Unparticle Physics

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2008年9月25日

1. Introduction and mini-review
2. Bounds on unparticle couplings to electron
3. Fermionic unparticle
4. Thermodynamics of unparticles
5. First attempt at gauging unparticles

Talk at KITPC on Sep 25th, 2008: 'New Phys Beyond Standard Model'
1. Introduction and mini-review

1.1 Original idea due to Georgi


- Some very high energy theory contains SM fields and the fields from a scale invariant (SI) sector
- The two interact via heavy particles of mass $M_U$
- Effective interactions below $M_U$: $\sim$ (SM operators) $\times$ (SI operators)
- Dimensional transmutation occurs at $\Lambda_U$ in SI sector: (SI operators) $\rightarrow U$: unparticle fields as new degrees of freedom
- Effective int. below $\Lambda_U$: $\sim$ (SM operators) $\times U$

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1.2 Basics on unparticles

- **No dispersion relation** for unparticle, $\mathcal{U}$
- Kinematical behavior largely fixed by **scaling dim.**, $d$, of $\mathcal{U}$ field

(1) **scale invar.** $\rightarrow$ **state density** of bosonic unparticle of momen. $p$

$$\propto \theta(p_0)\theta(p^2)(p^2)^{d-2}$$

Reminiscent of state density of a system of $d$ massless particles, suggesting

normalization factor

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d + \frac{1}{2})}{\Gamma(d - 1)\Gamma(2d)}$$

No known restrictions on $d$ but from analysis of unitary rep in conformal theory; e.g., $d \geq 1$ for scalar $\mathcal{U}$

(2) Via **unitarity**, state density implies the **propagator**:

$$\frac{A_d}{2\sin(\pi d)} \frac{i}{(-p^2 - i\epsilon)^{2-d}}$$

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Comments:

- Extendable to fields with spin, both tensors and spinors
- Theoretical considerations prefer $1 < d < 2$ for scalar $U$
- Enough info for pheno. analysis in effective field theory

1.3 Status

195 papers as of Sep 24, 08 quoted Georgi’s 1st paper in Mar 07

- Collider phys
- Low energy effects (FCNC, CP, neutrinos, precision tests, etc)
- Astrophys and cosmology
- Theoretically oriented approaches including various topics, e.g.:

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understanding $U$ via continuum mass, deconstruction, relation to hidden sectors, CFT, breaking of scale invar at low energies, etc

- Interplay with electroweak SSB and Higgs
- 5th force
- Supersymmetric $U$
- Thermodynamics
- Fermionic $U$
- Gauged $U$

This talk is based on:

Liao: 0804.0752; Liao: 0804.4033

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2. Bounds on unparticle couplings to electron

2.1 Why unparticle-electron interactions?

Want to say something certain about relevance of unparticle phys

Theoretically, leptons as the cleanest fundamental particles connect directly to theory
Experimentally, electron systems are most precisely tested

⇒ Aims: (1) precision QED; (2) long-ranged forces

Language: effective field theory with interactions:

\[
L_{\text{int}} = C_S \bar{\psi} \psi U_S + C_P \bar{\psi} i \gamma_5 \psi U_P + C_V \bar{\psi} \gamma_\mu \psi U_\mu^V + C_A \bar{\psi} \gamma_\mu \gamma_5 \psi U_\mu^A
\]

\( \psi \): electron field

\( U_{S,P,V,P} \): unparticle fields of scaling dim \( d \)

\( C_{S,P,V,A} = \pm \Lambda_{S,P,V,A}^{1-d} \) couplings and effective energy scales

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2.2 Electron $g - 2$

$U_{S,P,V,A}$ contribute to $(g - 2) = 2a$ as follows ($m = \text{electron mass}$):

\[
\begin{align*}
    a_S &= -\frac{A_d}{2 \sin(\pi d)} \frac{(C_S m^{d-1})^2}{8\pi^2} \frac{3\Gamma(2d - 1)\Gamma(2 - d)}{\Gamma(2 + d)} \quad \text{also in 0704.3532} \\
    a_P &= +\frac{A_d}{2 \sin(\pi d)} \frac{(C_P m^{d-1})^2}{8\pi^2} \frac{\Gamma(2 - d)\Gamma(2d)}{\Gamma(2 + d)} \\
    a_V &= -\frac{A_d}{2 \sin(\pi d)} \frac{(C_V m^{d-1})^2}{4\pi^2} \frac{\Gamma(3 - d)\Gamma(2d - 1)}{\Gamma(d + 2)} \quad \text{also in 0704.2588} \\
    a_A &= +\frac{A_d}{2 \sin(\pi d)} \frac{(C_A m^{d-1})^2}{\pi^2} \frac{\Gamma(2d - 2)\Gamma(3 - d)}{\Gamma(2 + d)}
\end{align*}
\]
deviation between SM and expt $|\delta a| < 15 \times 10^{-12}$  PRL97

$\Rightarrow \Lambda_S > 110$ TeV, $\Lambda_P > 73$ TeV, $\Lambda_V > 37$ TeV, $\Lambda_A > 146$ TeV for $d = 1.5$

**Comments**

- Bounds are weakened as $d$ increases.
- Partial cancellation occurs but not more can be gained by treating $U_{S,P,V,A}$ together since $d$’s are generally different.

2.3 Exotic positronium decays

**Positronium:** EM bound state of $e^- e^+$, with orbital $\ell$ and spin $s$, have $C = (-1)^{\ell+s}$, $P = (-1)^{\ell+1}$

**Spin-1:** ortho-positronium ($o$-Ps); $C = P = -1$ for ground-state

**Spin-0:** para-positronium ($p$-Ps); $-C = P = -1$ for ground-state
Dominant decays by $C$ and $P$ conservation:

\[
\begin{align*}
o\text{-Ps} & \rightarrow \gamma\gamma\gamma - \text{good place to discover new phys} \\
p\text{-Ps} & \rightarrow \gamma\gamma
\end{align*}
\]

$L_{\text{int}}$ also conserves $C$ and $P$, thus dominant new modes are

invisible decays : $o\text{-Ps} \rightarrow U_V$; \\
                   $p\text{-Ps} \rightarrow U_P$

exotic decays : $o\text{-Ps} \rightarrow \gamma U_S, \gamma U_P, \gamma U_A$; \\
                $p\text{-Ps} \rightarrow \gamma U_V - \text{too small}$
Results:

\[
\text{Br}(o-Ps \rightarrow \mathcal{U}_V) = \frac{3 \cdot 2^{2d-6}}{(\pi^2 - 9)\alpha^3} A_d \left( \frac{C_V}{m^{1-d}} \right)^2
\]

\[
\text{Br}(p-Ps \rightarrow \mathcal{U}_P) = \frac{2^{2d-5}}{\pi\alpha^2} A_d \left( \frac{C_P}{m^{1-d}} \right)^2
\]

\[
\text{Br} (o-Ps \rightarrow \gamma\mathcal{U}_{S,P,A}) = \frac{\Gamma \left( d + \frac{1}{2} \right)}{\Gamma(2d)\Gamma(d+1)} \frac{3}{4(\pi^2 - 9)\alpha^2} \frac{3^{\frac{3}{2} - 2d}}{4\pi^{\frac{3}{2}}} \left( \frac{C_{S,P,A}}{m^{1-d}} \right)^2
\]

Data:

\[
\text{Br}(o-Ps \rightarrow \text{invisible}) \leq 4.2 \times 10^{-7} \text{ (90\% C.L)} \quad \text{PRD75}
\]

\[
\text{Br}(p-Ps \rightarrow \text{invisible}) \leq 4.3 \times 10^{-7} \text{ (90\% C.L)} \quad \text{PRD75}
\]

\[
\text{Br}(o-Ps \rightarrow \gamma \text{X}^0) \leq 1.1 \times 10^{-6} \text{ (90\% C.L)} \quad \text{PRL66}
\]

Bounds at \( d = 1.5 \):

Invisible:\quad \Lambda_V \geq 4.3 \times 10^5 \text{ TeV,} \quad \Lambda_P \geq 5.6 \times 10^2 \text{ TeV}

Exotic:\quad \Lambda_{S,P,A} \geq 5.1 \times 10^2 \text{ TeV}

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Comments

- 1 → 1 transition impossible between particles of different masses due to kinematics
But 1 particle → 1 unparticle possible, and no resonance expected

⇒ very strong bound from invisible o-Ps decays

- Continuous spectrum of (instead of a monochromatic) photon in exotic 2-body decays.

- Bounds more stringent than from $g - 2$

2.4 Long-range forces between electrons

2.4.1 Motivations

- Known long-range (macroscopic) forces: EM and gravity
Interpretation in quantum theory:

- force quanta are massless, i.e., $p^2 = 0$ (particles) 
  $\leftrightarrow$ propagator $\propto 1/p^2$ (fields) 
  $\Rightarrow V \propto \frac{1}{r}$

- physical space is 3-dim

Due to its fundamental importance, search for extra (5th) long-range forces has been a great tradition in physics

- No further surprise for forces mediated by particles: spin-indep or spin-dep, multiple exchange of quanta, etc
  $\Rightarrow$ Always $V \propto r^{-n}$ for integral $n$ at macroscopic $r$

- Unparticles can be different because of strange-looking propagator or lack of dispersion relation.
  $\Rightarrow$ Different $V$ expected at macroscopic $r$. 

- Microscopic particle-unparticle interactions must be feeble enough

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to evade particle phys detection.
Interactions due to unparticles can be amplified between macroscopic bodies if they are long-ranged like gravity.

⇒ Stronger constraints on particle-unparticle interaction may result!

• Why spin-spin interactions of electron?

If \((e, U)\) interact, very likely also \((N, U)\) do. The latter is perhaps more important for macroscopic bodies because they contain more \(N\) than \(e\).

But \((e, U)\) is cleaner and connected more directly to theory.

⇒ Remove contamination from \(N\)!

The only data not involving \(N\) are for spin-spin interactions of \(e\).
2.4.2 $e^-e^-$ interaction potential due to $\mathcal{U}$

$t$-channel exchange of $\mathcal{U}$ yields

$$U_t^{-+}(r) = U_{\text{spin}}^{-+}(r) + U_{\text{non}}^{-+}(r)$$

$$U_{\text{spin}}^{-+}(r) = \frac{A_d}{4\pi^2} \frac{1}{r^{2d-1}} \left\{ -C_A^2 \Sigma_s \Gamma(2(d - 1)) - C_A^2 \Sigma_s \frac{\Gamma(2d)}{4m^2 r^2} (2 - d) ight.$$

$$- C_P^2 \frac{1}{4m^2 r^2} \frac{\Gamma(2d)}{2(d - 1)} [\Sigma_s - (2d + 1) \Sigma_a]$$

$$+ (C_A^2 - C_V^2) \frac{1}{4m^2 r^2} \frac{\Gamma(2d)}{2(d - 1)} [(1 - 2d) \Sigma_s + (2d + 1) \Sigma_a] \left\}$$

$$U_{\text{non}}^{-+}(r) = \frac{A_d}{4\pi^2} \frac{1}{r^{2d-1}} \left\{ (C_V^2 - C_S^2) \Gamma(2(d - 1)) + [(2 - d) C_V^2 - (3 - d) C_S^2] \frac{\Gamma(2d)}{4m^2 r^2} \right\}$$

$$\Sigma_s = \sigma_1 \cdot \sigma_2, \quad \Sigma_a = \sigma_1 \cdot \hat{r} \sigma_2 \cdot \hat{r}, \quad \hat{r} = r/r, \quad m = \text{electron mass}$$

Comments

• Even higher terms not important for $mr \gg 1$. 

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• Standard results for exchange of a massless particle are recovered as $d \to 1$ up to contact terms.

• Dominant spin-spin interaction: $\propto C_A^2 \sigma_1 \cdot \sigma_2 \ r^{1-2d}$ generally non-integral power law!

2.4.3 Bounds from macroscopic spin-spin interaction of $e^-$

• 4 precise yet reliable experiments:
  * 2 with torsion pendulum
  * 2 by induced para-magnetization


• Results:

**Largest term** $\sim r^{1-2d} C_A^2 \Sigma_s$

$$\left( \frac{\Lambda_A}{\text{TeV}} \right)^{2(d-1)} \geq 3.17 \times 10^4 \ \frac{\Gamma(d - \frac{1}{2})}{(2\pi)^{2d}\Gamma(d)} \left( \frac{10^{16}}{0.1973 \text{ cm}} \right)^{2(2-d)} \ \ r_0 : \text{typical distance of two samples}$$
• For $1.5 < d < 2$, $\Lambda_A$ is very stringently bounded.

• For $1 < d < 1.5$, practically $C_A \sim 0$: should consider next terms.

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\log_{10}(\Lambda_A/\text{TeV})$</th>
<th>$\log_{10}(\Lambda_P/\text{TeV})$</th>
<th>$\log_{10}(\Lambda_V/\text{TeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>1.3</td>
<td>$\times$</td>
<td>6.44</td>
<td>5.77</td>
</tr>
<tr>
<td>1.4</td>
<td>$\times$</td>
<td>0.126</td>
<td>$-0.307$</td>
</tr>
<tr>
<td>1.5</td>
<td>20.3</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>1.6</td>
<td>13.7</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>1.7</td>
<td>9.04</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>1.8</td>
<td>5.53</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 1: Bounds on $\Lambda_{A,P,V}$ as a function of $d$ for $r_0 = 25$ cm.

$\times$: scales far in excess of the Planck scale;

$-$: scales too low to be useful.
Next terms $\sim r^{-1-2d}C_{P,V}^2 m^2 \Sigma_s$ for $1 < d < 1.5$

$$\left(\frac{\Lambda_P}{\text{TeV}}\right)^{2(d-1)} \geq 6.07 \times 10^{16} \frac{\Gamma(d + \frac{1}{2})}{(2\pi)^{2d}\Gamma(d)} \left(\frac{0.1973 \text{ cm}}{10^{16} r_0}\right)^{2(d-1)}$$

$$\left(\frac{\Lambda_V}{\text{TeV}}\right)^{2(d-1)} \geq 1.52 \times 10^{16} \frac{\Gamma(d + \frac{1}{2})}{(2\pi)^{2d}\Gamma(d)} (2d - 1) \left(\frac{0.1973 \text{ cm}}{10^{16} r_0}\right)^{2(d-1)}$$

2.4.4 Bounds from hyperfine splitting of positronium

- **hfs**: level splitting between o-Ps and p-Ps. For ground-states:

  QED: $E(1^3S_1) - E(1^1S_0) = +203.39169 (41 \text{ or } 16)$ GHz  \hspace{1cm} \text{PRL 85(2000), PRL 86 (2001)}

  expt: $E(1^3S_1) - E(1^1S_0) = +203.3875(16)$ GHz \hspace{1cm} \text{PRL 34 (1975), PR A27 (1983)}

- **Potential for $e^-e^+$, $U^{-+}(r)$**: 2 contributions
  
  - $t-$channel exchange of $\mathcal{U}$: obtained from $U_{t^{-+}}(r)$ by $C_V^2 \rightarrow -C_V^2$

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• \( s \)-channel annihilation to \( U \):

\[
U_{s}^{-+}(r) = \frac{A_{d}}{4 \sin(\pi d)} \frac{1}{(-4m^{2}c^{2} - i\epsilon)^{2-d}} \delta^{3}(r) \\
\times \left[ (3C_{V}^{2} + C_{P}^{2} + C_{A}^{2}) + (C_{V}^{2} - C_{P}^{2} - C_{A}^{2})\sigma_{-} \cdot \sigma_{+} \right]
\]

Higher order in \( \alpha \) than \( U_{t}^{-+}(r) \) for \( 1 < d < 2 \), ignored below.

• Results:

\[
E(1^{3}S_{1}) - E(1^{1}S_{0}) = -m\alpha^{2d-1} \left( \frac{C_{A}}{m^{1-d}} \right)^{2} \frac{A_{d}}{2\pi^{2}} \Gamma(2(d-1))\Gamma(2(2-d))
\]

\( \Rightarrow \quad \Lambda_{A} \geq 21 \text{ TeV at } d = 1.5 \)
Summary of this section

- 1 particle to 1 unparticle transition possible $\Rightarrow$ invisible decays
- Continuous spectrum of 2-body decays containing an unparticle
- Strange-looking propagator $\Rightarrow$ new macroscopic forces

- Global pattern of constraints on $ee\mathcal{U}$ interactions

- Long-ranged spin-spin forces:
  - For $1 < d < 1.5$, $\mathcal{U}_A$ cannot couple to $ee$.
  - For $1.5 < d < 2$, $\Lambda_A > 10^5$ TeV.

- Invisible positronium decays:
  - $\Lambda_V > 4.3 \times 10^5$ TeV, $\Lambda_P > 5.6 \times 10^2$ TeV

Exotic positronium decays: $\Lambda_S > 5.1 \times 10^2$ TeV

- $g - 2$ and positronium hfs provide milder bounds
3. Fermionic unparticle

3.1 Motivations

Why bosonic $U$ first?

- It is easy to couple them as a SM singlet to particles
- Their propagators were known at the very start

No fundamental reason against fermionic $U$:

- QCD analogue: mesons and baryons from quarks and gluons
- If we imagine that fermionic $U$ are charged under SM, they can couple to particles as easily as bosonic ones.
- Not more difficult to construct a propagator for them from scale and Lorentz symmetry

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3.2 Basic considerations

- $U$ is free when its interactions with particles and itself are ignored

$\Rightarrow$ field and propagator is determined up to
- unknown $d$ (given by fundamental theory at a high scale)
- arbitrary normalization (not affecting phys)

compared to canonical QFT:
- mass as a given physical parameter
- normalization arbitrary though canonical one popular

Actually the only thing available for constructing $U$ and its propagator that is also sufficient and necessary is scale and Lorentz symmetry.

- It is highly desired that $U$ and its propagator have a massless fermionic particle limit as $d \to \frac{3}{2}$.

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3.3 Fermionic $U$ field

Can be built as in the canonical case:

$$U(x) = \int \frac{d^4 p}{(2\pi)^4} \Xi_d(p) \sum_s \left( a^s_p u^s(p) e^{-ip \cdot x} + b^{s\dagger}_p v^s(p) e^{ip \cdot x} \right)$$

$$\Xi_d(p) = F(d) \theta(p^0) \theta(p^2) (p^2)^{d-\frac{5}{2}}$$

- **Scale symmetry incorporated:** $U(\lambda x) = \lambda^{-d} U(x)$
- **Dimensions:** $[a] = [b] = \frac{1}{2} - d$, $[u] = [v] = \frac{1}{2}$
- $a^s_p$, $b^s_p$ assumed to satisfy the anti-commutation relations, e.g.,

$$\Xi_d(p) \Xi_d(q) \{ a^r_p, a^{s\dagger}_q \} = \Xi_d(p) (2\pi)^4 \delta^4(p - q) \delta^{rs}$$

with normalized single-unparticle states, e.g.,

$$|p, s\rangle = \Xi_d(p) a^{s\dagger}_p |0\rangle$$
\[ \langle p, r | q, s \rangle = \Xi_d(p)(2\pi)^4 \delta^4(p - q) \delta^{rs} \]

- The above \( \mathcal{U} \) is completely similar to a complex scalar \( \mathcal{U} \) in the absence of \( u, v \), with the correspondence:
  \[(d - \frac{1}{2}) \text{ (fermionic)} \leftrightarrow d \text{ (bosonic)}\]

suggesting the normalization:
\[ F(d) = A_{d-\frac{1}{2}} \equiv B(d - \frac{1}{2}) \]
\[ \Rightarrow \text{Correct canonical limit of all formulas guaranteed with } F(d) \]

**Comment:** absolute normalization immaterial

### 3.3.1 Construction of \( u(p), v(p) \)

**Point:** Although dispersion relation is missing for \( \mathcal{U} \), the standard procedure still works by the definition of its Lorentz properties.

- Physical unparticle has \( p^2 > 0, \ p_0 > 0 \): go to its rest frame
  \( \sqrt{p^2} \) plays the same role of mass as for a canonical fermion
• Make a boost to a general $p$. The wavefunction then satisfies

$$(\phi - \sqrt{p^2})u(p) = 0, \quad u(p)\bar{u}(p) = \phi + \sqrt{p^2}, \quad \text{etc}$$

spin

K-G eqn is a trivial identity due to the lack of dispersion relation.

3.4 Propagator

Start from definition:

$$\tilde{S}_F^{U\alpha\beta}(x - y) = \langle 0|TU\alpha(x)\bar{U}_\beta(y)|0\rangle$$

where, e.g.,

$$\langle 0|U\alpha(x)\bar{U}_\beta(y)|0\rangle = +i(\bar{\phi}^x)_{\alpha\beta} \frac{d^4p}{(2\pi)^4} \Xi_d(p) e^{-ip\cdot(x-y)}$$

$$+ \delta_{\alpha\beta} \frac{d^4p}{(2\pi)^4} \Xi_d(p) (p^2)^{\frac{1}{2}} e^{-ip\cdot(x-y)}$$
Final Answer after some manipulation

\[
\tilde{S}_F^U(x - y) = \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-y)} S_F^U(p),
\]

\[
S_F^U(p) = \frac{i F(d)}{2 \sin(d\pi)} \left[ \frac{1}{(-p^2 - i\epsilon)^{2-d}} - \tan(d\pi) \frac{\phi}{(-p^2 - i\epsilon)^{\frac{5}{2}-d}} \right]
\]

\[
\text{non-}\gamma \text{ term} \quad \gamma \text{ term}
\]

\[
S_F^U(p) \text{ has correct discontinuity across the cut } p^2 > 0:
\]

\[
F(d)(p^2)^{d - \frac{5}{2}}(\phi + \sqrt{p^2})
\]

Features

- Relative phase \(\frac{\pi}{2}\) between non-\(\gamma\) and \(\gamma\) terms for \(p^2 > 0\)
  \(\Rightarrow\) New interference possible beyond the one for bosonic \(U\)

- Ordinary mass term replaced by \(p\)-dependent non-\(\gamma\) term
Can enhance chirality-flipped transitions when \( \mathcal{U} \) appears in loop together with a heavy bosonic particle:

\[
\begin{array}{c}
\mathcal{X} \\
\Rightarrow \quad m \\
\Rightarrow \quad p \sim m_{\text{boson}}
\end{array}
\]

We demonstrate this feature by an example.

3.5 Chirality-flipped EM transitions due to fermionic \( \mathcal{U} \)

Consider the effective interaction:

\[
\mathcal{L}_{\text{int}}^{\mathcal{U}} = \Lambda_1^{3-d} \mathcal{U} (a_j + b_j \gamma_5) \psi_j \varphi + \Lambda_2^{3-d} \mathcal{U}^\dagger (a_j^* - b_j^* \gamma_5) \psi_j \varphi
\]

(1)

- \( \varphi, \psi_j \): ordinary particle fields of mass \( m_\varphi \) and \( m_j; j = 1, 2 \)
- \( \Lambda_\mathcal{U} \) unparticle scale; \( a_j, b_j \) unknown pure numbers
- \( \mathcal{U} \) chosen to be neutral: \( Q(\varphi) = -Q(\psi_j) \equiv -Q_j \) in units of \( e > 0 \)
3.5.1 Amplitudes for $\psi_1 \rightarrow \psi_2\gamma$

On-shell amplitude for $m_{\varphi} \gg m_{1,2}$:

$$A_\mu = -\frac{1}{(4\pi)^2} \frac{Q_1 e^\Lambda U^{3-2d} F(d)}{2 \sin(d\pi)} \frac{m_{2(d-2)} F_\mu}{m_{\varphi}}$$

$$F_\mu = \frac{\tan(d\pi)}{6m_{\varphi}} \frac{\Gamma(d + 1/2) \Gamma(7/2 - d)}{\Gamma(d)}$$

$$\times \left[ -\bar{m} (a_1 a_2^* + b_1 b_2^*) i\sigma_{\mu\nu} q^\nu + \Delta m (a_2^* b_1 + a_1 b_2^*) i\sigma_{\mu\nu} q^\nu \gamma_5 \right]$$

$$+ \frac{1}{2} \Gamma(d) \Gamma(3 - d) \left[ (a_1 a_2^* - b_1 b_2^*) i\sigma_{\mu\nu} q^\nu + (a_2^* b_1 - a_1 b_2^*) i\sigma_{\mu\nu} q^\nu \gamma_5 \right]$$

with $\bar{m} = (m_1 + m_2)/2$, $\Delta m = (m_1 - m_2)/2$
Non-\(\gamma\) term enhanced by \(\frac{m_\varphi}{m_j}\) compared to the ordinary case!

### 3.5.2 Transition rates

Keeping only the enhanced term and assuming \(m_1 \gg m_2\)

\[
\Gamma(\psi_1 \rightarrow \psi_2 \gamma) = 2^{-7} m_1 \alpha \left[ f(d) Q_1 \left( \frac{m_\varphi}{\Lambda_u} \right)^{2d - 3} \frac{m_1}{m_\varphi} \right]^2 X_{12}
\]

\[
X_{ij} = |a_i a_j^* - b_i b_j^*|^2 + |a_i^* b_i - a_i b_j^*|^2
\]

\[
f(d) = \frac{1}{2^{2d} \pi^{2d - \frac{3}{2}} \sin(d \pi)} \frac{[\Gamma(d)]^2 \Gamma(3 - d)}{\Gamma(d - \frac{3}{2}) \Gamma(2d - 1)}
\]
3.5.3 EM dipole moments

\[ a_{\psi_1} = Q_1 \frac{2m_1 \Lambda u^{3-2d} F(d)}{(4\pi)^2} m_\varphi^{2(d-2)} \left[ \frac{1}{2} \Gamma(d)\Gamma(3-d)(|a_1|^2 - |b_1|^2) \right. \]

\[ \left. - \tan(d\pi) \frac{m_1}{6m_\varphi} \Gamma \left( d + \frac{1}{2} \right) \Gamma \left( \frac{7}{2} - d \right) (|a_1|^2 + |b_1|^2) \right] \]

\[ d_{\psi_1} = -Q_1 \frac{1}{(4\pi)^2} \frac{e\Lambda u^{3-2d} F(d)}{2\sin(d\pi)} m_\varphi^{2(d-2)} \Gamma(d)\Gamma(3-d) \Im(a_1^*b_1) \]

Compare to **pure particle case** assuming \( m_\varphi \gg m_\chi, m_\psi \):

\[ \mathcal{L}^\chi_{\text{int}} = \bar{\chi}(a + b\gamma_5)\psi\varphi + \bar{\psi}(a^* - b^*\gamma_5)\chi\varphi^\dagger \]

\[ \Rightarrow a_\psi = -\frac{Q_\psi m_\psi}{(4\pi)^2 m_\varphi^2} \left[ m_\chi(|a|^2 - |b|^2) + \frac{1}{3} m_\psi(|a|^2 + |b|^2) \right] \]

\[ d_\psi = \frac{Q_\psi e}{(4\pi)^2 m_\varphi^2} \Im(a^*b) \]
3.5.4 Constraints from EM dipole moments

\[ |\delta a_e| < 15 \times 10^{-12} \text{ [PRL97(2006)]}, \quad \delta a_i \equiv a_i^{\text{expt}} - a_i^{\text{SM}} \]
\[ \delta a_\mu = 22(10) \times 10^{-10} \text{ [PDG]} \]
\[ d_e = (0.07 \pm 0.07) \times 10^{-26} \text{ e cm [PDG]} \]
\[ d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm [PDG]} \]
\[ d_n < 0.63 \times 10^{-25} \text{ e cm [PDG]} \]

<table>
<thead>
<tr>
<th>(d)</th>
<th>1.6</th>
<th>1.7</th>
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<tr>
<td>(10^3(</td>
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<tr>
<td>(10^3(</td>
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<td>b</td>
<td>^2)_\mu)</td>
</tr>
<tr>
<td>(-10^9 \Im(a^*b)_e)</td>
<td>8.9</td>
<td>7.8</td>
<td>7.9</td>
<td>6.5</td>
</tr>
<tr>
<td>(-\Im(a^*b)_\mu)</td>
<td>4.7</td>
<td>4.1</td>
<td>4.2</td>
<td>3.4</td>
</tr>
<tr>
<td>(-10^7 \Im(a^*b)_n)</td>
<td>8.0</td>
<td>7.0</td>
<td>7.1</td>
<td>5.9</td>
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</tbody>
</table>

**Table 2:** Bounds for \(\Lambda_U = 1 \text{ TeV}, m_\varphi = 200 \text{ GeV.}\)
3.5.4 Constraints from lepton flavor changing radiative decays

\[ \text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \ [\text{MEGA}] \]
\[ \text{Br}(\tau \rightarrow \mu\gamma) < 4.5 \times 10^{-8} \ [\text{Belle}], \quad \text{Br}(\tau \rightarrow e\gamma) < 1.2 \times 10^{-7} \ [\text{Belle}] \]

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>(10^{12} X_{\mu e})</td>
<td>3.3</td>
<td>2.6</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>(10^5 X_{\tau e})</td>
<td>5.3</td>
<td>4.1</td>
<td>4.2</td>
<td>2.9</td>
</tr>
<tr>
<td>(10^5 X_{\tau \mu})</td>
<td>2.0</td>
<td>1.5</td>
<td>1.6</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3: Bounds on \(X_{ij}\). Same input parameters as above.

Comments

- High quality in \(X_{\mu e}\) not only from better precision but also from less power in \(m_\mu\).
- Bound on \(X_{bs}\) from \(b \rightarrow s\gamma\) less stringent.

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Summary of this section

- The propagator of fermionic $U$ is determined by scale and Lorentz symmetry. It has the correct canonical limit.
- Two (γ and non-γ) terms appear in propagator.
- Relative phase of $\frac{\pi}{2}$ between the two in time-like regime. ⇒ Expect new interference phenomenon beyond the known one.
- Non-γ term can effect chirality flip without the usual suppression of light fermion mass. Instead, it can enhance the flip by the mass of a virtual boson in loop.
4. Thermodynamics of unparticles

4.1 Motivations and main results

• Density of states for \( U \), very different from particles, could result in very different thermodynamics even if no other differences occur.

• \( U \) effects have hitherto been studied either at \( T = 0 \) (few-body systems) or in astrophysical environments with \( T \neq 0 \).

Thermodynamics of photons is taken for granted in the latter case. Is this justified?

We find:

• Equation of state (EoS) parameter \( \omega_U = \frac{1}{2d+1} \) — Some new form of energy between matter (\( \omega_M = 0 \)) and radiation (\( \omega_\gamma = \frac{1}{3} \))

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• \( \omega_\mathcal{U} < \omega_\gamma \Rightarrow \mathcal{U} \), once produced and decoupled, evolve more slowly than photons in the expansion of universe!

Possible to obtain **significant relic \( \mathcal{U} \) in the present** even if the universe started with a low \( \rho_\mathcal{U} \)!

### 4.2 Thermodynamics

**Basic idea**

Start from bosonic particles of mass \( \mu \) whose partition function is

\[
\ln Z(\mu^2) = -g_s V \frac{d^4 p}{(2\pi)^4} 2p^0 \theta(p^0) \delta(p^2 - \mu^2) \ln(1 - e^{-p^0 \beta}), \quad \beta = T^{-1}
\]

No such dispersion-relation constraint on \( \mathcal{U} \) whose density of states is

\[
\propto \frac{d^4 p}{(2\pi)^4} 2p^0 \theta(p^0) \theta(p^2)(p^2)^{d-2}
\]
It may be interpreted as a **continuous collection of particles** with the spectral function $\varrho(\mu^2) \propto \theta(\mu^2)(\mu^2)^{d-2}$:

$$\frac{d^4p}{(2\pi)^4}2\pi^2p^0\theta(p^0)\delta(p^2 - \mu^2)\varrho(\mu^2)d\mu^2$$

$\mu^2$ serves as a new quantum number to be summed over with weight $\varrho(\mu^2)$!

**Normalization of $\varrho(\mu^2)$**

$\cal U$ exists only below $\Lambda_{\cal U}$:
- $\varrho(\mu^2)$ **must terminate** at $\mu^2 = \Lambda_{\cal U}^2$
- **Our results apply only to** $\beta\Lambda_{\cal U} > 1$. Beyond certain $T$, may be resolved – no more suitable degrees of freedom to cope with

**Result:**

$$\varrho(\mu^2) = (d-1)\Lambda_{\cal U}^{2(1-d)}\theta(\mu^2)(\mu^2)^{d-2}$$

**Thermodynamics of $\cal U$**

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Its partition function is

\[
\ln Z = \int_0^{\Lambda_U^2} d\mu^2 \varrho(\mu^2) \ln Z(\mu^2)
\]

\[
= -\frac{g_s V (d_U - 1)}{4\pi^2 \beta^3 (\beta \Lambda_U)^2 (d_U - 1)} \int_0^{(\beta \Lambda_U)^2} dy \ y^{d-2} \int_y^\infty dx \ \sqrt{x - y} \ln(1 - e^{-\sqrt{x}})
\]

For \(\beta \Lambda_U > 1\),

\[
\ln Z = \frac{g_s V}{\beta^3 (\beta \Lambda_U)^2 (d-1)} \frac{C(d)}{4\pi^2}, \quad C(d) = B(3/2, d) \Gamma(2d + 2) \zeta(2d + 2)
\]

Then

\[
p_U = \frac{1}{\beta} \frac{\partial}{\partial V} \ln Z = g_s T^4 \left( \frac{T}{\Lambda_U} \right)^{2(d-1)} \frac{C(d)}{4\pi^2},
\]

\[
\rho_U = -\frac{1}{V} \frac{\partial}{\partial \beta} \ln Z = (2d + 1) g_s T^4 \left( \frac{T}{\Lambda_U} \right)^{2(d-1)} \frac{C(d)}{4\pi^2}
\]

\[
\omega_U = \frac{p_U}{\rho_U} = \frac{1}{2d + 1}
\]
Comments

- The case of massless particles is recovered as \( d \to 1 \).
- EoS parameter \( \omega_U \) lies between \( \omega_M = 0 \) and \( \omega_\gamma = \frac{1}{3} \).
- Important impacts on cosmology.

4.3 Cosmological implications

Why \( w_U \) important

It determines the evolution of \( \rho(T) \) or \( \rho(R) \) after decoupling from equilibrium in the expanding universe of Friedmann-Robertson-Walker (FRW),

\[
\rho(R) = \rho(R_D) \left( \frac{R_D}{R} \right)^{3(1+\omega)}
\]

\( R \): scale factor, \( D \): decoupling

For photon, \( R_D/R = T_\gamma/T_D \), thus

\[
\rho_U(T_\gamma) = \rho_U(T_D) \left( \frac{T_\gamma}{T_D} \right)^{3(1+\omega_U)}, \quad \rho_\gamma(T_\gamma) = \rho_\gamma(T_D) \left( \frac{T_\gamma}{T_D} \right)^4
\]
Comments:

- If $\mathcal{U}$ always in equilibrium with photon, they share the same $T$ so that
  - always $\rho_\mathcal{U} < \rho_\gamma$ due to extra power in $T/\Lambda_\mathcal{U} < 1$ and
  - $\rho_\mathcal{U}$ drops faster than $\rho_\gamma$ as the universe cools down

- If $\mathcal{U}$ decouples at $T_D$, it evolves afterwards more slowly than photon — this case more interesting:
  We could have a significant $\rho_\mathcal{U}(T_\gamma)$ today, albeit $\rho_\mathcal{U}(T_D)$ is very small!

Feasibility reserved for a global fit of cosmological and astrophysical data. We studied a simple scenario with the photon-$\mathcal{U}$ interaction:

$$\mathcal{L} = \frac{\lambda}{\Lambda_\mathcal{U}^d} F^{\mu\nu} F_{\mu\nu} \mathcal{U}$$
Summary of results

- First study on thermodynamics of unparticles and surprising results obtained.

\[ \rho_U, p_U: \text{higher powers in } T \text{ for } U \text{ than for photons} \]

\[ \omega_U = \frac{1}{2d+1} \] lies between those of matter and photon: new form of energy in the universe.

- Different evolution of \( U \) from photons could result in a significant \( U \) relic today after taking into account strong constraints from star-cooling and BBN, etc.

- Actual relevance of \( U \) to cosmology relies more on numerical fitting of data and a survey of interactions.

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5. First attempt at gauging unparticles

Unparticles must interact with particles to be physically relevant.
⇒ It seems natural they are charged under SM gauge group.

Gauging $U$ was already challenged in Georgi’s original paper —
‘but I have no idea whether this is possible’

Why non-trivial? Non-integral $d$ or $d > 2$ ⇒ non-local theory!

Minimal coupling for local fields doesn’t work.

First attempt by Cacciapaglia, Marandella, Terning, 0708.0005
with the help of Wilson line and lessons from non-local chiral quark model

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Free theory

\[
S_0 = d^4x \, d^4y \, U^\dagger(x) \tilde{D}^{-1}(x - y) U(y),
\]

\(\tilde{D}^{-1} : \) Fourier transf of \(D^{-1}(p) \propto (m^2 - p^2)^{2-d}\)

Gauge theory

\[
S = d^4x \, d^4y \, U^\dagger(x) \tilde{D}^{-1}(x - y) P \exp \left[ -i g T^a \int_x^y A_\mu \, dw^\mu \right] U(y),
\]

\(P : \) path ordering in both \(A_\mu^a\) and \(T^a\)

Feynman rules

Complicated but systematically manageable Terning 1991, Holdom 1992

Examples:

\[
A U U^\dagger \quad i g \Gamma^{a\mu}(-(p + q), p; q) = i g T^a (2p + q)^\mu E_1(p; q)
\]
\[ A A U U^\dagger \quad i g^2 \Gamma^{a b \mu \nu} (- (p + q_1 + q_2), p; q_1, q_2) \]
\[ = i g^2 \left\{ \{ T^a, T^b \} g^{\mu \nu} E_1 (p; q_1 + q_2) \right. \]
\[ + T^a T^b (2p + q_2) \nu (2p + 2q_2 + q_1) \mu E_2 (p; q_2, q_1) \]
\[ + T^b T^a (2p + q_1) \mu (2p + 2q_1 + q_2) \nu E_2 (p; q_1, q_2) \left\} \right. \]

where

\[ E_0 (p) = D^{-1} (p), \]
\[ E_1 (p; q_1) = \frac{E_0 (p + q_1) - E_0 (p)}{(p + q_1)^2 - p^2}, \]
\[ E_2 (p; q_1, q_2) = \frac{E_1 (p; q_1 + q_2) - E_1 (p; q_1)}{(p + q_1 + q_2)^2 - (p + q_1)^2}. \]

**Particle case arises as a limit:**
\[ d \to 1 \Rightarrow D (p) \to i (p^2 - i \epsilon)^{-1}, \quad E_1 \to 1, \quad E_2 \to 0 \]
But there are **serious problems** with this gauge model.

**(2 − d) rule**

I found for 2- and 3-point Green’s functions of gauge bosons at one-loop that

\[
\begin{align*}
\mathcal{A}_{\alpha\beta}^{ab} \text{ unparticle} & = (2 - d) \mathcal{A}_{\alpha\beta}^{ab} \text{ particle} \\
\mathcal{A}_{\alpha\beta\gamma}^{abc} \text{ unparticle} & = (2 - d) \mathcal{A}_{\alpha\beta\gamma}^{abc} \text{ particle}
\end{align*}
\]

I suspect it holds true generally. The model looks trivial!

**Unitarity violation**

Unitarity is preserved by non-gauge interactions of \( \mathcal{U} \) because

- propagator has the correct cut structure by construction and
- interactions are Hermitian
But above gauge interactions are non-Hermitian in time-like region! ⇒ may signal failure of unitarity

Example:

\[ \text{Im} \left[ \begin{array}{c}
\ \ \ |
\ \ \ |
\ \ \ |
\ \ \ |
\end{array} \right] \neq \sim \Gamma \left[ \begin{array}{c}
\ \ \ |
\ \ \ |
\ \ \ |
\ \ \ |
\end{array} \right] \]

finite and odd in \((2 - d)\) \quad non-integrable and even in \((2 - d)\)

Violation of Ward identities for physical amplitudes

Ward-Takahashi identities for unphysical amplitudes hold true by construction. For example:

\[ q^\mu \Gamma_\mu^a(-p - q, p; q) = T^a[D^{-1}(p + q) - D^{-1}(p)] \]
At first glance it may look strange that identities are broken by physical amplitudes.

**Example:** \( A^a_\alpha(k_1) A^b_\beta(k_2) \rightarrow \mathcal{U}(p_1)\overline{\mathcal{U}}(p_2) \) scattering

I found

\[
k_1^\alpha k_2^\beta \mathcal{A}_{\alpha\beta}^{ab} = \frac{T^a T^b}{D(p_1) D(p_2)} \left( D(k_1 - p_2) - \frac{1}{2} [D(p_1) + D(p_2)] \right) \\
+ \frac{T^b T^a}{D(p_1) D(p_2)} \left( D(k_2 - p_2) - \frac{1}{2} [D(p_1) + D(p_2)] \right) \\
\neq 0
\]
Note: cannot be avoided even for Abelian case.

Why?
Dispersion relation for particles is missing for $U$!

Require $D^{-1}(p_1) = 0$ for a physical $U$? Dispersion relation for $U$?
Not consistent — what’s the difference between physical particle and unparticle?

Conclusion

• First gauge model for $U$ unsuccessful:
  breaks unitarity and Ward identities

• Georgi’s challenge persists: Is it possible to gauge unparticle?
  Or, is there a consistent concept for gauging unparticle?

谢谢！