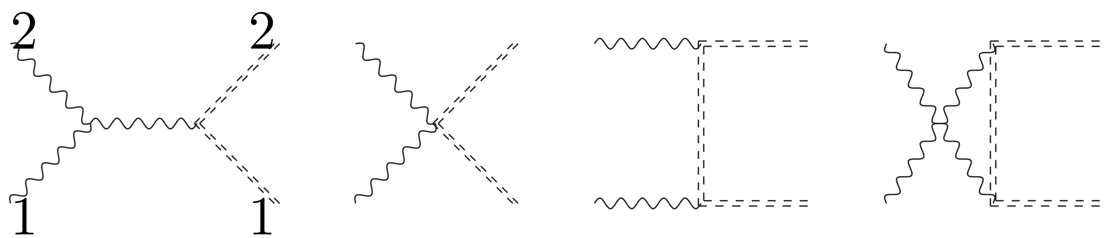
Violation of Ward identities for physical amplitudes

Ward-Takahashi identities for unphysical amplitudes hold true by construction. For example:

$$q^{\mu}\Gamma^{a}_{\mu}(-p-q,p;q) = T^{a}[D^{-1}(p+q) - D^{-1}(p)]$$

At first glance it may look strange that identities are broken by physical amplitudes.

Example: $A^a_\alpha(k_1)A^b_\beta(k_2) \to \mathcal{U}(p_1)\bar{\mathcal{U}}(p_2)$ **scattering**



I found

$$\begin{aligned} k_1^\alpha k_2^\beta \mathcal{A}^{ab}_{\alpha\beta} &= \frac{T^aT^b}{D(p_1)D(p_2)}\left(D(k_1-p_2)-\frac{1}{2}\left[D(p_1)+D(p_2)\right]\right) \\ &+ \frac{T^bT^a}{D(p_1)D(p_2)}\left(D(k_2-p_2)-\frac{1}{2}\left[D(p_1)+D(p_2)\right]\right) \\ &\neq 0 \end{aligned}$$

Note: cannot be avoided even for Abelian case.

Why?
Dispersion relation for particles is missing for $\mathcal{U}$!

Require $D^{-1}(p_1) = 0$ for a physical $\mathcal{U}$? Dispersion relation for $\mathcal{U}$?
Not consistent — what's the difference between physical particle and unparticle?

Conclusion

- **First gauge model for $\mathcal{U}$ unsuccessful:**
breaks unitarity and Ward identities

- **Georgi's challenge persists: Is it possible to gauge unparticle?**
Or, is there a consistent concept for gauging unparticle?

谢谢!