Minimal Lepton Flavor Violation in Randall-Sundrum Model

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Based on work done with Hai-Bo Yu, arXiv:0804.2503

KITPC, Beijing, China, September 19, 2008
Gauge Hierarchy Problem

- SM: precision data indicates a light Higgs
- Gauge Hierarchy Problem:
  - quantum corrections to Higgs mass $\sim \Lambda^2$

$\frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$

$(200 \, GeV)^2 = m_{H_0}^2 + \left[ -(2 \, TeV)^2 + (700 \, GeV)^2 + (500 \, GeV)^2 \right] \left( \frac{\Lambda}{10 \, TeV} \right)^2$

- Require severe fine-tuning for high cut-off scale
- for $\Lambda \sim M_{pl}$: fine-tuning at level $\sim 10^{-30}$
Solution to Gauge Hierarchy Problem

- Randall-Sundrum model
  - **5D metric:** \( ds^2 = e^{-2kr(x)\phi} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 \)

- TeV brane-localized Higgs: 5D action
  \[
  \int d^4x \int dy \sqrt{-g} \, \delta(y - \pi R) \left[ g^{\mu\nu} D_\mu H(x) D_\nu H(x) - \lambda \left( |H(x)|^2 - v_5^2 \right)^2 \right]
  \]

- 4D effective action
  \[
  H(x) \rightarrow e^{\pi k R} \tilde{H}(x)
  \]
  \[
  \int d^4x \eta^{\mu\nu} D_\mu \tilde{H}(x) D_\nu \tilde{H}(x) - \lambda \left( |\tilde{H}(x)|^2 - e^{-2\pi k R} v_5^2 \right)^2
  \]

- 4D symmetry breaking scale related to 5D symmetry breaking scale
  \[
  v = e^{-\pi k R} v_5
  \]

- with \( kR \approx 11 - 12 \):
  \[
  v_{ew} \sim e^{-\pi k R} M_{pl}
  \]
EW Constraints

- Precision Constraints
  - expand the bulk gauge symmetry to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
  - custodial symmetry preserved
  - 1st KK mass - 3 TeV allowed by EWPT

- Constraints on flavor sector:
  - model independent analysis: with $O(1)$ couplings:

$$\Lambda > (10^2-10^3) \text{ TeV}$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Limit on $\Lambda_F$ (TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re$C^1_K$</td>
<td>$1.0 \cdot 10^3$</td>
</tr>
<tr>
<td>Re$C^4_K$</td>
<td>$12 \cdot 10^3$</td>
</tr>
<tr>
<td>Re$C^5_K$</td>
<td>$10 \cdot 10^3$</td>
</tr>
<tr>
<td>Im$C^1_K$</td>
<td>$15 \cdot 10^3$</td>
</tr>
<tr>
<td>Im$C^4_K$</td>
<td>$160 \cdot 10^3$</td>
</tr>
<tr>
<td>Im$C^5_K$</td>
<td>$140 \cdot 10^3$</td>
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</tbody>
</table>
Flavor Sector

- SM fermions on the TeV brane
  - cutoff scale ~ 1 TeV
  - leads to dangerously large FCNC

- SM fermions and gauge fields in the bulk
  - generate fermion mass hierarchy by wave function localization
  \[ \psi(0) \sim e^{(1/2-c)k_y} \]
  - two sources of flavor violations:
    - 5D Yukawa coupling constants
    - 5D bulk mass terms
    - generally independent

Gherghetta & Pomarol, 2000
Flavor Violations

- 5D Lagrangian

\[ \mathcal{L}_{5D} \supset \bar{\psi} C \psi + \bar{\psi}_u C \psi_u \psi_u + \bar{\psi}_d C \psi_d \psi_d + H \bar{\psi} \lambda_U \psi_u + \overline{H^*} \lambda_D \psi_d \]

- unitary transformations: rotate to diagonal basis for bulk mass matrices

- decomposition of fermion field

\[ \psi(x, y) = \sum_n \frac{e^{2krc|x|}}{\sqrt{r_c}} \psi_n(x) f_n(\phi; c) \]

- couplings between zero mode fermions and gauge boson KK modes:

\[ \sum_n G^n (\bar{\psi}_0^\dagger f^2_0 \Psi^0 + \bar{\psi}_u^0 f^2_u \psi_u^0 + \bar{\psi}_d^0 f^2_d \psi_d^0) \]

- \( f \): profile of zero-mode along 5th dimension
**Flavor Violations**

- effective 4D Yukawa interactions
  \[ H \Psi^0 f^\dagger f^0 \lambda^5 \tilde{f} \psi^0 \tilde{f}^0 + \overline{H} \Psi^0 f^\dagger f^0 \lambda^5 \tilde{f} \psi^0 \tilde{f}^0 \]

- effective 4D Yukawa couplings
  \[ \lambda^4 = f^\dagger f^0 \lambda^4, \quad \lambda^4 = f^\dagger f^0 \lambda^4 \]

- diagonalized by the following chiral rotations:
  \[ \Psi^0 \rightarrow V \Psi^0, \quad \psi^0_u \rightarrow W_u \psi^0_u, \quad \psi^0_d \rightarrow W_d \psi^0_d \]

- in mass eigenstates of SM fermions: fermion-gauge couplings
  \[ \sum_{\Psi^0} G^m (\Psi^0 f^\dagger f^0 V \Psi^0 + \psi^0 f^\dagger f^0 W_u \psi^0 + \psi^0 W_d \psi^0) \]

- non-universal \( f \): leads to tree-level FCNCs

- RS-GIM mechanism:
  - light fermions close to Planck brane
  - gauge zero-mode flat
  - somewhat alleviate flavor constraints, though not enough
Flavor Violations in Quark Sector

- anarchical flavor structure:
  - in diagonal bulk mass basis, 5D Yukawa matrices have no structures, i.e. all elements of the same order

Contributions to $\Delta F = 2$ processes from KK gluon exchange.

taken from Agashe, Perez, Soni, 2004
Bounds on Cutoff Scale

Csaki, Falkowski & Weiler, 2008

\[ \Lambda > 21 \text{ TeV} \]

Generally: \( \Lambda > O(10) \text{ TeV} \)
Lepton Flavor Violations

Contributions from FCNCs:
- presence even in the limit of massless neutrinos
- at tree level:
  - tri-lepton decays: $\mu \rightarrow 3\ e$, etc.
  - $\mu$-e conversion

Contributions from charged currents:
- at one-loop:
  - $\mu \rightarrow e + \gamma$

[Agashe, Blechman and Petriello 2006]
Lepton Flavor Violation

Agashe, Blechman, Petriello, 2006

- constraints on cutoff scale:

<table>
<thead>
<tr>
<th></th>
<th>Brane Higgs</th>
<th>$\nu = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(\mu \rightarrow 3e)$</td>
<td>2.5 TeV</td>
<td>2.0 TeV</td>
</tr>
<tr>
<td>$B_{\text{conv}}$</td>
<td>5.9</td>
<td>4.7</td>
</tr>
<tr>
<td>$BR(\tau \rightarrow 3\mu)$</td>
<td>0.40</td>
<td>0.33</td>
</tr>
<tr>
<td>$BR(\tau \rightarrow \mu ee)$</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>$BR(\tau \rightarrow 3e)$</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>$BR(\tau \rightarrow e\mu\mu)$</td>
<td>0.09</td>
<td>0.08</td>
</tr>
</tbody>
</table>

- tension between tree-level and one-loop processes:
  - opposite dependence on Yukawa couplings
  - tree-level FCNC $\sim \frac{1}{\lambda_{5D}}$
  - one-loop charged current contributions $\sim \lambda_{5D}^2$

<table>
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<tr>
<th></th>
<th>$Y = 1$</th>
<th>$Y = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\text{conv}}$</td>
<td>6.7 TeV</td>
<td>4.7 TeV</td>
</tr>
<tr>
<td>$BR(\mu \rightarrow e\gamma)$</td>
<td>8.0</td>
<td>15.8</td>
</tr>
</tbody>
</table>
**Minimal Flavor Violation**

D'Ambrosio, Giudice, Isidori, Strumia, 2002

- assume Yukawa couplings the only source of flavor violation
- SM: absence of Yukawa couplings (with massless neutrinos)

\[
G_F \equiv \text{SU}(3)_q^3 \otimes \text{SU}(3)_\ell^2 \quad \text{SU}(3)_q^3 = \text{SU}(3)_{Q_L} \otimes \text{SU}(3)_{U_R} \otimes \text{SU}(3)_{D_R} \quad \text{SU}(3)_\ell^2 = \text{SU}(3)_{L_L} \otimes \text{SU}(3)_{E_R}.
\]

- promote Yukawa couplings to be auxiliary fields

\[
Y_U \sim (3, \bar{3}, 1)_{\text{SU}(3)_q^3}, \quad Y_D \sim (3, 1, \bar{3})_{\text{SU}(3)_q^3}, \quad Y_E \sim (3, \bar{3})_{\text{SU}(3)_\ell^2}
\]

- can rotate to

\[
Y_D = \lambda_d, \quad Y_L = \lambda_\ell, \quad Y_U = V^\dagger \lambda_u \quad \text{V: CKM matrix}
\]

- effects of flavor violation:

\[
(\lambda_{FC})_{ij} = (Y_U Y_U^\dagger)_{ij} \approx \lambda_\ell^2 V_{3i}^* V_{3j} \quad i \neq j
\]
Minimal Flavor Violation

- bounds on cutoff scale loosened

<table>
<thead>
<tr>
<th>Minimally flavour violating dimension six operator</th>
<th>main observables</th>
<th>( \Lambda ) [TeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_0 = \frac{1}{2}(Q_L\lambda_{FC}\gamma_\mu Q_L)^2 )</td>
<td>( \epsilon_K, \Delta m_{B_s} )</td>
<td>6.4 5.0</td>
</tr>
<tr>
<td>( O_{F1} = H^\dagger(D_R\lambda_{dFC}\sigma_{\mu\nu}Q_L)F_{\mu\nu} )</td>
<td>( B \to X_s\gamma )</td>
<td>9.3 12.4</td>
</tr>
<tr>
<td>( O_{G1} = H^\dagger(D_R\lambda_{dFC}\sigma_{\mu\nu}T^aQ_L)G_{\mu\nu}^a )</td>
<td>( B \to X_s\gamma )</td>
<td>2.6 3.5</td>
</tr>
<tr>
<td>( O_{r1} = (Q_L\lambda_{FC}\gamma_\mu Q_L)(\tilde{L}<em>L\gamma</em>\mu L_L) )</td>
<td>( B \to (X)\ell\bar{\ell}, \ K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell} )</td>
<td>3.1 2.7</td>
</tr>
<tr>
<td>( O_{r2} = (Q_L\lambda_{FC}\gamma_\mu \tau_\mu Q_L)(\tilde{L}<em>L\gamma</em>\mu \tau_\mu L_L) )</td>
<td>( B \to (X)\ell\bar{\ell}, \ K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell} )</td>
<td>3.4 3.0</td>
</tr>
<tr>
<td>( O_{H1} = (Q_L\lambda_{FC}\gamma_\mu Q_L)(H^\dagger iD_\mu H) )</td>
<td>( B \to (X)\ell\bar{\ell}, \ K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell} )</td>
<td>1.6 1.6</td>
</tr>
<tr>
<td>( O_{g5} = (Q_L\lambda_{FC}\gamma_\mu Q_L)(D_R\gamma_\mu D_R) )</td>
<td>( B \to K\pi, \ \epsilon'/\epsilon, \ldots )</td>
<td>( \sim 1 )</td>
</tr>
</tbody>
</table>

- in RS model:
  - the MFV assumption relates 5D Yukawa matrices & bulk mass terms
  - implementation in quark sector

D’Ambrosio, Giudice, Isidori, Strumia, 2002
Fitzpatrick, Perez, Randall, 2007
MFV in Lepton Sector -- Massless Neutrinos

Massless neutrino case:

- relevant 5D Lagrangian

\[ \mathcal{L}_5^{\text{lep}} \supset \overline{L} C_L L + \overline{e} C_e e + \overline{H} \overline{L} Y_e e \]

- Implementation of MFV:
  - only sources of flavor violation are Yukawa couplings

\[ C_e = a Y_e \dagger Y_e, \quad C_L = b Y_e Y_e \dagger \]

- parameters a & b: O(1) proportionality constants

- MFV allows simultaneous diagonalization of \( C_e, C_L \) and \( Y_e \)
MFV in Lepton Sector -- Massless Neutrinos

- simultaneous diagonalization:
  - field transformations
    \[ L \rightarrow V L, \ e \rightarrow W e \quad \text{thus} \quad V^\dagger Y_e W \rightarrow \hat{Y}_e \]
  \[ V^\dagger Y_e Y_e^\dagger V \rightarrow \hat{Y}_e \hat{Y}_e^\dagger, \quad W^\dagger Y_e Y_e W \rightarrow \hat{Y}_e \hat{Y}_e \]
- diagonal 5D Yukawa: \[ \hat{Y}_e = \text{diag}(Y_{e_1}, Y_{e_2}, Y_{e_3}) \]
- diagonal bulk masses: \[ C_e = a\hat{Y}_e^\dagger \hat{Y}_e \ \text{and} \ C_L = b\hat{Y}_e \hat{Y}_e^\dagger \]

- NO tree-level FCNCs
- intrinsically different from the anarchical assumption
\[ \hat{Y}_e = \text{diag}(Y_{e_1}, Y_{e_2}, Y_{e_3}) \]
MFV in Lepton Sector -- Massless Neutrinos

- charged lepton masses

\[ m_l \simeq v F_L Y_e F_e \]

\( F_L \) and \( F_e \) are the values of the zero-mode profiles on the TeV brane.

- eigenvalues of \( F_L \) and \( F_e \):

\[ f_{L_i} = \sqrt{\frac{1 - 2c_{L_i}}{1 - \epsilon^{1-2c_{L_i}}}} \]
\[ f_{e_i} = \sqrt{\frac{1 - 2c_{e_i}}{1 - \epsilon^{1-2c_{e_i}}}} \]

\[ \epsilon = e^{-\pi k r_c} \simeq 10^{-15} \]

\( c_{L_i} \) and \( c_{e_i} \) are eigenvalues of the 5D bulk mass \( C_L \) and \( C_e \).
MFV in Lepton Sector -- Massless Neutrinos

- numerical results:

\[ Y_e = \text{diag}(Y_{e1}, Y_{e2}, Y_{e3}), \]
\[ C_L = \text{diag}(b|Y_{e1}|^2, b|Y_{e2}|^2, b|Y_{e3}|^2) \]
\[ C_e = \text{diag}(a|Y_{e1}|^2, a|Y_{e2}|^2, a|Y_{e3}|^2). \]

\[ a = 1 \text{ and } b = 1 \]

\[ Y_{e1} \approx 0.816, \ Y_{e2} \approx 0.759 \text{ and } Y_{e3} \approx 0.720, \]

- resulting charged lepton masses

\[ m_e \approx 0.511 \text{ MeV}, \ m_\mu \approx 105.6 \text{ MeV and } m_\tau \approx 1.77 \text{ GeV} \]
MFV in Lepton Sector -- Massive Neutrinos

M.-C.C, H.B. Yu, 2008

Massive neutrino case:

- introduce three RH neutrinos
- small Dirac neutrino masses by localizing RH neutrinos toward Planck brane
- relevant 5D Lagrangian

\[ \mathcal{L}_{5D}^{\text{lep}} \supset \bar{L} C_L L + \bar{r} C_r e + \bar{N} C_N N + \bar{H} \bar{L} Y_e e + H \bar{L} Y_\nu N \]

- Implementation of MFV:
  - only sources of flavor violation are Yukawa couplings

\[ C_e = a Y^\dagger e, \quad C_N = d Y^\dagger \nu, \quad C_L = c(\xi Y^\dagger \nu + Y_e Y^\dagger) \]

- parameters a & c & d: O(1) proportionality constants
MFV in Lepton Sector -- Massive Neutrinos

- can rotate to basis where either $Y_e$ or $Y_\nu$ is diagonal
- without loss of generality: work in $Y_e$ diagonal basis $Y_e = \hat{Y}_e$

$$ Y_\nu = V_{5D} \hat{Y}_\nu, \quad V_{5D} : \ 5D \text{ Leptonic Mixing Matrix} $$

- in this basis, both $C_N$ and $C_e$ are diagonal, but not $C_L$

$$ \hat{C}_N \equiv d\hat{Y}_\nu \hat{Y}_\nu^\dagger \text{ and } \hat{C}_e \equiv a\hat{Y}_e \hat{Y}_e^\dagger $$

$$ C_L \simeq (\xi V_{5D} \hat{C}_N V_{5D}^\dagger + \hat{C}_e) $$

- eigenvalues of $C_L$: zero mode localizations of SU(2) doublets
- leads to a set of constraints on 5D bulk mass parameters
MFV in Lepton Sector -- Massive Neutrinos

- non-diagonal term - $\xi$ in $C_L$: source of FCNC in charged lepton sector
- contributions to FCNC: depends on $\xi$
- $V_{5D}$ unknown: taking the trace:
  \[
  \text{Tr}(C_L) \simeq c(\xi \text{Tr}(C_N) + \text{Tr}(C_e))
  \]
- small neutrino masses $\Rightarrow$ small $\xi$
- realistic charged lepton masses:
  \[C_{L_i}, C_{e_i} \sim (0.4 - 0.6)\]
- small neutrino masses: RH neutrinos close to Planck brane
  \[C_{N_i} \sim (1.2 - 1.5)\]
- The trace relation then implies \(\xi \sim (0 - 0.1)\)
- FCNC contributions suppressed by \(\xi^2 \sim O(0 - 10^{-2})\)
Charged Current Contributions

- in the presence of massive neutrinos:
  - charged current contributions to LFV
- MFV does not suppress charged current contributions to LFV
Numerical Results (massive neutrino case)

\[ \xi \approx 0 \quad a = c = d = 4. \]

\[ Y_{e_1} \approx 0.405, \quad Y_{e_2} \approx 0.375 \quad Y_{e_3} \approx 0.354, \]

\[ \theta_{12} \approx 1.383, \quad \theta_{23} \approx 1.358, \quad \theta_{13} \approx 1.338, \]

\[ Y_{\nu_1} \approx 0.713, \quad Y_{\nu_2} \approx 0.5634 \text{ and } Y_{\nu_3} \approx 0.5475, \]

\[ \hat{Y}_\nu = \text{diag}(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) \quad Y_\nu \equiv V_{5D} \hat{Y}_\nu \approx \begin{pmatrix} 0.0307 & 0.128 & 0.533 \\ -0.275 & -0.504 & 0.123 \\ 0.657 & -0.217 & 0.0267 \end{pmatrix}. \]

- resulting masses and mixing angles:

\[
\begin{align*}
\sin^2 \theta_{12} & \approx 0.28, \\
\sin^2 \theta_{23} & \approx 0.49, \\
\sin^2 \theta_{13} & \approx 0.023 \\
\Delta m^2_{21} & \approx 7.4 \times 10^{-5} \text{eV}^2, \\
\Delta m^2_{31} & \approx 2.7 \times 10^{-3} \text{eV}^2
\end{align*}
\]

- for \( \text{Br}(\mu \rightarrow e\gamma) \approx 10^{-12} \) 1st KK mass - 3 TeV allowed
Comments

- mild hierarchy among 5D parameters: $\sim O(25)$ needed for large neutrino mixing

$$Y_{\nu} \equiv V_{5D} \hat{Y}_{\nu} \simeq \begin{pmatrix} 0.0307 & 0.128 & 0.533 \\ -0.275 & -0.504 & 0.123 \\ 0.657 & -0.217 & 0.0267 \end{pmatrix}.$$  

generic anarchy case:

$$V_{ij} \sim f_{L_i}/f_{L_j}$$

large atm & solar mixing angles: $f_{L_1}/f_{L_2} \sim 1$ and $f_{L_2}/f_{L_3} \sim 1$.

- MFV with $\xi = 0$, $f_{L_i}/f_{L_j}$ is fixed by $\sqrt{m_i/m_j}$

$$f_{L_1}/f_{L_2} \simeq 0.07 \quad f_{L_2}/f_{L_3} \simeq 0.24.$$  

- some structure in 5D Yukawa needed to accommodate mixing angles and mass ratios simultaneously
Comments

- counting the number of independent parameters that determine 5D Yukawa and bulk masses in lepton sector: (without CPV)
  - anarchical case: 27 parameters
    - $3 \times 3$ (diagonal $C$) + $2 \times 9$ (general $Y$)
      \[
      \mathcal{L}_{5D}^{\text{lep}} \supset \overline{L}C_LL + \overline{\nu}C_{\nu} + \overline{N}C_{N}N + \overline{HLY_{\nu}}e + HLY_{\nu}N
      \]
  - MFV: 12 parameters
    - $3 \times 3$ (eigenvalues of $Y$) + $4$ (prop. constants) - $1$ (trace relation)
      \[
      Y_{\nu} = V_{5D} \hat{Y}_{\nu},
      \]
      \[
      \hat{C}_{N} \equiv d\hat{Y}_{\nu}\hat{Y}_{\nu}^{\dagger} \quad \text{and} \quad \hat{C}_{e} \equiv a\hat{Y}_{e}\hat{Y}_{e}^{\dagger}
      \]
  - allow suppression in FCNC & charged current contributions to LFV
    - not possible in anarchical case, due to opposite dependence on 5D Yukawas
Conclusions

- RS model: provides novel way to generate fermion mass hierarchy
  - additional sources of flavor violation: the bulk parameters
  - leads to tree-level FCNC

- with Minimal Flavor Violation:
  - tree-level FCNCs suppressed due to small neutrino masses
  - massless neutrino limit: NO tree-level FCNCs

- solutions exist that give realistic lepton masses and mixing angles
- 1st KK mass ~ 3 TeV allowed
  - viable solution to gauge hierarchy problem
  - testability at collider experiments