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TECHNICOLOUR

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Contents:

Abstract:

The ideas of Technicolour or Dynamical Symmetry Breaking for the weak interactions are reviewed. The need for technicolour is established because of the gauge hierarchy problem. The obvious phenomenological consequences are explored with emphasis on the possible existence of light (less than 100 GeV) scalar particles. The extended technicolour or sideways generalizations of technicolour are discussed. The relations between technicolour and grand unified theories, groups breaking themselves and vacuum alignment are also explored. Finally there is a discussion of technicolour and the possible composite structure of quarks and leptons.

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1. Introduction

According to current thinking, the spectrum of masses of particles originates in a hierarchy of symmetry breaking scales [1]. At the lowest energies the world appears exactly symmetric under a symmetry group $SU(3) \times U(1)$, i.e., the strong interaction colour symmetry and the symmetry of electromagnetism. Various additional approximate symmetries such as isospin are also manifest but these appear to be accidental consequences of the mass spectrum. For example it is believed that the origin of isospin and chiral symmetry is the fact that the u and d quarks happen to be a good deal lighter than the scale Λ_{QCD} of quantum chromodynamics [2].

Nevertheless the real symmetry of the laws of nature is probably much bigger than $SU(3) \times U(1)$ but most of this symmetry is hidden by spontaneous breakdown. The spontaneous breakdown of a symmetry is a phenomenon which occurs at a characteristic scale and it is these symmetry breaking scales which appear to control the particle spectrum. Typically, the spontaneous breakdown of a symmetry is a low energy phenomenon in the sense that amplitudes involving momenta larger than the characteristic breakdown scale appear almost symmetric while below this scale amplitudes are grossly asymmetric. Thus Nature can be viewed in terms of a hierarchy of spontaneously broken symmetries. As the energy increases the size of the apparent symmetry group always increases [3].

For example, at energies in excess of a few hundred GeV the standard Weinberg–Salam [4] electroweak theory predicts that the manifest symmetry expands from $SU(3)_{QCD} \times U(1)_{EM}$ to $SU(3)_{QCD} \times \{SU(2) \times U(1)\}_{EW}$. Speculations about a further increase to SU(5) at $\sim 10^{15}$ GeV are very popular [5].

Typically the breakdown of a symmetry is signalled by an *order parameter*. An order parameter is a non-vanishing vacuum expectation value of some local field which transforms non-trivially under the symmetry group. For example in massless QCD with two flavours u, d the order parameter is

$$\langle \bar{u}u + \bar{d}d \rangle \neq 0 \tag{1.1}$$

which breaks chiral $SU(2) \times SU(2)$. In the standard $SU(2)_{LEFT} \times U(1)_{HYP}$ electroweak theory [4] the symmetry breaking order parameter is $\langle \varphi \rangle$, the expectation value of the Higgs doublet.

The order parameter is generally a dimensional quantity. Thus in QCD the *condensate* $\langle \bar{u}u + \bar{d}d \rangle$ has the dimensions of mass cubed while the Higgs field in the Weinberg–Salam theory has dimensions of mass. The value of the order parameter is determined by the symmetry breaking scale. In QCD $\langle \bar{u}u + \bar{d}d \rangle \sim (250\ \text{MeV})^3$ and in the Weinberg–Salam theory $\langle \varphi \rangle \sim (250\ \text{GeV})$.

An exact unbroken symmetry often implies certain definite properties or relationships in a theory. For example the equality of masses within a multiplet or, in the case of chiral symmetries, the vanishing of certain masses can result from unbroken symmetries. However, the spontaneous breakdown of the symmetry will usually cause violations of such relationships. Nevertheless, the fact that the symmetry breaking is a low energy phenomenon prevents the violations of symmetry relations from being arbitrarily large. Generally the masses or mass differences resulting from symmetry breakdown will be on the order of the breaking scale or smaller.

In this light it is interesting to realize that all the ordinary fundamental particles, quarks, leptons and intermediate vector bosons would have vanishing masses if the electroweak symmetry were unbroken. The fermions form left-handed $SU(2)_{LEFT}$ doublets and right-handed singlets. Fermion mass terms are products of left- and right-handed fields and cannot be $SU(2)_{LEFT}$ invariant. Fermion masses can only arise out of the $SU(2)_{LEFT}$ breakdown. Similarly the intermediate vector bosons $W^{\pm}$, Z being gauge

bosons must be massless in the absence of symmetry breaking. It follows that the masses of these particles should be on the order of or smaller than the symmetry breaking scale ~250 GeV. Any particle whose mass is in excess of 250 GeV must have an $SU(2)_{LEFT} \times U(1)_{HYP}$ allowed mass. We will say it is "unprotected" by $SU(2)_{LEFT} \times U(1)_{HYP}$. In fact, it is a reasonable expectation that any particle which is unprotected will have a mass determined by some larger scale and accordingly will be very heavy. This idea suggests that the presently observable spectrum is merely the "tip of the iceberg" with many unprotected masses at much higher scales.

Look carefully at the breaking of $SU(2)_{LEFT} \times U(1)_{HYP}$ and how it gives masses to the $W^{\pm}$, Z and the fermions. According to the standard theory [4] in addition to quark fields $q = (u, d)$, lepton fields $\ell = (\nu, e)$ and $SU(2)_{LEFT} \times U(1)_{HYP}$ gauge fields W^{α}, B there exists a scalar $SU(2)_{LEFT}$ doublet $\varphi = (\varphi^{\circ}, \varphi^{-})$. The $SU(2)_{LEFT} \times U(1)_{HYP}$ gauge symmetry is presumed to be spontaneously broken by the non-vanishing vacuum expectation value $\langle\varphi^{\circ}\rangle \simeq 250$ GeV. This order parameter sets the scale for all symmetry breaking phenomena including fermion and $W^{\pm}$, Z masses. Fermion masses are, of course, excluded from the Lagrangian by $SU(2)_{LEFT} \times U(1)_{HYP}$ invariance. However, Yukawa couplings of the form:

$$y_u \bar{q}_L \cdot \varphi u_R = y_u(\bar{u}_L u_R \varphi^{\circ} + \bar{d}_L u_R \varphi^{-})$$

and

$$y_d \bar{q}_L \cdot \tilde{\varphi} d_R = y_d(\bar{d}_L d_R \varphi^{\circ *} - \bar{u}_L d_R \varphi^{+}) \tag{1.2}$$

are allowed. ($\tilde{\varphi}_i$ is defined as $\varepsilon_{ij}\varphi_j^*$.) The breaking of $SU(2)_{LEFT} \times U(1)_{HYP}$ by $\langle\varphi^{\circ}\rangle$ implies that quarks move through the vacuum with effective masses:

$$m_u = y_u\langle\varphi^{\circ}\rangle, \qquad m_d = y_d\langle\varphi^{\circ}\rangle. \tag{1.3}$$

Here we have an example of protected particles receiving masses on the order of symmetry breaking scale.

Next consider the generation of mass of the electroweak gauge bosons. These particles, being gauge bosons, are also protected in the symmetry limit. To see how they gain mass, first consider the Higgs sector when the gauge couplings are turned off. The Lagrangian of the Higgs field,

$$\mathscr{L} = \partial_\mu \varphi^* \cdot \partial^\mu \varphi + V(\varphi^* \cdot \varphi) \tag{1.4}$$

is chosen so that the vacuum expectation value of φ° is ~250 GeV. The Lagrangian $\mathscr{L}$ has a four-dimensional rotation invariance which is made manifest by writing:

$$i\varphi^{-} = \varphi_1 + i\varphi_2, \qquad i\varphi^{\circ} = \varphi_3 + i\varphi_4. \tag{1.5}$$

So

$$\mathscr{L} = \partial_\mu \varphi_i \, \partial^\mu \varphi_i + V(\varphi_i \varphi_i). \tag{1.6}$$

The expectation value of φ may be taken as:

$$\langle\varphi_1\rangle = \langle\varphi_2\rangle = \langle\varphi_3\rangle = 0$$
$$\langle\varphi_4\rangle = F = 250\ \text{GeV} \tag{1.7}$$

which breaks O(4) down to O(3). This breaking requires three massless Goldstone bosons of charge $\pm 1, 0$ which transform as a three-vector under the unbroken O(3). Let us call these Goldstone bosons $\Pi^{\pm}$ and Π°.

These Goldstone bosons have non-vanishing couplings to the weak currents which couple to the $W^{\pm}$, W° and B. In particular

$$\langle 0|J_\mu^{\pm,\circ}|\Pi^{\pm,\circ}\rangle = Fq_\mu$$

and

$$\langle 0|J_\mu^{Y}|\Pi^{\circ}\rangle = Fq_\mu \tag{1.8}$$

where $J_\mu^{\pm,\circ}$ are the three $SU(2)_{LEFT}$ currents and J_μ^{Y} is the hypercharge current. The W's couple to the currents $J^{\pm,\circ}$ with strength $g_2/2$ and the B couples to J_μ^{Y} with strength $g_1/2$ so the diagrams in which gauge bosons turn into Goldstone bosons have the values $g_1F/2$ and $g_2F/2$ and are shown in fig. 1.

Now consider the propagator of the gauge bosons $W^{\pm}$. As in electrodynamics it can be expressed as

$$\Delta_{\mu\nu} = \frac{g_{\mu\nu} - q_\mu q_\nu/q^2}{q^2(1+\Pi(q^2))} \tag{1.9}$$

where $\Pi(q^2)$ is the vacuum polarization. The interesting term in $\Pi(q^2)$ comes from intermediate states involving a single Goldstone boson. Accordingly we write:

$$\Pi(q^2) = g_2^2F^2/4q^2 \tag{1.10}$$

where the pole in q^2 is due to the massless intermediate Goldstone boson. Evidently the occurrence of a massless pole in $\Pi(q^2)$ cancels the massless pole in $\Delta_{\mu\nu}$ and shifts it to [6]:

$$m_W^2 = \tfrac{1}{4}g_2^2F^2\,. \tag{1.11}$$

Thus we again see how the symmetry breaking scale F controls the mass of a protected particle.

The neutral gauge boson sector is slightly more complicated due to mixing between the W° and B. The problem is solved in terms of a mass matrix (see section 2), the entries of which come from the graphs in fig. 2. The result is the familiar superpositions of B and W°, the Z and photon. An important consequence of the standard Higgs mechanism is the empirically established relation:

$$\frac{m_Z}{m_W}\cos\theta_W = 1 + \theta(\alpha)\,. \tag{1.12}$$

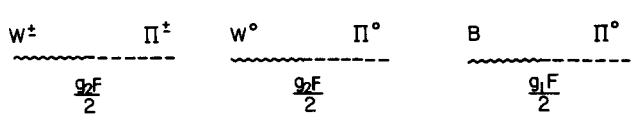

Fig. 1.

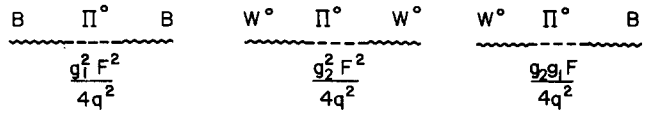

Fig. 2.

Certain aspects of the above scenario are unquestionably correct. The need for spontaneous breaking of $SU(2)_{LEFT} \times U(1)_{HYP}$ at a scale ~ 250 GeV is established. What is less clear is that the driving force which breaks the symmetry must be a canonical scalar $\lambda\varphi^4$ type theory. Fundamental scalars may only be part of a provisional low energy theory analogous to the σ model for low energy pion physics [7]. In both cases a symmetry is spontaneously broken and the low energy behaviour is adequately described by a phenomenological Lagrangian. Perhaps, as in pion physics, the fundamental Higgs bosons must eventually be replaced by much richer composite structures. This is the main assumption of the technicolour theory of dynamical symmetry breakdown.

In fact there is a very serious difficulty with the existence of fundamental scalar fields which is called the gauge hierarchy problem [8]. To explain this problem we must suppose that there exists a symmetry breaking scale at some very high energy such as the one that occurs in grand unified theories [1]. The problem which arises is how to arrange the parameters of the theory to ensure a gap of many orders of magnitude between the low and high energy symmetry breaking scales. Generally there is a very strong tendency for quantum effects to pull the two scales together unless the parameters of the theory are adjusted to absurd precision.

We shall illustrate this situation in the context of the SU(5) theory of Georgi and Glashow [5] but the reader should realize that it is very general. In the SU(5) theory two stages of symmetry breaking must occur, one at energies $\sim 10^{15}$ GeV which breaks SU(5) down to $SU(3) \times SU(2) \times U(1)$. This breakdown is usually accomplished with a 24-dimensional Higgs field Φ which must obtain a vacuum expectation value $\sim 10^{15}$ GeV. The second stage of breakdown from $SU(3) \times SU(2) \times U(1)$ down to $SU(3) \times U(1)_{EM}$ is initiated by a five-dimensional Higgs field φ with vacuum expectation value $\sim 10^2$ GeV. Thus we must account for a ratio of scales $\sim 10^{-13}$. How can this be done?

The obvious proposal is to construct a field potential $V(\Phi, \varphi)$ with minimum at

$$\langle \Phi \rangle = 10^{13} \langle \varphi \rangle . \tag{1.13}$$

For example

$$V(\Phi, \varphi) = \lambda_1 (\Phi^2 - \langle \Phi \rangle^2)^2 + \lambda_2 (\varphi^2 - \langle \varphi \rangle^2)^2 . \tag{1.14}$$

The difficulty is that quantum radiative corrections renormalize V and introduce couplings between φ and Φ. The simplest source of such corrections in SU(5) comes from the fact that both φ and Φ interact with the SU(5) gauge bosons. For example graphs like the one in fig. 3 give corrections to V of the form:

$$\sim \frac{g^4}{8\pi^2} \varphi^2 \Phi^2 . \tag{1.15}$$

The effect is devastating. The minimum of the potential is shifted from $\langle \Phi \rangle$, $\langle \varphi \rangle$ to $\sim \langle \Phi \rangle$, $(g^2/4\pi\lambda_2^{1/2}) \times$

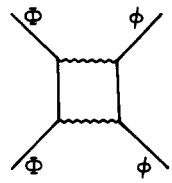

Fig. 3.

$\langle\Phi\rangle$. Thus instead of the desired ratio of 10^{-13} we get

$$\langle\varphi\rangle/\langle\Phi\rangle \sim g^2/4\pi\lambda_2^{1/2} \tag{1.16}$$

which is $\sim\alpha$.

To compensate for these radiative effects it is necessary, in each order, to go back and delicately retune the parameters in V. In the present case the required precision is one part in 10^{26}.

In contrast to this situation consider the scale for chiral symmetry breakdown in the QCD sector of the SU(5) theory. According to our current understanding of quantum field theory, the QCD coupling constant α_s is a function of the momentum scale satisfying [9]

$$\frac{\partial\alpha_s}{\partial \ln q^2} = \beta(\alpha_s) = \frac{-1}{4\pi}\left(\frac{11\cdot 3}{3}\right)\alpha_s^2 + \cdots . \tag{1.17}$$

When SU(5) breaks down to SU(3) × SU(2) × U(1) at 10^{15} GeV the coupling is small so the QCD sector experiences no non-perturbative effects at this scale. As the energy scale decreases α_s increases according to eq. (1.17) until it becomes large at the scale Λ_{QCD}. At this point non-perturbative effects presumably trigger chiral breakdown. The scale Λ_{QCD} can be found by integrating eq. (1.17) and finding the value of q^2 for which $\alpha_s \approx 1$. The result is

$$\Lambda_{QCD} \sim M \exp(-11/8\pi\alpha_0) \tag{1.18}$$

where M is the scale ($\sim 10^{15}$ GeV) at which SU(5) → SU(3) × SU(2) × U(1) and α_0 is the gauge coupling at the scale. The idea is shown in fig. 4 which depicts the evolution of the QCD coupling from the unification scale M down to the scale Λ_{QCD}.

The important point to note about eq. (1.18) is that with a rather ordinary value of α_0 the ratio Λ_{QCD}/M can easily and naturally be $\sim 10^{-15}$. It would obviously be desirable if the weak interaction scale at ~ 100 GeV could be generated in an equally natural way. This is the goal of the technicolour theory of symmetry breaking.

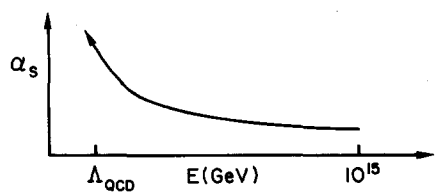

Fig. 4.

Now return to the case of symmetry breaking by fundamental scalars and to the radiative corrections to $V(\Phi, \varphi)$ illustrated in fig. 3. Another way to think about the diagram is as a self-energy correction to the φ field. We illustrate the effect in fig. 5 where the external Φ lines of fig. 3 are absorbed by the vacuum expectation value $\langle\Phi\rangle$. Evidently the diagram is a correction to the mass of φ which is of order $\langle\Phi\rangle^2$. The main contribution to this self-energy graph comes from large loop momenta of order $\langle\Phi\rangle$. If we can suppress the large loop momenta in fig. 5 so that momenta no larger than a few TeV could flow, then the induced correction to the mass of φ is innocuous. The problem is similar to obtaining a finite and small electromagnetic pion mass shift [10] when the simplest graph diverges. For the pion the solution lies in the form factors which result from the composite nature of hadrons. The same solution can work for the Higgs field φ. If it is a composite object bound of fermion pairs with an inverse radius about 10^3 GeV, then the radiative corrections from very short distances will not be important. This is the essential motivation for the technicolour [11] idea which is described in the following sections of this article.

Our article begins with the simplest technicolour model and ends with the most speculative ideas. Some of the claims made near the end of the article are based more on intuition than on calculation. However, we feel that all of the consequences of dynamical symmetry breakdown must be pursued to gain a true understanding of particle physics [12].

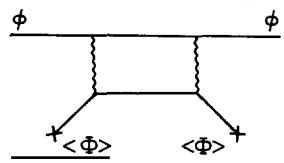

Fig. 5.

2. Technicolour

2.1. A simple model without scalars

In order to illustrate how the technicolour mechanism works we will start with a familiar system [13]. Our discussion follows closely the discussion of the Higgs phenomenon in section 1. Consider a massless doublet of quarks u and d interacting through ordinary QCD and neglect all other interactions. The Lagrangian of this theory has a $SU(2)_{LEFT} \times SU(2)_{RIGHT} \times U(1)$ symmetry. The U(1) is associated with baryon conservation and is irrelevant to this discussion. Although it has not been proven it is universally assumed that QCD spontaneously breaks the $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ symmetry down to $SU(2)_{ISOSPIN}$. Associated with this spontaneous breaking is the non-zero vacuum expectation value of the field operators

$$\langle \bar{u}u + \bar{d}d \rangle \neq 0 . \tag{2.1}$$

Again, these are often called *condensates* in analogy with similar phenomena in many-body physics. Note that the condensate is not invariant under $SU(2)_{LEFT}$ or $SU(2)_{RIGHT}$ but only under the group making equal left- and right-handed rotations, i.e., $SU(2)_{ISOSPIN}$.

The vacuum now has less symmetry than the Lagrangian. The Goldstone theorem [14] tells us that there will be a massless spin 0 boson for each broken generator. In this case this means three massless

pions, Π_a, forming an isotriplet. This picture is the basis for many of the successful low energy pion theorems [15].

An important quantity is the pion decay constant f_π defined by:

$$\langle 0|J^\mu_{5a}|\Pi_b\rangle = f_\pi q^\mu \delta_{ab} \tag{2.2}$$

where J^μ_{5a} are the three axial isospin currents. Note that f_π defined in this way is a purely strong interaction quantity independent of weak interactions. Its magnitude is controlled by the universal scale parameter of QCD, Λ_{QCD}. f_π has been measured through pion decays and it has the value:

$$f_\pi \sim 93\,\mathrm{MeV}\,. \tag{2.3}$$

The reason that each pion has the same f_π in eq. (2.2) is the unbroken isospin symmetry.

Let us complicate the picture. Imagine turning on the ordinary $\mathrm{SU(2)_{LEFT}} \times \mathrm{U(1)_{HYP}}$ electroweak interactions but without the fundamental scalar fields which are usually introduced to give mass to $W^\pm$ and Z. You might think that the spectrum of this theory includes four massless gauge bosons: W^+_μ, W^-_μ, W°_μ associated with $\mathrm{SU(2)_{LEFT}}$ and B_μ associated with hypercharge. This is not true [16]. In fact the massless pions of our previous discussion replace the conventional scalar fields and appear as longitudinal components of *massive* bosons $W^\pm$ and Z. The argument parallels the discussion of the previous section. Focus attention on the charged weak gauge boson propagator. We are particularly interested in the hadronic contributions to the vacuum polarization diagrams of fig. 6. Such graphs modify the propagator from

$$\frac{g^{\mu\nu} - q^\mu q^\nu/q^2}{q^2} \to \frac{g^{\mu\nu} - q^\mu q^\nu/q^2}{q^2(1+\Pi(q^2))}\,. \tag{2.4}$$

If $\Pi(q^2)$ is smooth near $q^2 = 0$ then the corrected propagator has a pole at $q^2 = 0$. However, in this case $\Pi(q^2)$ develops a pole due to massless pions. The boson $W^\pm_\mu$ has an interaction $\frac{1}{2}g_2 W^\mp_\mu J^\mu_{\pm 5}$ where g_2 is the $\mathrm{SU(2)_{LEFT}}$ gauge coupling constant. The current $J^\mu_{\pm 5}$ couples to the $\Pi^\mp$ with strength $f_{\pi^\pm}$ as in eq. (2.2). $\Pi(q^2)$ has a contribution

$$\Pi(q^2) \xrightarrow[q^2\to 0]{} \frac{g_2^2 f^2_{\pi^\pm}}{4q^2} \tag{2.5}$$

as is seen in fig. 7. The zero of the inverse propagator has shifted to $\frac{1}{4}g_2^2 f^2_{\pi^\pm}$ so the $W^\pm$ has a mass of $\frac{1}{2}g_2 f_{\pi^\pm}$.

For the neutral gauge bosons we need a mass matrix in the 2×2 spaces of B and W°. The entries can be read from the diagrams in fig. 8. The resulting matrix is:

$$m^2 = \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \frac{f^2_{\pi^\circ}}{4} \tag{2.6}$$

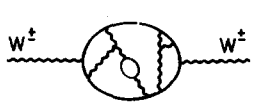

Fig. 6.

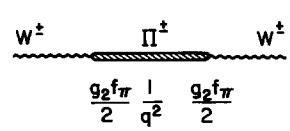

Fig. 7.

Fig. 8.

where g_1 is the U(1) coupling constant. The eigenvalues of this matrix are

$$\begin{aligned} m_\gamma^2 &= 0 \\ m_Z^2 &= \tfrac{1}{4}(g_1^2 + g_2^2) f_{\pi^\circ}^2 \,. \end{aligned} \tag{2.7}$$

The states which diagonalize this matrix are identified as the usual photon and weak neutral boson Z. Notice that the ratio

$$\frac{m_W}{m_Z} = \frac{g_2}{(g_1^2 + g_2^2)^{1/2}} \frac{f_{\pi^+}}{f_{\pi^\circ}} = \cos\theta_w \frac{f_{\pi^+}}{f_{\pi^\circ}} \tag{2.8}$$

where θ_w is the usual weak mixing angle defined in terms of coupling constants. We have already argued that $f_{\pi^+} = f_{\pi^\circ}$ from isospin conservation so we recover the empirically successful relation

$$m_W/m_Z = \cos\theta_w \,. \tag{2.9}$$

This relation is a tree level result in the ordinary Higgs picture. One may question its general validity if the Higgs sector is strongly interacting. The significance of the above argument is that the relation (2.9) follows from a symmetry of the strong interactions and is therefore only modified by electroweak radiative corrections [17]. (In ordinary weak interaction models with a Higgs boson there is an O(4) symmetry of the Higgs potential protecting this relation [18].)

We are not advocating this as a picture of the world. First, this theory has no true pion since it now only appears as one degree of freedom of a vector boson. Second, the masses of the $W^\pm$ and Z are in the tens of MeV's given the known values of g_1, g_2 and f_π. We have illustrated a point. The spontaneous symmetry breaking known to occur in the strong interactions has the same effect on the weak gauge boson sector as the carefully constructed Higgs sector of the standard theory. The difference is roughly a factor of 2000 in the symmetry breaking scales.

2.2. Technicolour

Let us suppose that an electroweak doublet of fermions exists which also engages in a new strong interaction called technicolour [19] (TC). We call this doublet

$$T = \begin{pmatrix} A \\ B \end{pmatrix} . \tag{2.10}$$

We make no assumption about the colour, baryon number or lepton number of this doublet. However, T_{LEFT} is a $SU(2)_{LEFT}$ doublet while A_{RIGHT} and B_{RIGHT} are $SU(2)_{LEFT}$ singlets.

These fermions feel the techniforce and we call them technifermions. The interaction is like QCD in

its essential aspects except that it becomes strong at a much higher energy, near the scale of weak interaction symmetry breaking [20]. The (A, B) doublet is just like the (u, d) doublet of the previous subsection except that colour is replaced by technicolour. All of the arguments of the previous section apply, in particular the $W^{\pm}$ and Z become massive but their masses are proportional to the *technipion decay constant* F_π:

$$m_W = \tfrac{1}{2} g_2 F_\pi . \tag{2.11}$$

Using the measured values of m_W and g_2 we get $F_\pi \sim 246$ GeV if we want these interactions to replace the usual Higgs sector.

Assuming that technicolour dynamics QTD is a scaled up version of QCD we get

$$\Lambda_{TC}/\Lambda_{QCD} \sim F_\pi/f_\pi \sim 2600 \tag{2.12}$$

giving $\Lambda_{TC} \sim 500$ GeV if $\Lambda_{QCD} \sim 200$ MeV. This solves the problem of getting a large mass for the weak gauge bosons. As before the technipion disappears from the spectrum.

We have replaced the usual Higgs sector of the standard electroweak theory with a strongly interacting set of fermions. The vacuum expectation value of φ at the minimum of the Higgs potential $\langle\varphi\rangle = V$ has been replaced by the strong interaction symmetry breaking parameter F_π. Numerically they are equal.

If we had more than one technicolour doublet of fermions, say r of them, then there would be three technipions for each doublet. If each technipion has the same decay constant F_π then eq. (2.2) is modified to

$$m_W^2 = \tfrac{1}{4} r g_2^2 F_\pi^2 . \tag{2.13}$$

This would lower the technicolour scale by $1/\sqrt{r}$. An example we consider later has $r = 4$ so $\Lambda_{TC} \sim 1300\, \Lambda_{QCD}$.

Now consider a world with both QCD and coloured quarks as well as QTD and technifermions interacting with the weak gauge bosons of $SU(2)_{LEFT} \times U(1)$. With QCD alone we argued that the ordinary pions become the longitudinal $W^{\pm}$ and Z and disappear while with QTD alone it is the technipions that have this role. In the combined picture the linear combination [21]:

$$|\text{pion absorbed}\rangle = \frac{F_\pi|\text{technipion}\rangle + f_\pi|\text{QCD pion}\rangle}{\sqrt{F_\pi^2 + f_\pi^2}} \tag{2.14}$$

becomes the longitudinal gauge boson components while the orthogonal combination:

$$|\text{physical pion}\rangle = \frac{F_\pi|\text{QCD pion}\rangle - f_\pi|\text{technipion}\rangle}{\sqrt{F_\pi^2 + f_\pi^2}} \tag{2.15}$$

remains a massless pion in the spectrum. Since $F_\pi \gg f_\pi$ the physical pion is mostly QCD pion while the absorbed pion is mostly technipion.

To see how these linear combinations arise note that:

$$\langle 0|J^5_\mu|\text{QCD pion}\rangle = f_\pi q^\mu \tag{2.16a}$$

and

$$\langle 0|J^5_\mu|\text{technipion}\rangle = F_\pi q^\mu \tag{2.16b}$$

where J^5_μ is the full axial vector current. These imply

$$\langle 0|J^5_\mu|\text{pion absorbed}\rangle = \sqrt{F^2_\pi + f^2_\pi}\, q^\mu \tag{2.17a}$$

$$\langle 0|J^5_\mu|\text{physical pion}\rangle = 0\,. \tag{2.17b}$$

The weak gauge bosons couple to the pions through these axial currents so the physical pion has no coupling while the orthogonal combination is totally absorbed. In the standard theory the same mixing occurs and the physical pion has a minute admixture of elementary scalar.

We believe that the above combination of QCD and QTD may be an accurate explanation of the spontaneous breakdown of electroweak symmetry. However, there is physics missing. In particular additional mechanisms are required to give the quarks and leptons masses (and therefore the physical pion). We will discuss these mechanisms in a later section.

The dynamics of technicolour closely mimic the dynamics of QCD. Therefore the technicolour interactions are probably controlled by a non-Abelian gauge theory with an associated group G. Imagine that technicolour is unified with QCD at some high energy between the weak interaction scale and the Planck mass. In order to have $\Lambda_{\rm TC}$ bigger than $\Lambda_{\rm QCD}$, the technicolour coupling constant must grow faster than the QCD coupling constant as you come down in energy from where they are unified. To accomplish this the group G should be bigger than SU(3) so the associated β function is more negative. [See eqs. (1.17) and (1.18).]

Suppose the technicolour group is SU(N) with $N > 3$. We can use the results of the $1/N$ expansion [22] to gain information about the N dependence of the parameters in the technicolour sector. For example the $1/N$ expansion tells us that F_π goes like $\sqrt{N}$ times the fundamental scale as N gets very large. So eq. (2.12) is replaced by:

$$\sqrt{\tfrac{1}{3}N}\,\Lambda_{\rm TC}/\Lambda_{\rm QCD} \sim F_\pi/f_\pi \tag{2.18}$$

where F_π comes from eq. (2.13)

$$F_\pi = \frac{2m_{\rm W}}{\sqrt{r}\,g_2} = \frac{246\ \text{GeV}}{\sqrt{r}}\,. \tag{2.19}$$

The technicolour scale $\Lambda_{\rm TC}$ gets smaller as N grows. We will use other results from the $1/N$ expansion when they are useful.

2.3. Technispectrocopy

Suppose there exists at least one technicoloured fermion doublet and that technicolour becomes strong in the hundreds of GeV's. What are the obvious phenomenological implications of this assumption?

2.3.1. Technihadrons

There exists a rich spectrum [23] of particles called technihadrons which are bound by the technicolour force. We are assuming that the technicolour force is a non-Abelian gauge force whose spectrum resembles the usual QCD spectrum rescaled by $\Lambda_{TC}/\Lambda_{QCD}$ given by eq. (2.18). Thus for example there may exist a techni-ρ and a techni-ω with masses ~ 1 TeV. The spacings and widths of these particles would also scale up and be $\sim 10^3$ times larger than their QCD counterpart's values. This contrasts sharply with the possibility of having higher mass families of ordinary quarks which would have narrow, closely spaced states. The technicolour spectrum would be easily visible in e^+e^- annihilation and in fig. 9 we show a possible picture of R up to 10 TeV.

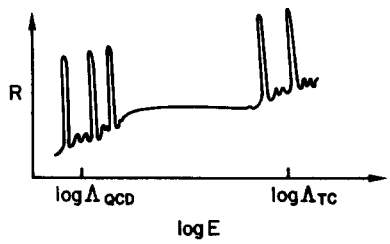

Fig. 9.

2.3.2. Longitudinal $W^\pm$ and Z

The $W^\pm$ and Z will have a strongly interacting component which will behave like a composite technihadron. This is because the longitudinal degree of freedom of these bosons is the composite technipion in disguise. This effect would show up in $e^+e^- \to W^+W^-$ cross-sections. The production of transverse $W^\pm$ would be the same as in the standard theory indicating a point-like $W^\pm$. However, the longitudinal production would show a complicated form factor including resonances.

The ordinary ρ likes to decay into pions. The techni-ρ with a mass near 1 TeV would like to decay strongly into W^+W^- pairs. This strong decay into weak bosons would be characteristic of technihadrons.

Jets of technihadrons could be produced in e^+e^- or $p\bar{p}$ collisions either through photons, weak bosons or gluons if some technifermions carry colour. These jets would look like ordinary hadronic jets but the mean transverse momentum would be scaled up by a factor of 10^3. These techni-jets could be copious sources of weak bosons.

2.3.3. Stable technibaryons

Among the technihadrons there can exist analogues of the baryons. If the technigroup is SU(N) these will consist of N technifermions [24] and carry a conserved technibaryon number of N if each technifermion carries one unit. These technibaryons will be fermions or bosons depending on whether N is odd or even. The mass of these particles can be estimated using scaling and large N techniques which tell us that the ratio of the mass of the baryon to the scale Λ grows as N. For example, the technicolour group SU(4) gives a mass of

$$\frac{4}{3}\frac{\Lambda_{TC}}{\Lambda_{QCD}} m_{\text{proton}} \sim \frac{2.8\,\text{TeV}}{\sqrt{r}} \tag{2.20}$$

to the technibaryon corresponding to the proton. Again r is the number of technidoublets contributing

to $W^{\pm}$ and Z masses. Like the ordinary proton, the lightest of these technibaryons will not be able to decay. However, there may exist technibaryon number violating interactions (discussed in section 4) which would allow this lightest technibaryon to decay.

2.4. Models and alternatives

Further details of the technispectrum require specific assumptions about the technigroup and the representations of the technifermions. We have no precise knowledge about what these should be but we will explore some reasonable general possibilities. In specific models the group properties of the technicoloured sector determine the pattern of light quark masses and this can be used as a guide. The search for the correct technigroup is an outstanding problem in this field.

Consider the $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$ quantum numbers of the technicoloured fermions. This group is anomaly free [25] in each family of ordinary fermions. By a standard family we mean a set of particles with the gauge quantum numbers of (up-quark, down-quark, electron, neutrino). We will require that the technicoloured fermions are in an anomaly free representation of $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$ so that the entire theory is anomaly free. Two possibilities should be considered immediately.

2.4.1. The one doublet model

Previously we introduced an illustrative example of two technifermions

$$T = \begin{pmatrix} A \\ B \end{pmatrix} \tag{2.21}$$

which form an $SU(2)_{LEFT}$ doublet but are colour singlets and transform as a non-trivial representation of technicolour. This is the minimal example which correctly breaks $SU(2)_{LEFT} \times U(1)_{HYP} \to U(1)_{EM}$. Suppose there are no other technifermions. Then in order to avoid the anomalies in fig. 10 which are

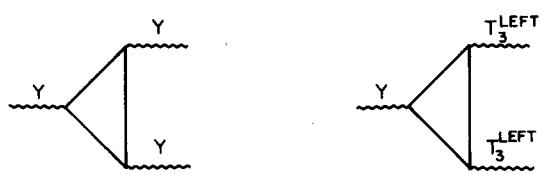

Fig. 10.

usually cancelled by other particles we make the hypercharge assignments:

$$Y = \begin{cases} 0 & T_{LEFT} \\ \frac{1}{2} & A_{RIGHT} \\ -\frac{1}{2} & B_{RIGHT}\,. \end{cases} \tag{2.22}$$

This leads to charges of $\frac{1}{2}$ and $-\frac{1}{2}$ for A and B. This model has a simple spectrum of technihadrons. It does not have any of the pseudo-Goldstone bosons which are abundant in the next model.

2.4.2. The one family model

Ordinary fermions appear to come in families. It is possible that the technicoloured fermions also

come in a standard family [26]. This would guarantee the cancellation of the $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$ anomalies. If one such family exists we could label it:

$$\begin{array}{cccc} \begin{pmatrix} U^\alpha_{red} \\ D^\alpha_{red} \end{pmatrix}_{LEFT} & \begin{pmatrix} U^\alpha_{blue} \\ D^\alpha_{blue} \end{pmatrix}_{LEFT} & \begin{pmatrix} U^\alpha_{yellow} \\ D^\alpha_{yellow} \end{pmatrix}_{LEFT} & \begin{pmatrix} N^\alpha \\ E^\alpha \end{pmatrix}_{LEFT} \\ (U^\alpha_{red})_{RIGHT} & (U^\alpha_{blue})_{RIGHT} & (U^\alpha_{yellow})_{RIGHT} & (N^\alpha)_{RIGHT} \\ (D^\alpha_{red})_{RIGHT} & (D^\alpha_{blue})_{RIGHT} & (D^\alpha_{yellow})_{RIGHT} & (E^\alpha)_{RIGHT}\,. \end{array} \tag{2.23}$$

The α is a technicolour index and we are assuming that all particles in this family have the same technitransformation properties. We have included a right-handed technineutrino which is a singlet under $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$. As will be seen this particle is necessary to ensure the relation $m_W/m_Z = \cos\theta_w$. We will refer to Q = (U, D) as techniquarks and L = (N, E) as technileptons.

As far as the weak interaction symmetry breaking is concerned, this model is like four-doublet models where the doublets are distinguished by the label red, blue, yellow, lepton. It is natural to assume that these doublets condense with equal strength so we get:

$$\begin{aligned} \langle \bar{U}_{red}\, U_{red}\rangle &= \langle \bar{D}_{red}\, D_{red}\rangle = \langle \bar{U}_{blue}\, U_{blue}\rangle = \langle \bar{D}_{blue}\, D_{blue}\rangle \\ &= \langle \bar{U}_{yellow}\, U_{yellow}\rangle = \langle \bar{D}_{yellow}\, D_{yellow}\rangle = \langle \bar{E}E\rangle = \langle \bar{N}N\rangle \neq 0\,. \end{aligned} \tag{2.24}$$

The factor of r in eq. (2.19) is 4 so the technicolour scale is $\sim 1300\sqrt{3/N}$ if technicolour is SU(N). If there were no right-handed technineutrino then in the technilepton sector there would not be the $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ symmetry which protects the relation $m_W/m_Z = \cos\theta_w$. Since this is a good symmetry in the techniquark sector the relation would be violated but not grossly.

This model has a rich spectrum of technihadrons with masses near 1 TeV. The techniforce would bind technifermions to anti-technifermions to make technimesons of spin 0, 1, 2 etc. Some of these would be coloured, for example there might be a spin 1 $\bar{U}E$ which would have charge $-\frac{5}{3}$ and be a colour triplet. Presumably it would be confined and form atoms with light quarks. If the technigroup is $SU(2n+1)$ we would have fermionic baryons. These could have unusual colour and charges as well. The physics at this scale is rich and worth exploring. However, we believe that the spin 0 mesons might have masses much below 1 TeV, some possibly below 100 GeV. Some of these particles might be visible in the next generation of accelerators. For this reason we devote the next sections to the spin 0 sector of this model.

2.4.3. Other models

We have discussed two simple models. The list of possibilities is endless. We urge the reader to explore all the variations until he or she finds the one that does it all.

3. Pseudo-Goldstone bosons

The existence of more than one technidoublet can lead to massless or light (on the technicolour scale) spin 0 particles [27]. In this section we illustrate how this occurs in the one family model. There is a rich spectrum of particles some of which may have masses in the tens of GeV's. New interactions just

above the technicolour scale could raise these masses to the hundreds of GeV's. These particles have unusual production and decay characteristics.

Our discussion of the spectrum of pseudos is almost completely dependent on the model we have chosen to analyze. There is no compelling reason to believe in this model except that it is a natural extension of low energy ideas.

3.1. Goldstone bosons and pseudo-Goldstone bosons

Suppose you have a theory which at the Lagrangian level is invariant under some internal global symmetry group G. If the ground state vacuum is invariant under a smaller group H, then for each group generator of G not in H you have a massless spin 0 particle. These are called Goldstone bosons and this result is the Goldstone theorem [14]. If G is an approximate symmetry then the Goldstone bosons are only approximately massless. The masses go to zero with the symmetry breaking parameter. These nearly massless particles are called pseudo-Goldstone bosons.

As an example consider the massless u, d quarks in QCD. This was discussed in section 2.1. The theory is $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ invariant but the vacuum is only $SU(2)_{ISOSPIN}$ invariant:

$$\langle \bar{u}u + \bar{d}d \rangle \neq 0 . \tag{3.1}$$

Corresponding to each broken generator (the three axial SU(2) generators), there is a massless pion.

In fact, $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ is only an approximate symmetry of the real world. Besides electromagnetism there are quark masses of the form

$$m_u \bar{u}u + m_d \bar{d}d \tag{3.2}$$

which violate $SU(2)_{LEFT} \times SU(2)_{RIGHT}$. The pions develop masses whose squares are proportional to m_u and m_d. The real pion is a pseudo-Goldstone boson.

3.2. Pseudos in the one family model

Consider the one family model of section 2.4.2 and ignore all forces except technicolour. This is a good approximation near the technicolour scale where colour and electroweak forces are small. There are eight left-handed and eight right-handed fields which have identical technicolour transformation properties. These particles can be labelled up-red, up-blue, up-yellow, down-red, down-blue, down-yellow, electron, neutrino in both the left- and right-handed lists. The technifermions are massless so this theory (again neglecting colour and electroweak forces) is $SU(8)_{LEFT} \times SU(8)_{RIGHT}$ invariant.

The technicolour force causes a spontaneous symmetry breakdown of the vacuum at the technicolour scale. We assume the condensates have the form of eq. (2.24). With each component condensing equally the vacuum is still $SU(8)_{VECTOR} = SU(8)_{LEFT} + SU(8)_{RIGHT}$ invariant. The symmetry has been reduced from $SU(8)_{LEFT} \times SU(8)_{RIGHT}$ to $SU(8)_{VECTOR}$ so there are $2 \times 63 - 63 = 63$ massless Goldstone bosons.

Most of these 63 particles are not strictly massless. If you no longer ignore $SU(3)_{COLOUR} \times SU(2) \times U(1)$ then the original $SU(8)_{LEFT} \times SU(8)_{RIGHT}$ symmetry is only approximate. Most of the Goldstone bosons are pseudo-Goldstone bosons whose masses vanish as colour and electroweak forces are turned off. In fact there are probably other forces which break the $SU(8)_{LEFT} \times SU(8)_{RIGHT}$ and these will be discussed in the next section. We now explore the properties of the 63 real and pseudo-Goldstone bosons.

3.3. The Goldstone spectrum

Neglecting the colour and electroweak forces the vacuum is invariant under the SU(8) vector symmetry $SU(8)_{LEFT} \times SU(8)_{RIGHT}$. For each of the axial SU(8) generators there is a Goldstone boson Π^a. These couple to the currents:

$$J^{\mu}_{5a} = \bar{F}\gamma^{\mu}\gamma_5 t_a F \tag{3.3a}$$

where

$$F = \begin{pmatrix} U_{red} \\ U_{blue} \\ U_{yellow} \\ D_{red} \\ D_{blue} \\ D_{yellow} \\ N \\ E \end{pmatrix} \tag{3.3b}$$

and the t_a are the 63 traceless SU(8) generators normalized so that

$$\mathrm{Tr}(t^a t^b) = \tfrac{1}{2}\delta^{ab}\,. \tag{3.4}$$

The Hermitean fields Π^a couple to the currents with strength F_π

$$\langle \Pi^a | J^{\mu}_{5b} | 0 \rangle = F_\pi q^\mu \delta^a_b\,. \tag{3.5}$$

Each Π^a has the same F_π because of the residual SU(8) vector symmetry.

In order to distinguish between the pseudos and the true Goldstone bosons it is convenient to choose a basis for the t_a matrices which exhibits the $SU(3)_{COLOUR}$, the isospin and the charge of the states. Let Q^c and L be techniquark and technilepton doublets given by

$$c = \text{red, blue, yellow} \qquad Q^c = \begin{pmatrix} U \\ D \end{pmatrix}^c$$

$$L = \begin{pmatrix} N \\ E \end{pmatrix}. \tag{3.6}$$

Let τ^i be the three isospin matrices acting on techniquark and technilepton doublets and let λ^a be the octet of colour matrices acting on colour indices. The 63 linear combinations of Π^a's which have definite colour, charge and isospin can be written as:

$$\theta^i_a \sim \bar{Q}\gamma_5\lambda_a\tau^i Q, \qquad a = 1, 8; \quad i = 1, 3 \tag{3.7a}$$

$$\theta_a \sim \bar{Q}\gamma_5\lambda_a Q, \qquad a = 1, 8 \tag{3.7b}$$

$$T^i_c \sim \bar{Q}^c\gamma_5\tau^i L; \qquad \bar{T}^i_c \sim \bar{L}\gamma_5\tau^i Q^c \tag{3.7c}$$

c = red, blue, yellow

$$T_c \sim \bar{Q}^c \gamma_5 L; \qquad \bar{T}_c \sim \bar{L}\gamma_5 Q^c \tag{3.7d}$$

$$\Pi^i \sim (\bar{Q}\gamma_5\tau^i Q + \bar{L}\gamma_5\tau^i L) \tag{3.7e}$$

$$P^{\pm} \sim (\bar{Q}\gamma_5\tau^{\pm} Q - 3\bar{L}\gamma_5\tau^{\pm} L) \tag{3.7f}$$

$$P^3 \sim (\bar{Q}\gamma_5\tau^3 Q - 3\bar{L}\gamma_5\tau^3 L) \tag{3.7g}$$

$$P^0 \sim (\bar{Q}\gamma_5 Q - 3\bar{L}\gamma_5 L)\,. \tag{3.7h}$$

The states couple to their respective currents with strength F_π. We will now discuss the properties of these states in turn.

3.3.1. *The octet pseudos*

These are the particles called θ_a^i and θ_a where the θ_a and θ_a^3 are electrically neutral and $\theta_a^{\pm}$ are charged. They are all colour octets. When we take into account colour SU(3) forces (as well as weak interactions) these particles are no longer massless. The main contribution to their mass comes from one gluon exchange as in fig. 11. A simple estimate of the mass can be made by scaling arguments applied to the electromagnetic mass difference of ordinary pions. In the case of the pions single photon exchange gives the difference between the squares of masses [10] $m_{\pi^\pm}^2 - m_{\pi^0}^2$. The gluon exchange in the case of the octet pseudo is structurally identical except for a different coupling constant and a colour group theory factor. Therefore

$$\frac{m_\theta^2}{m_{\pi^\pm}^2 - m_{\pi^0}^2} = \frac{\Lambda_{\mathrm{TC}}^2}{\Lambda_{\mathrm{QCD}}^2} \cdot \frac{\alpha_{\mathrm{strong}}(\Lambda_{\mathrm{TC}})}{\alpha_{\mathrm{em}}} \cdot \frac{3}{1}\,. \tag{3.8}$$

The factor $(\Lambda_{\mathrm{TC}}/\Lambda_{\mathrm{QCD}})^2$ arises because the bound state structure, form factors etc., are defined by the TC scale instead of the QCD scale. The 3 is the colour Casimir for the octets. This model has four doublets so $r = 4$ in eq. (2.19) and using a value of $\alpha_s(\Lambda_{\mathrm{TC}}) \sim 0.1$ we get

$$m_\theta \sim \sqrt{\frac{4}{N}}\, 260\ \mathrm{GeV} \tag{3.9}$$

where the technicolour group is SU(N).

Bjorken and others [28] have estimated this mass using the current algebra techniques which work well for the ordinary pions and their estimates agree with ours. Other interactions will contribute to the mass of the octet but are probably smaller. Equation (3.9) is certainly a lower bound on the octet mass.

The colour octets $\theta_a^{\pm}$, θ_a^3 form a techni-isospin triplet while the θ_a is a techni-isospin singlet. The triplet is called a colour octet technipion while the singlet is called a colour octet techni-eta. Ordinary quarks, leptons and gluons are techni-isospin singlets while the electroweak interactions violate

Fig. 11.

techni-isospin. Thus we expect decays like

$$\begin{aligned}\theta_a &\to \bar{q}, \bar{q}\\ &\to \text{gluon, gluon}\\ &\to \text{gluon}, \gamma\\ &\to \text{gluon, Z}\\ &\to \text{gluon, gluon, gluon etc.}\end{aligned} \tag{3.10a}$$

$$\begin{aligned}\theta_a^3 &\to \bar{q}, q\\ &\to \text{gluon}, \gamma\\ &\to \text{gluon, Z etc.}\end{aligned} \tag{3.10b}$$

$$\begin{aligned}\theta_a^{\pm} &\to \bar{q}, q\\ &\to \text{gluon}, W^{\pm} \text{ etc.}\end{aligned} \tag{3.10c}$$

With the physics so far introduced the direct decay of a spin 0 boson into a fermion–antifermion pair is forbidden. To see this note that in the Lagrangian the light fermions are only coupled through the axial and vector gauge couplings of $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$. These couplings are invariant under the chiral operation:

$$\psi \to \gamma_5 \psi . \tag{3.11}$$

Furthermore this symmetry of light fermions is not broken by the technicolour condensate. As in the case of the ordinary pion, a spin 0 particle cannot decay into a fermion pair without violating chirality. The same chirality invariance prevents the light fermions from getting a mass and must be violated by other interactions not yet discussed [29]. These interactions will allow the pseudo-Goldstone bosons to decay into fermion–antifermion pairs and as in the case of the ordinary pion the amplitude is proportional to the mass of the fermion. By dimensional considerations the coupling is then

$$g_{Pf\bar{f}'} \sim (m_f + m_{f'})/F_\pi . \tag{3.12}$$

[Actually in specific models the coupling may go like the mass difference but we will work with (3.12).]

Given this coupling you see that the octet pseudos prefer to decay into heavy quarks. (Lepton pairs are not allowed because they cannot make a colour octet.) The rate is roughly [30]

$$\Gamma(\theta \to f\bar{f}') \sim \tfrac{1}{2} g^2_{Pf\bar{f}'} \frac{m_\theta}{8\pi} \tag{3.13}$$

where an extra $\frac{1}{2}$ is included as a colour group theory factor. For example the rate for $\theta \to b\bar{b}$ would be $\sim$30 MeV given eq. (3.9) with $N = 4$. This may well be the dominant decay mode of the colour octet pseudos.

The coupling of an octet pseudo to two gluons or two other vector fields can be calculated in a manner similar to the calculation of $\Pi^0 \to \gamma\gamma$. Various groups [31] have calculated the rate for $\theta_a \to g_b + g_c$ and they get

$$\Gamma(\theta_a \to g_b g_c) = \frac{5}{384} \frac{\alpha^2_{\text{strong}}}{\pi^3} \frac{m_\theta^3}{F_\pi^2} N^2 \tag{3.14}$$

which for technicolour $N = 4$ and $m_\theta \sim 260$ GeV gives a rate of 80 MeV. If two gluons are produced in this way we expect to see two highly energetic jets whose total invariant mass is that of the pseudo.

The decays of the octet pseudos into the other vector pairs listed in eq. (3.10) have also been calculated [31]. They are all smaller than the estimate of 80 MeV for two gluons because one strong coupling constant is replaced by a weak coupling constant and the group theory factors are smaller.

Perhaps the easiest way to produce the θ_a is through two gluon annihilation [32], the inverse of eq. (3.14). This equation and estimates for gluon distributions inside protons give at tevatron energies ($\sqrt{s} = 2000$ GeV)

$$\sigma(p\bar{p} \to \theta_a + X) \sim 10^{-35}\,\text{cm}\,. \tag{3.15}$$

If the θ_a decays mostly into hadrons it will be difficult to see above the background. Perhaps the best hope is to look in the gluon + photon signal which would have a different angular distribution than the background and might stick out just enough to be seen [33].

An octet pseudo along with a vector boson might be produced in e^+e^- annihilation at some super LEP. For example fig. 12 shows the production of an θ_a^3 along with a gluon. Unfortunately these reactions typically contribute to R only $\sim 10^{-3}$ but they would be fairly clean signals of individual octets [34].

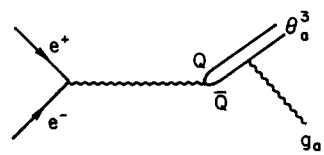

Fig. 12.

3.3.2. The colour triplets

The particles called T_c and T_c^i and their antiparticles in eq. (3.7) are colour triplets and carry technibaryon and technilepton numbers. The techni-isotriplet (T_c^+, T_c^3, T_c^-) has charges (1/3, −2/3, −5/3) and the isosinglet T_c has charge −2/3. The main contribution to the masses of these states is through single gluon exchange as for the octets. The estimate of the mass follows the octet estimate except that the group theory factor of 3 is replaced by 4/3 in eq. (3.8) so the mass of the triplet is

$$m_T = \tfrac{2}{3} m_\theta \tag{3.16}$$

which is $\simeq 170$ GeV for $N = 4$.

The T_c^+, T_c^3, T_c^- and T_c all receive different contributions to their masses from electroweak interactions. These mass differences squared are on the order of a few GeV^2 (see the section on charged axions) so the mass differences are quite small. We do expect

$$m_{T_c^-} \gtrsim m_{T_c^3} \simeq m_{T_c} \gtrsim m_{T_c^+}\,. \tag{3.17}$$

These particles will be produced by the strong and electroweak interactions of ordinary particles and since these forces conserve technilepton and technibaryon numbers the triplets must be pair produced. Since these particles are relatively stable we can imagine them separating after production with a light quark–antiquark pair appearing for colour neutralization. This is illustrated in fig. 13.

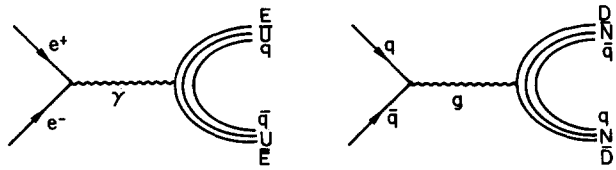

Fig. 13.

The dominant decay of the colour triplets will be into ordinary fermion pairs. As for the colour octets we expect the coupling to go as

$$(m_f + m_{f'})/F_\pi \tag{3.18}$$

so the triplets prefer heavy fermions.

The interactions responsible for the decay must respect colour and be $SU(2)_{LEFT} \times U(1)$ invariant so we expect decays like:

$$T_c^+ \to \bar{b}_c + \nu \tag{3.19a}$$

$$T_c^3 \to \bar{t}_c + \nu, \bar{b}_c + \tau \tag{3.19b}$$

$$T_c \to \bar{t}_c + \nu, \bar{b}_c + \tau \tag{3.19c}$$

$$T_c^- \to \bar{t}_c + \tau \tag{3.19d}$$

where again c is a colour index. The rate for $T_c \to \bar{b}\tau$ would be roughly

$$\Gamma \sim \frac{1}{8\pi} \frac{(m_b + m_\tau)^2}{F_\pi^2} m_T \sim 20\,\text{MeV}\,. \tag{3.20}$$

If these decays are found it will be interesting to study them in detail and see if there is information about family number conservation. For example, will the T_c decays into $\bar{b}\tau$, $\bar{b}e$, $\bar{b}\mu$ all be allowed? Will there be complicated mixing angles?

Again the decays of the triplets into ordinary fermion pairs require new interactions which violate γ_5 invariance. If these interactions were turned off the heavier triplets could still β decay into the lighter ones. For example we expect

$$T_c^- \to T_c^3 + \mu^- + \nu \tag{3.21}$$

just as $\pi^+ \to \pi^0 + e^+ + \nu$. The rates for individual channels would be extremely small $\sim 10^{-11}$ eV. The lightest triplet would then be absolutely stable because of conservation of technilepton and technibaryon number.

3.3.3. The eaten pions

These are the three particles, Π^i, which are colour neutral and couple maximally to the $SU(2)_{LEFT} \times U(1)_{HYP}$ interactions. They disappear from the spectrum via the Higgs phenomenon as discussed in section 2. Each can be thought of as the sum of four equally weighted fermion pairs labelled red, blue, yellow and lepton. Since they vanish from the spectrum we have only 60 pseudos.

3.3.4. The charged axions

The $P^{\pm}$ in eq. (3.7f) are the charge states which are colour neutral and orthogonal to the $\Pi^{\pm}$. This orthogonality accounts for the factor of 3 in their definition. Since they are colourless their masses do not come from gluon exchange but from other interactions. Electroweak contributions have been estimated by various groups using the current algebra formalism which works well for $m^2_{\pi^+} - m^2_{\pi^0}$. They all get [35]

$$(m^{\mathrm{em}}_{P^+})^2 \approx \frac{3\alpha}{4\pi} m^2_Z \log\left(\frac{m^2_{TC}}{m^2_Z}\right) \tag{3.22}$$

where m_Z is the mass of the Z boson and m_{TC} is the mass of a technimeson probably near 1 TeV. Thus the electroweak contribution is (5–10 GeV)^2. In section 4 we will discuss and estimate the possible contributions to the mass squared from effective four Fermi interactions amongst the technifermions. These contributions could be as large as (100 GeV)^2. Other authors get lower numbers. We feel that there is great uncertainty in the mass of these particles and they could vary from 5 GeV to 100 GeV.

These charged axions will decay through the additional interactions responsible for breaking the chiral invariance of ordinary fermions. Again using the coupling of eq. (3.18) we get preferential decays into heavy quarks or leptons. For example $P^+ \to c + \bar{b}$ would have a rate of 3 MeV for a charged axion mass of 30 GeV. If the additional interactions couple techniquarks to ordinary quarks and technileptons to ordinary leptons with equal strength then the rate into lepton pairs may be enhanced by a factor of 3 relative to the rate into quark pairs [36]. [See eq. (3.7f).] The decay into a fermion–antifermion would probably be parity non-conserving, like $\pi \to \mu\nu$, but not necessarily maximally parity violating.

If a P^+P^- pair is produced in e^+e^- annihilation it would contribute 1/4 unit to R. They would have a threshold effect of β^3 where β is the velocity. Another intriguing source of charged axions is in toponium decays. If $m_{P^+} < m_{top} - m_{bottom}$ then you would expect $t \to P^+ + b$ again with a coupling proportional to the fermion masses. This should compare favourably with the ordinary weak decays of a top quark. A toponium state would then decay as follows

$$\begin{aligned}(\bar{t}t) &\to P^+ + \bar{t} + b \\ &\qquad\quad \hookrightarrow P^- + \bar{b}\,.\end{aligned} \tag{3.23}$$

This might be the best source of charged axions [37].

3.3.5. The neutral axions

The particles defined in eqs. (3.7g) and (3.7h) are charge zero and colourless [38]. In fact they get no mass [39] at all from electroweak interactions much like the π^0. First consider the axion P^3 which couples to

$$\frac{1}{\sqrt{48}}\langle 0|\bar{Q}\gamma^\mu\gamma_5\tau^3 Q - 3\bar{L}\gamma^\mu\gamma_5\tau^3 L|P^3\rangle = F_\pi q^\mu\,. \tag{3.24}$$

The current is not conserved because of electroweak interactions. But the P^3 also couples to this current plus a vector current:

$$\frac{1}{\sqrt{48}}\langle 0|\bar{Q}\gamma^{\mu}(1+\gamma_5)\tau^3 Q - 3\bar{L}\gamma^{\mu}(1+\gamma_5)\tau^3 L|P^3\rangle = F_{\pi}q^{\mu}\,. \tag{3.25}$$

Since this current is made of right-handed fields it commutes with the $SU(2)_{LEFT}$ part of the Hamiltonian. It is easy to check that it also commutes with the $U(1)_{HYP}$ so it is a conserved current. Taking the divergence of both sides of (3.25) gives

$$0 = F_{\pi}m^2_{P^3} \tag{3.26}$$

so the axion is massless.

The addition of other interactions can make this non-zero. In the next section we estimate such effects and, as for the P^+, we get

$$0 \lesssim m^2_{P^3} \lesssim (100\ \mathrm{GeV})^2\,. \tag{3.27}$$

Similar arguments apply to the P^o. In the absence of additional interactions, the current it couples to is conserved so:

$$0 = \partial_{\mu}\frac{1}{\sqrt{48}}\langle 0|\bar{Q}\gamma^{\mu}\gamma_5 Q - 3\bar{L}\gamma^{\mu}\gamma_5 L|P^0\rangle = F_{\pi}m^2_{P^o}\,. \tag{3.28}$$

Again it gets mass from other interactions and we estimate:

$$0 \lesssim m^2_{P^o} \lesssim (100\ \mathrm{GeV})^2\,. \tag{3.29}$$

These particles should be able to decay into fermion pairs just like the other pseudos. For a light axion, say 5 GeV, decaying into an s, $\bar{s}$ pair we expect a rate [30] of around 1 keV. The decays will violate parity since the amplitude generally contains a scalar and pseudoscalar coupling [37]. This is in contrast to the fermionic decay of the neutral Higgs boson [40] which in the standard model is a scalar particle with only scalar couplings to fermions [41]. The fermion couplings of the neutral Higgs particle are completely determined by the fermion mass matrix so we expect:

$$\Gamma(\mathrm{Higgs}\to\bar{q}q)/\Gamma(\mathrm{Higgs}\to\bar{\ell}\ell) = 3m^2_q/m^2_{\ell}\,. \tag{3.30}$$

If the Higgs is replaced by a P^o or P^3 we expect the mass squared in eq. (3.30) to be the same but the 3 will change perhaps to a 27 or 1/3 depending on the model [36]. [In a model with more than one Higgs doublet eq. (3.30) need not hold.]

The P^o will also decay into two gluons whereas the P^3 will not. This is due to the techni-isospin symmetry. This decay has been computed and the rate is [30]

$$\Gamma(P^o\to gg) \approx 30\ \mathrm{eV}\,(m_{P^o}/2\ \mathrm{GeV})^3(N/4)^2\,. \tag{3.31}$$

For a heavy P^o this would be the dominant mode. The inverse of this process, $gg\to P^o$, could lead to the production of a P^o in proton–proton or proton–antiproton collisions.

Perhaps the best way to produce a P^o or P^3 is in e^+e^- annihilation via a resonant quarkonium state [42]. For example we expect

$$e^+e^- \to \bar{t}t \to P^o, P^3 + \gamma . \tag{3.32}$$

The decay rate of a quarkonium state into a pseudo plus photon is comparable to the leptonic rate, i.e., $(\bar{t}t) \to e^+e^-$ and the exact ratio would be different if the pseudo was a Higgs.

Another characteristic of pseudos in e^+e^- annihilation is the smallness of processes like [42]:

$$\begin{aligned} &e^+e^- \to P^o, P^3 + Z \\ &e^+e^- \to P^o, P^3 + \gamma \\ &Z \to P^o, P^3 + \mu^+\mu^- \end{aligned} \tag{3.33}$$

relative to the rates if the spin 0 particle is the neutral Higgs. The reason is that the processes in eq. (3.33) proceed through technifermion loops while the comparable Higgs diagrams are at the tree level. The absence of processes like those in eq. (3.33) could signal technicolour.

The P^o and P^3 are the lightest of the possible pseudo-Goldstone bosons in this one family model. The discovery of these or other pseudos would indicate that many of the technicolour ideas are correct. Their absence from the spectrum would reveal that our simple extrapolations from low energies are too naive.

4. Effective four Fermi interactions

The physics we have introduced so far has included fermions and gauge bosons, without fundamental scalars, transforming under the gauge group $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR. This cannot be a complete description. In particular it fails to account for the masses of the light fermions [43]. Recall the discussion of γ_5 invariance in section 3.3.1 and eq. (3.11). A light fermion mass term is not γ_5 invariant whereas the gauge interaction of the light fermions is γ_5 invariant. The γ_5 invariance of light fermions must be violated for the light fermions to get a mass. Evidently new interactions are required. These new interactions should also contribute to the masses of the pseudo-Goldstone bosons and should eliminate any empirically unacceptable massless Goldstone bosons.

The observed fermions have a wide variety of masses all well below the technicolour scale. To account for this you might imagine that the new interactions induce fermion masses which are of order α, α^2, α^3, etc., times the technicolour scale, where α is some weak coupling constant [44]. Order α^0 masses would be prevented by special symmetries. This possibility was first considered by Weinberg and we explore it in a different context in the discussion of nearly massless composite fermions in section 6.

Another possibility is that the new interactions occur at a variety of scales all above the technicolour scale. In this section we will show how this can give rise to a complex pattern of fermion masses below the technicolour scale.

4.1. Other interactions and scales

Let us assume that at an energy scale even higher than Λ_{TC} a new scale of interactions, Λ_E, exists

whose precise nature we will not discuss now. (These new interactions may come from the exchange of heavy particles.) At low energies these interactions take the form of non-renormalizable multiparticle vertices with dimensionful coupling constants. This is illustrated by two familiar examples:

(i) The renormalizable Weinberg–Salam theory of electroweak interactions involves the exchange of heavy vector bosons. At low energies this theory is described by an effective four Fermi interaction with a coupling constant of dimension $(\text{mass})^{-2}$.

(ii) Proton decay in grand unified theories is mediated by extremely heavy bosons, and at low energies these interactions are also described by a four Fermi effective interaction with a dimensionful coupling.

More generally a long or infinite hierarchy of non-renormalizable interactions may be needed to describe low energy physics [45]. In particular an n Fermi interaction will require a coupling of dimension $(\text{mass})^{4-3n/2}$ to give the Lagrangian dimension of $(\text{mass})^4$. It is usually assumed that the coupling constant

$$g \sim \theta(1)/M^{3n/2-4} \tag{4.1}$$

where M is the scale of the interaction. Thus the higher the dimension of the operator the smaller the induced effect.

At the scale Λ_E above Λ_{TC} we expect new interactions producing effective low energy many fermion coupling. These interactions at the scale Λ_E must respect the unbroken symmetries at this scale. The induced low energy effective operators will then also respect the unbroken symmetries at Λ_E. This means that the low energy effective interactions must be $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR invariant.

Let us look at the operator with the lowest dimensions which by our previous argument has the largest coupling constant. Our low energy theory only has fermions and gauge bosons so the lowest dimension Lorentz invariant operator is the two Fermi Lorentz scalar $\bar{\psi}_1\psi_2$. This alone is a mass term so it is prevented by $SU(2)_{LEFT}$ invariant.

4.2. Four Fermi interactions and quark masses

The next lowest dimension operators of interest are the four Fermi operators of dimension six. These operators can be $SU(2)_{LEFT}$ invariant and give mass to the light fermions. For example, consider the one doublet model of section 2.4.1 with a technidoublet (A, B) along with a doublet of ordinary quarks (u, d). Colour is an irrelevant complication and can be ignored. Define a 2×2 technicolour singlet matrix

$$M_T = \mathbb{1}(\bar{T}T) + i\boldsymbol{\tau}\cdot\bar{T}\gamma_5\boldsymbol{\tau}T \tag{4.2}$$

where T is the technidoublet

$$T = \begin{pmatrix} A \\ B \end{pmatrix}. \tag{4.3}$$

Under $SU(2)_{LEFT}$

$$M_T \to UM_T \tag{4.4}$$

where U is a member of $SU(2)_{LEFT}$ and under $U(1)_{HYP}$

$$M_T \to M_T e^{i\tau_3\theta/2}. \tag{4.5}$$

Then an $SU(2)_{LEFT} \times U(1)_{HYP}$ invariant coupling between technifermions and ordinary quarks is:

$$\frac{a}{\Lambda_E^2}\bar{q}_L M_T q_R + \frac{b}{\Lambda_E^2}\bar{q}_L M_T \tau_3 q_R + \text{h.c.} \tag{4.6}$$

When the technifermions condense so that

$$\langle \bar{T}T\rangle = \langle \bar{A}A + \bar{B}B\rangle \tag{4.7}$$

these interactions will give mass to the light quarks through diagrams like those in fig. 14.

The propagators for the fermions have masses due to the condensate [eq. (4.7)] and the evaluation of the diagrams gives:

$$m_u \sim \frac{\langle \bar{A}A + \bar{B}B\rangle}{\Lambda_E^2}(a+b) \tag{4.8a}$$

$$m_d \sim \frac{\langle \bar{A}A + \bar{B}B\rangle}{\Lambda_E^2}(a-b). \tag{4.8b}$$

To estimate Λ_E we assume that:

$$\frac{\langle \bar{T}T\rangle}{2} \approx \sqrt{\frac{3}{N}}\langle \bar{q}q\rangle \left(\frac{F_\pi}{f_\pi}\right)^3 \tag{4.9}$$

where $\langle \bar{q}q\rangle$ is the one flavour QCD condensate and we have the N dependence for an SU(N) technicolour. Standard [46] QCD analysis gives $\langle \bar{q}q\rangle/f_\pi^3 \approx 17$. In the one doublet model $F_\pi \simeq 250$ GeV and taking $N = 4$ gives $\langle \bar{T}T\rangle/2 \simeq (600\,\text{GeV})^3$. In the one family model F_π is reduced by $\frac{1}{2}$ and $\langle \bar{T}T\rangle/2 \simeq (300\,\text{GeV})^3$. If a and b are of order unity and we want quark masses of 1 GeV then Λ_E is around 20 TeV for the one doublet model and around 7 TeV for the one family model. This is a new scale of interactions.

In the real world we must account for the vast difference in masses between the generations, the most extreme ratio being m_e/m_{bottom} or m_ν/m_{top}. One possibility is a variety of scales Λ_E governing different quartic interactions. Alternatively there could be a variety of scales of condensates which

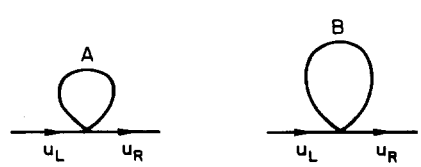

Fig. 14.

separately contribute to ordinary masses. A third possibility is that the lightest observed fermions cannot get masses from four Fermi interactions because of special symmetries and only receive their masses from six or higher Fermi interactions which give rise to extra powers of (Λ_{TC}/Λ_E). In the standard weak interaction theory with fundamental scalars the quark and lepton masses are determined by arbitrary Yukawa couplings and no explanation is offered for their wide range of values.

Now consider the one family model of section 2.4.2 and also include one family of ordinary fermions:

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \ell = \begin{pmatrix} \nu \\ e \end{pmatrix}. \tag{4.10}$$

Defining

$$\begin{aligned} M_Q &= \mathbb{1}(\bar{Q}Q) + i\tau \cdot \bar{Q}\gamma_5\tau Q \\ M_L &= \mathbb{1}(\bar{L}L) + i\tau \cdot \bar{L}\gamma_5\tau L \end{aligned} \tag{4.11}$$

we can have at least eight independent couplings of the form:

$$\begin{aligned} &\bar{q}_L M_Q q_R,\ \bar{q}_L M_Q \tau_3 q_R,\ \bar{q}_L M_L q_R,\ \bar{q}_L M_L \tau_3 q_R\,, \\ &\bar{\ell}_L M_Q \ell_R,\ \bar{\ell}_L M_Q \tau_3 \ell_R,\ \bar{\ell}_L M_L \ell_R,\ \bar{\ell}_L M_L \tau_3 \ell_R \end{aligned} \tag{4.12}$$

with respect $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR and which contribute to light fermion masses. (Here the labels L and R on q and ℓ mean left and right.) Thus the ordinary quarks get mass from techniquarks and technileptons. These types of mixed operators can eliminate unwanted massless Goldstone bosons.

In models with more than one light or technifermion family, the couplings (4.12) can be generalized to matrices. This gives rise to mass matrices which mix families and produce Cabibbo angles.

4.3. Four Fermi interactions and Goldstone bosons

In addition to giving light fermions their masses, effective four Fermi interactions can eliminate unwanted massless Goldstone bosons [47]. For example consider the neutral axions P^3 and P^o of section 3.3.5. In a theory in which the only interactions are $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR these two particles are strictly massless because they couple to conserved currents. These currents are linear combinations of techniquark chiral currents and technilepton chiral currents.

Consider a possible interaction of the form

$$I = \frac{1}{\Lambda_E^2}[\bar{Q}Q\bar{L}L - \bar{Q}\gamma_5\tau^a Q\bar{L}\gamma_5\tau^a L]\,. \tag{4.13}$$

This new interaction respects all of the gauge symmetries but it violates the separate techniquark and technilepton chiralities. It gives the neutral axions P^o and P^3, their masses. We can estimate these masses by using Dashen's formula:

$$m^2 = \frac{1}{F^2}\langle 0|[J^o, [J^o, H']]|0\rangle \tag{4.14}$$

where J° is the charge associated with the current that couples to the pseudo-Goldstone boson with strength F. H' is the part of the Hamiltonian that violates the conservation of the current. In the case of the P° we use the current from eq. (3.7h)

$$J^\circ = \frac{1}{\sqrt{48}}(Q^+\gamma_5 Q - 3L^+\gamma_5 L) \tag{4.15}$$

and the interaction which violates this current is the one just introduced in eq. (4.13). Evaluating the commutators gives

$$m^2_{P^0} = \frac{1}{F^2_\pi}\frac{1}{\Lambda^2_E}\frac{40}{48}\langle 0|\bar{Q}Q\bar{L}L|0\rangle . \tag{4.16}$$

The interaction of eq. (4.13) will give the same contribution to the mass of the P^3 if we use the current defined in eq. (3.7g).

Now $\bar{Q}Q$ is the sum of three coloured doublets while $\bar{L}L$ is one doublet so

$$\langle 0|\bar{Q}Q\bar{L}L|0\rangle \sim 3\langle\bar{T}T\rangle^2 \tag{4.17}$$

where $\langle\bar{T}T\rangle$ is the vacuum expectation value of one coloured doublet as in eq. (4.7). Using the values for the one family model of $\langle\bar{T}T\rangle/2 \simeq (300\,\text{GeV})^3$, $\Lambda_E \simeq 7\,\text{TeV}$ and $F_\pi \simeq 125\,\text{GeV}$ we get $m^2 \simeq 100\,\text{GeV}$. More generally we expect

$$m^2 \approx R^2\langle\bar{T}T\rangle^2/F^2_\pi\Lambda^2_E \tag{4.18}$$

where R is a number of order unity. (In the case just considered R is $\sqrt{5/2}$.) So the lightest pseudo-Goldstone bosons, who get all their mass from these kinds of interactions will have masses

$$m \simeq R \cdot 60\,\text{GeV} . \tag{4.19}$$

Other authors have obtained lower estimates and one is discussed in the next section. They assume a different interaction than the one we have used in eq. (4.13) but more crucially they assume a larger scale Λ_E for the interactions. To make a precise estimate you need a specific model in which all of the interactions are known. In conclusion if the light pseudos exist they should have masses below 100 GeV. The search for pseudos at all energies below the technicolour scale should be conducted.

5. The larger picture

We have shown that the introduction of a new set of fermions, technifermions, interacting through a new force, technicolour, can successfully replace the Higgs sector of the standard electroweak theory. The technicolour and ordinary forces are mediated by gauge boson exchange. Ordinary fermion masses can be generated through effective four Fermi interactions between ordinary and technicoloured particles. In this section we show that the effective four Fermi couplings might also come from gauge boson exchange. Technicoloured and ordinary particles would then sit in the same representation of

some group and the consequences of this must be explored. In particular problems can arise related to neutral strangeness changing currents.

If the Lagrangian of the world only has fermions and gauge bosons then it is natural to look for one big gauge group which contains all of the forces including technicolour. The difficulties of this attempt are also examined in this section.

Gauge theories with fermions but without fundamental scalars must dynamically break themselves at a variety of scales in order to correctly describe the world. The dynamics of spontaneously broken gauge theories is a difficult subject but recently some progress has been made. We will examine the questions of gauge group breaking themselves and the alignment of the vacuum in dynamical theories.

In section 6 on massless composites we stress a different point of view. The ordinary and technicoloured fermions are viewed as composites and they interact through their constituents. The nature of their effective interaction is different and it might be necessary to enlarge the weak gauge group to give light fermions their masses.

5.1. Extended technicolour or sideways

In a technicolour scheme effective four Fermi interactions may arise from the exchange of gauge bosons which mediate transitions from ordinary to technifermions [29] as in fig. 15. These ordinary fermion–technifermion–gauge boson vertices do not preserve the γ_5 invariance of the ordinary fermions. Diagrams like those in fig. 16 are responsible for light fermion masses. The *constituent* mass of the technifermion is fed down by boson exchange to the *current algebra* mass of the light fermions. If

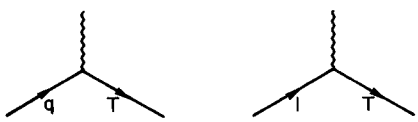

Fig. 15.

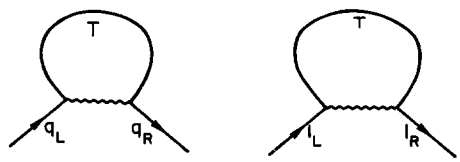

Fig. 16.

the mass, m_B, of the boson exchanged is greater than Λ_{TC} and the coupling at the vertices is g then these diagrams give light fermion masses:

$$m \sim \frac{g^2}{m_B^2} \langle \bar{T}T \rangle . \tag{5.1}$$

Thus m_B/g corresponds to the scale Λ_E of section 4. This boson exchange has been called *sideways* to distinguish it from *horizontal* which takes ordinary family to ordinary family.

The transitions from techni to ordinary fermions may arise in the following way. For simplicity imagine that the technicolour group is SU(N) and some chiral technifermions transform in the fundamental $\underline{N}$ dimensional representation. Now suppose this SU(N) is embedded in a larger group, extended technicolour, say SU($N+1$) and this SU($N+1$) somehow breaks down to SU(N) at a scale Λ_E. The fermions in the fundamental $\underline{N+1}$ of SU($N+1$) decompose into a technicolour $\underline{N}$ and a technicolour singlet, an ordinary fermion. The broken generators couple to massive gauge bosons which mediate transitions from technicoloured to ordinary particles.

The fundamental $(N+1)$ of extended technicolour looks like

$$\text{extended technicolour}\left\{\begin{pmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_N \\ q \end{pmatrix}\right\}\begin{array}{l}\text{technicolour}\\ \\ \leftarrow\text{technicolour singlet}.\end{array} \tag{5.2}$$

This scenario can be generalized. Suppose extended technicolour is $SU(N+m)$ which breaks down to $SU(N)$ technicolour. The fundamental $\underline{N+m}$ representation of $SU(N+m)$ would break into a technicolour $\underline{N}$ and m technicolour singlets. These m singlets might be m generations of a particular flavour.

Another possibility is $SU(N+3)$ for extended technicolour which breaks down into $SU(N)$ technicolour $\times$ $SU(3)$ colour. Then a fundamental $\underline{N+3}$ of extended technicolour breaks into a colour neutral technifermion and a quark (technicolour singlet).

The extended technicolour idea can be summarized as follows. The technicolour group is contained in a larger group which at some scale Λ_E breaks down into technicolour and others either broken or unbroken groups. The representations of the extended technicolour group decompose into technicoloured objects and technicolour singlets (ordinary fermions). The broken generators mediate transitions from ordinary to technifermions. The scale Λ_E is larger (see section 4) than the scale Λ_{TC}. Thus the extended technicolour group which is a good symmetry at energies above Λ_E must also respect $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR.

It is also possible that at the scale Λ_E there exist broken generators which mediate transitions from technifermion type to technifermion type. For example there might be gauge bosons of mass $\sim\Lambda_E$ which connect techniquarks to technileptons. These might be responsible for the effective four Fermi interaction of eq. (4.13) which could give mass to the light axions. There could also exist interactions at the scale Λ_E amongst ordinary fermions. This could lead to small but non-zero probabilities for unusual decay modes of stable particles, e.g., $K \to \mu e$, $\pi \to e^+e^+e^-$, etc. The experimental discovery of an unexpected decay mode would be a source of information about physics between the weak interaction scale and the grand unification scale.

Let us illustrate these ideas with some examples. First consider the one family model of section 2.4.2 along with one family of ordinary particles (u, d, e, ν). Let the extended technicolour group be $SU(5)$ which breaks down to technicolour $SU(4)$ leaving one technicolour $\underline{4}$ and one technicolour singlet in each $\underline{5}$ of $SU(5)$. The multiplets look like:

$$\begin{array}{ll}
\begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{red}}_{\text{LEFT}} & \begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{red}}_{\text{RIGHT}} \\
\begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{blue}}_{\text{LEFT}} & \begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{blue}}_{\text{RIGHT}} \\
\begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{yellow}}_{\text{LEFT}} & \begin{pmatrix} U & U & U & U & u \\ D & D & D & D & d \end{pmatrix}^{\text{yellow}}_{\text{RIGHT}} \\
\begin{pmatrix} N & N & N & N & \nu \\ E & E & E & E & e \end{pmatrix}_{\text{LEFT}} & \begin{pmatrix} N & N & N & N & \nu \\ E & E & E & E & e \end{pmatrix}_{\text{RIGHT}}.
\end{array} \tag{5.3}$$

Assuming the condensation of eq. (2.24) we get equal masses for the technifermions U, D, E and N. These feed masses down to u, d, e and ν so we also get $m_u = m_d = m_e = m_\nu$. Ignoring this problem we can look at the spectrum of Goldstone bosons associated with technicolour. The spectrum is that discussed in section 3. If there are no additional interactions the two neutral axions are massless. To avoid this, imagine that colour and lepton number are part of a vector SU(4) Pati–Salam group. This SU(4) breaks down to SU(3) colour and six of the broken generators mediate quark lepton transitions. The coupling to heavy gauge bosons χ_c^μ is of the form

$$gX_c^\mu[\bar{U}^c\gamma^\mu N + \bar{D}^c\gamma^\mu E + \bar{u}^c\gamma^\mu\nu + \bar{d}^c\gamma^\mu e] \tag{5.4}$$

where c is a colour index to be summed over.

This interaction will violate the conservation of the currents which kept the neutral axions massless. If m_B is the mass of gauge boson then the contribution to the mass of the axion is estimated [48] as:

$$\Delta m^2 \approx \frac{2g^2}{m_B^2}\frac{1}{F_\pi^2}\langle\bar{T}T\rangle^2 . \tag{5.5}$$

If there is more than one light generation then gauge bosons of this kind can mediate $K_L \to \mu e$ with the aid of the additional coupling

$$gX_c^\mu[\bar{s}^c\gamma^\mu\mu] . \tag{5.6}$$

Using experimental bounds on $K_L \to \mu e$ the authors of ref. [48] estimate $m_{\text{axion}}^2 \lesssim (2.5\ \text{GeV})^2$. In models of this kind the axion-like objects could be very light and observable with present machines. Of course this is not inevitable and only occurs in this very specific type of model.

For an alternate example consider a model with SU(4) technicolour with one doublet of technifermions as in section 2.4.1. Consider also two light families (u, d, e, ν) and (c, s, μ, ν). Imagine extended technicolour is SU(12) and there are four fundamental $\underline{12}$'s of SU(12)

$$\begin{pmatrix} A & B \\ A & B \\ A & B \\ A & B \\ u & d \\ u & d \\ u & d \\ \nu & e \\ c & s \\ c & s \\ c & s \\ \nu & \mu \end{pmatrix}_{\text{LEFT}} \begin{matrix} \left.\right\} \text{technicolour} \\ \left.\right\} \text{colour} \\ \left.\right\} \text{colour} \end{matrix} \left\{ \begin{pmatrix} A \\ A \\ A \\ A \\ u \\ u \\ u \\ \nu \\ c \\ c \\ c \\ \nu \end{pmatrix}_{\text{RIGHT}} \right. \left\{ \begin{pmatrix} B \\ B \\ B \\ B \\ d \\ d \\ d \\ e \\ s \\ s \\ s \\ \mu \end{pmatrix}_{\text{RIGHT}} \right. \tag{5.7}$$

and also suppose that the SU(12) extended technicolour breaks at the scale Λ_E down to $SU(4)_{\text{TECHNICOLOUR}} \times SU(3)_{\text{COLOUR}}$. The $SU(3)_{\text{COLOUR}}$ is the diagonal sum of two SU(3)'s which sit directly in SU(12). Now there is only one doublet of technifermions. There are no extra Goldstone bosons at all. There is no axion and no paraxion.

Of course this model is also in trouble because the u and c quarks are treated symmetrically so you expect them to have the same mass. These two models illustrate the difficulties of giving masses of light fermions in models with extended technicolour. The symmetries of the unbroken extended technicolour Lagrangian are ultimately reflected in degeneracies in the light fermion masses. To account for the observed pattern of quark and lepton masses you need a complicated symmetry breaking pattern. If extended technibosons with different masses connect to different light fermions, these light fermions will no longer be degenerate. In general, an extended technicolour model will have a variety of different mass extended technicolour bosons being exchanged between various types of ordinary and technicolour fermions. Consider the extended technicolour interactions of the down quarks d^i ($d^1 = d$, $d^2 = s$, $d^3 = b$, etc.) with technifermions T^α where α is the set of all the indices carried by a technifermion. The most general gauge interaction with gauge bosons E_a is of the form

$$g^a_{i\alpha}\bar{d}^i\gamma_\mu E^\mu_a(1-\gamma_5)T^\alpha + g'^a_{i\alpha}\bar{d}^i\gamma^\mu E^\mu_a(1+\gamma_5)T^\alpha \tag{5.8}$$

where a runs over the possible types of extended technibosons. Of course this interaction must be $U(1)_{HYP} \times SU(2)_{LEFT} \times SU(3)_{COLOUR} \times$ TECHNICOLOUR invariant. This type of interaction gives rise to a down quark mass matrix through the diagram in fig. 17. If the fields d^i are mass eigenstates then the mass matrix is diagonal in the indices i and j.

The general interaction of eq. (5.8) can lead to trouble with neutral flavour changing currents [49]. The diagrams in fig. 18 are the source of the problem. Unless these diagrams have the structure $\delta_{ij}\delta_{Kl}$ or $\delta_{il}\delta_{Kj}$ there will be neutral flavour changing processes. A part of the diagram in fig. 18, involving left- and right-handed quarks, can be related to the quark mass matrix of fig. 17 and this part will be flavour diagonal. But there is no reason to expect that the parts involving only left- or right-handed quarks are flavour diagonal or suppressed.

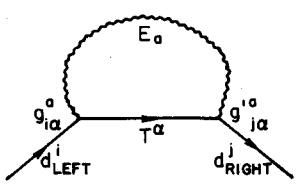

Fig. 17.

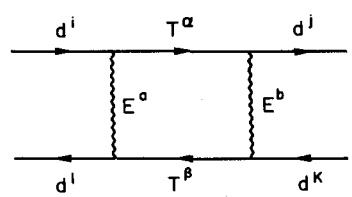

Fig. 18.

Consider the contribution of such a diagram in fig. 19 to the K_L, K_S mass difference. This is very similar to the original calculation of this mass difference [50] involving the GIM [51] mechanism but here an extended techniboson replaces the weak boson and a techniquark replaces the up-quarks. In our case there is no cancellation of the leading piece and the effective four Fermi operator, leading to a

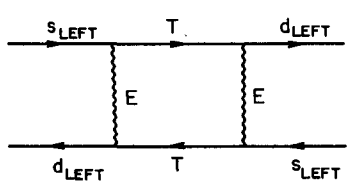

Fig. 19.

$K_L K_S$ mass difference, can be written [52] as

$$\frac{g^4}{32\pi^2 m_B^2}\,\bar{s}\gamma^\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d \tag{5.9}$$

where m_B is the mass of the exchanged techniboson. To be consistent with the observed $K_L K_S$ mass difference, the coefficient [50] in eq. (5.9) must be less than $5\times 10^{-13}\,\mathrm{GeV}^{-2}$ so m_B/g^2 is greater than 80 TeV. Since $m_B \approx g\Lambda_E$ this becomes

$$\Lambda_E/g > 80\,\mathrm{TeV}\,. \tag{5.10}$$

In the one family model the scale of interactions Λ_E required to give a one GeV quark mass is around 7 TeV. [See section 4.2.] For the strange quark it would be closer to 20 TeV which is still inconsistent with the limit of eq. (5.10) if $g \simeq 1$.

An extended technicolour model can also have neutral strangeness changing processes through single boson exchange [52]. (These are analogous to ordinary flavour changing processes through Z exchange which again vanish because of the GIM mechanism.) Consider a toy model with extended technicolour being SU($N+2$) which breaks down to SU(N) technicolour. A fundamental $\underline{N+2}$ might break into a technicoloured fermion D and two technicolour singlets d′ and s′ where the prime means weak interaction eigenstates

$$\mathrm{SU}(N+2)\ \text{etc.}\left\{\begin{pmatrix} \left.\begin{matrix} D_1 \\ D_2 \\ \vdots \\ D_N \end{matrix}\right\}\text{technicolour SU}(N) \\ \left.\begin{matrix} d' \\ s' \end{matrix}\right\}\text{technicolour singlets}\,. \end{pmatrix}\right. \tag{5.11}$$

The broken generator connecting the last two components would mediate d′ to s′ transitions. If we write the weak interaction eigenstates in terms of mass eigenstates

$$\begin{aligned} d'_L &= d_L\cos\theta_L + s_L\sin\theta_L \\ s'_L &= s_L\cos\theta_L - d_L\sin\theta_L \\ d'_R &= d_R\cos\theta_R + s_R\sin\theta_R \\ s'_R &= s_R\cos\theta_R - d_R\sin\theta_R \end{aligned} \tag{5.12}$$

where L, R mean left and right, then the diagram in fig. 20a would lead to the neutral strangeness changing diagram in fig. 20b with angle factors like $\cos^2\theta_L \sin^2\theta_R$ depending on the handedness of the particles involved. The effective neutral strangeness changing operator would have the form

$$\frac{g^2}{m_B^2}\cos^2\theta_{\substack{L\\R}}\sin^2\theta_{\substack{L\\R}}\,\bar{s}\gamma_\mu(1\pm\gamma_5)d\bar{s}\gamma^\mu(1\pm\gamma_5)d\,. \tag{5.13}$$

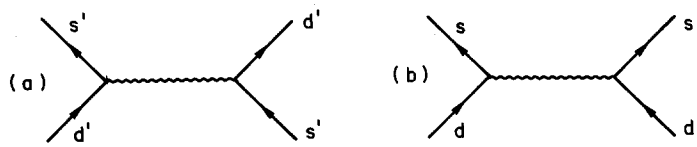

Fig. 20.

Again, to be consistent with the $K_L K_S$ mass difference the coefficient must be less than $5 \times 10^{-13}\,\mathrm{GeV}^{-2}$ so

$$\Lambda_E \sim m_B/g \gtrsim 1500\,\mathrm{TeV}\cos^2\theta_{\substack{L\\R}}\sin^2\theta_{\substack{L\\R}}\,. \tag{5.14}$$

Even for $\theta_L \simeq \theta_{\mathrm{Cabibbo}} \simeq 13°$ the scale Λ_E used for fermion mass generation violates the inequality.

The situation can be summarized as follows: two generators of extended technicolour connecting the same technifermion to two different ordinary fermions can have a commutator that connects the two ordinary fermions. The boson associated with this generator mediates *horizontal* currents and the limits on horizontal generators [53] are often higher than the scale Λ_E we expect for extended technicolour. This problem along with the problem of neutral flavour changing currents associated with double boson exchange, might be eliminated if there are GIM [51] type mechanisms at work in the extended technicolour models. Unless the needed suppression factors can be discovered, the extended technicolour mechanism is being severely challenged by low energy phenomenology.

5.2. Grand unified theories and technicolour

Grand unified theories [54] unify the electroweak and strong interactions by making $SU(3)_{\mathrm{COLOUR}}$, $SU(2)_{\mathrm{LEFT}}$ and $U(1)_{\mathrm{HYP}}$ all subgroups of one group. Quarks and leptons sit in the same irreducible representations of the grand unifying group. These theories are elegant and have certain striking phenomenological successes.

The extended technicolour schemes [43] put technifermions and ordinary fermions in the same representations of the extended technicolour group. It is natural to ask if technifermions, quarks and leptons can all be put in the same representation of a super-unifying group. This group would contain the known gauge groups as well as the technicolour group as subgroups.

One of the simplest ways to implement this idea is to generalize the SU(5) Georgi–Glashow grand unification scheme. Suppose technicolour is $SU(N)$ and the super-unifying group is $SU(5+N)$. This $SU(5+N)$ contains the $SU(N)_{\mathrm{TECH}} \times SU(3)_{\mathrm{COLOUR}} \times SU(2)_{\mathrm{LEFT}}$ embedded in the obvious way

$$\left[\begin{array}{c|c|c} SU(N) & & \\ \hline & SU(3) & \\ \hline & & SU(2) \end{array}\right]. \tag{5.15}$$

The $U(1)_{\mathrm{HYP}}$ would be a diagonal generator as it is in SU(5) without technicolour. Various representations of $SU(5+N)$ would have technicolour singlets (ordinary fermions) as well as technicoloured non-singlets (technicolour fermions).

The super-unified group would be a good symmetry only above some very high energy. Then the super-unified group would break down, perhaps through a sequence of breakings, until at a scale Λ_E

above Λ_{TC} the symmetry is (TECHNICOLOUR) $\times$ $SU(3)_{COLOUR} \times SU(2)_{LEFT} \times U(1)_{HYP}$. Then at Λ_{TC} the technicolour force becomes strong and the $SU(2)_{LEFT} \times U(1)_{HYP}$ group breaks down to $U(1)_{EM}$.

Using SU(7) as the super-unifying group [55], the authors of this paper tried to implement these ideas. Other models [56] have been discussed in the literature and we will not discuss any model in detail here. However, we will list the typical successes and problems with these schemes.

Successes:

(i) The low energy Higgs sector responsible for the breaking of the electroweak group can be replaced with fermions. These fermions are super-unified with ordinary fermions.

(ii) More than one family of light fermions can be naturally incorporated in the large representation of the super-unified group.

Problems:

(i) It is difficult to implement the required symmetry breaking because it requires many stages. Perhaps a *tumbling* gauge theory [57] is the solution. There are also difficulties with vacuum alignment [58] for certain choices of the technicolour group (see section 5.4).

(ii) There are often a large number of technifermions which carry $SU(3)_{COLOUR}$, $SU(2)_{LEFT}$ and $U(1)_{HYP}$ quantum numbers. These fermions contribute to the β functions which determine the coupling constant renormalizations. In the grand unified picture the coupling constant renormalization sets the scale for proton decay and determines the low energy weak mixing angle θ_W [1]. The addition of technifermions (especially many of them) can ruin these predictions. A super-unified model with many technifermions may also have a techniforce which is not asymptotically free [59]. Then it is hard to imagine the techniforce becoming strong as you come down in energy from the unification point.

(iii) The mass relations amongst the light fermions are usually wrong. This is similar to the standard SU(5) model with a Higgs where $m_{\text{electron}} = m_{\text{down}}$.

The idea of super-unifying technicolour with the other known force is so attractive that it should be pursued despite the difficulties.

5.3. Groups breaking themselves

In the previous two subsections we explored the possibility of placing the technicolour group inside of a larger group which breaks down to technicolour at some scale Λ_E perhaps on the order of 10 TeV. Then at some scale Λ_{TC} the technicolour force breaks $SU(2)_{LEFT} \times U(1)_{HYP}$. The electroweak group and the colour group are believed to break apart at the grand unification scale $\approx 10^{15}$ GeV.

All of these ideas require a sequence of symmetry breaking scales between the grand unification point and present energies. The technicolour philosophy prohibits the existence of fundamental scalars. The question arises of whether or not a gauge theory with only fermions and gauge bosons can exhibit this kind of sequential breaking. Recent ideas indicate that this is possible [60].

Start with a gauge group and a set of fermions in various representations. The number of fermions should be small enough to keep the theory asymptotically free. At the tree level the fermions will exchange a gauge boson and be attracted or repelled depending on the sign of a group theory factor (see fig. 21). This factor depends on the representation of the initial fermions and the representation of the combined channel. For example in colour SU(3) a quark ($\underline{3}$) and antiquark ($\underline{3}^*$) are attracted if they scatter in a singlet but repulsed if they scatter in the octet ($\underline{8}$). The strength of the attraction (or repulsion) for fermions f_1, f_2 to scatter into a state of representation r is

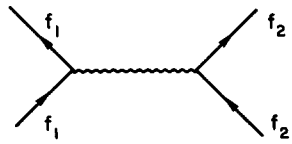

Fig. 21.

$$c(\mathrm{f}_1, \mathrm{f}_2, r)\, g^2(E) \tag{5.16}$$

where c is the group theory factor (positive for repulsion, negative for attraction) and $g(E)$ is the running coupling constant of the theory.

It is possible that if the combination in eq. (5.16) is big enough and the final state is a Lorentz invariant then condensation will occur. For example in QCD at low enough energies the coupling constant is big enough so that there is a condensation in the quark–antiquark singlet channel. (See section 2.1.) This causes a spontaneous breakdown of $SU(2)_{LEFT} \times SU(2)_{RIGHT}$. Since the attraction is in a colour singlet channel, colour is still a good symmetry.

Given a set of fermion representations of a group there will be a most attractive channel [57], MAC, that is to say a choice of two fermions for the initial state and a representation for the final state for which eq. (5.16) is most negative. As you come down in energy the coupling constant grows and this channel will condense first. If the condensate is not a singlet of the group, then the group will break itself. For example if ordinary colour is the force of attraction between a set of fermions which are all colour $\underline{3}$'s (no $\underline{3}^*$'s) then the MAC is two $\underline{3}$'s to attract into a $\underline{3}^*$ state. If condensation occurs the group colour SU(3) would break itself down into colour SU(2).

Imagine a non-Abelian gauge group G and a set of fermion representations with the gauge group unbroken at some high scale E_0. If the group is asymptotically free, then the coupling constant increases as you lower the energy. In fact $1/g^2$ goes as $\log E$. At some scale E_1 condensation will occur in the most attractive channel and G will break to a subgroup G_1. Now the fermions should be described under their representation of G_1. If there are no symmetries to forbid them fermion mass terms will appear at the scale E_1 so some subset of fermions will become massive with masses $\sim E_1$ and these fermions will only weakly influence physics at energies below E_1. Now track the couplings associated with the gauge group G_1. If one or more of them is asymptotically free there will be a scale E_2 at which condensation occurs in a new MAC and G_1 will break to G_2. New fermion mass terms appear, etc., and the process continues. In this way a gauge group G can break in a series of steps $G \supset G_1 \supset G_2 \ldots \supset G_N$ at scales $E_1 > E_2 \cdots > E_N$. The logs of these scales are separated by numbers of order unity so the scales differ by factors of ~ 10 or $\sim 10^2$ or $\sim 10^3$, etc. At the end of the sequence the group G_N cannot break itself further. This picture is called a *tumbling* gauge theory.

The tumbling idea implies a sequence of scales of symmetry breaking between the low energy world and the grand unification point. No fundamental scalars are needed to implement symmetry breaking. Models which tumble have been constructed and we urge the reader to consult ref. [57] for specific examples.

5.4. Vacuum alignment

In technicolour models a spontaneous symmetry breaking is induced by the technicolour force which breaks the weak interaction symmetry group. The dynamics of these systems is extremely complicated

and it is usually only an assumption that the desired symmetry breaking pattern occurs. However, it is often possible to compare the energies of the different vacua (or ground states) which result for different symmetry breaking patterns and to choose the correct symmetry breaking pattern by minimizing the energy.

When the technicolour force or another strong force causes a fermion condensate to form, the global symmetry group of the theory is generally reduced. For example QCD reduces the symmetry of a two quark world from $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ down to $SU(2)_{VECTOR}$. (See section 2.1.) If the original global symmetry group is G and the unbroken group is $H \subset G$ then the vacuum is left invariant by any group element $h \in H$, i.e.,

$$h|\text{vacuum}\rangle = |\text{vacuum}\rangle \tag{5.17}$$

whereas

$$b|\text{vacuum}\rangle \neq |\text{vacuum}\rangle \tag{5.18}$$

if $b \in G$ but $b \notin H$. In the absence of other interactions we can use eq. (5.18) to define a new vacuum:

$$b|\text{vacuum}\rangle = |\text{vacuum}\rangle' \tag{5.19}$$

and the new vacuum, $|\text{vacuum}\rangle'$, will be invariant under a new subgroup H′ of G. In general H′ is equivalent to H. To be less abstract, suppose our original Lagrangian is $O(n)$ symmetric and the vacuum points in the n direction spontaneously breaking $O(n)$ down to $O(n-1)$. If we rotate the vacuum to some other direction the theory will still have a left over $O(n-1)$ symmetry. As long as all directions are equivalent the choice of vacuum direction is arbitrary and the unbroken groups are equivalent.

However, the weak interactions gauge a subgroup G_w of G so all directions in the group G are not equivalent. The energy of the vacuum will be a function of its orientation relative to G_w. The true vacuum is the one with the lowest energy and the problem that must be solved is how to find the true vacuum. Weinberg [61] first set up the formalism to solve this problem in general and two independent papers by Peskin and Preskill [58] have applied these ideas to systems which are quite relevant to technicolour model building. We will show two examples from their papers to see how important this problem is. The first example starts with a technicolour group like $SU(N)$ and two technidoublets with conventional weak interaction properties but different charges:

$$\begin{array}{llll} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}_{LEFT} & Y = q_1 & (A_1)_{RIGHT} & Y = q_1 + \frac{1}{2} \\ & & (B_1)_{RIGHT} & Y = q_1 - \frac{1}{2} \\ \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}_{LEFT} & Y = q_2 & (A_2)_{RIGHT} & Y = q_2 + \frac{1}{2} \\ & & (B_2)_{RIGHT} & Y = q_2 - \frac{1}{2}. \end{array} \tag{5.20}$$

Neglecting weak interactions the world looks $SU(4)_{LEFT} \times SU(4)_{RIGHT}$ symmetric and we usually assume the condensation

$$\langle \bar{A}_1 A_1 + \bar{B}_1 B_1 + \bar{A}_2 A_2 + \bar{B}_2 B_2 \rangle \neq 0 \tag{5.21}$$

which reduces the symmetry to $SU(4)_{VECTOR}$. As far as technicolour interactions are concerned any

$SU(4)_{LEFT} \times SU(4)_{RIGHT}$ rotation of the vacuum of eq. (5.21) should leave the vacuum energy invariant. However, when weak interactions are considered the energy is extremized by only two directions. One direction is that of eq. (5.21) and the other is the vacuum:

$$\langle \bar{A}_1 A_1 + \bar{B}_1 A_2 + \bar{A}_2 B_1 + \bar{B}_2 B_2 \rangle \neq 0 \tag{5.22}$$

which is left invariant by a different SU(4). The physics of these two vacua is very different. The first choice breaks the $SU(2)_{LEFT} \times U(1)_{HYP}$ gauge group down to $U(1)_{EM}$ which is the conventional breaking. The second vacuum leaves no unbroken group and the photon becomes massive. The energetics is such that if

$$0 < |q_1 - q_2| < 1 \tag{5.23}$$

the second is preferred.

This result is quite surprising. Two technidoublets of slightly different charge will not line up and leave the photon massless. In building technicolour models the charge assignments of the technifermions, contributing to the weak interaction symmetry breaking, are crucial.

We have been generally assuming that the technifermions transform under a complex representation of the technicolour group. For example, if technicolour is $SU(N)$ technifermions usually transform as $\underline{N}$'s while anti-technifermions transform as $\underline{N}^*$'s. If we assume that the technifermions transform under a real representation of the technicolour group then technifermions and anti-technifermions have the same transformation properties. This can affect the original global symmetries of the group as well as the pattern of symmetry breakdown. Our next example illustrates these points.

Consider one doublet of technifermions (A, B) with an arbitrary mean charge q. We will use a notation where all fields are left-handed so right-handed fields are replaced by left-handed charge conjugates:

$$\begin{pmatrix} A \\ B \end{pmatrix}_{LEFT} \quad Y = q \qquad \begin{matrix} A^C_{LEFT} & Y = -q - \frac{1}{2} \\ B^C_{LEFT} & Y = -q + \frac{1}{2}. \end{matrix} \tag{5.24}$$

If the fermions transform under a complex representation R of the technicolour group, then A_{LEFT} and B_{LEFT} will transform as two R's and A^C_{LEFT} and B^C_{LEFT} will transform as two R*'s. The global symmetry of technicolour is $SU(2) \times SU(2)$. However, if the representation is real then we have four R's and the global symmetry is enlarged to SU(4).

Let us assume that the technifermions transform under a real representation R with the additional assumption that the antisymmetric product of two R's makes a singlet. For example if technicolour is SU(2) and technifermions are doublets we know that $(1/2 \times 1/2)_{ANTISYMMETRIC}$ is a singlet. This will also be true for fundamental representations of $SP(N)$ groups. Now the condensate we usually assume has the form:

$$\langle A^C_{LEFT} A_{LEFT} - A_{LEFT} A^C_{LEFT} + B^C_{LEFT} B_{LEFT} - B_{LEFT} B^C_{LEFT} \rangle \neq 0 . \tag{5.25}$$

The minus signs come from:

(a) antisymmetrizing in technicolour indices to make a technicolour singlet
(b) antisymmetrizing in spin indices to make a Lorentz scalar
(c) Fermi statistics.

This condensate breaks SU(4) down to SP(4). The weak interactions break from $SU(2)_{LEFT} \times U(1)_{HYP}$ down to $U(1)_{EM}$. We must compare the energy of this condensate with the energy obtained by making all possible SU(4) rotations on the condensate. The result is that the energy is extremized by two condensates: the one in eq. (5.25) and also by:

$$\langle A_{LEFT}B_{LEFT} - B_{LEFT}A_{LEFT} + A^c_{LEFT}B^c_{LEFT} - B^c_{LEFT}A^c_{LEFT}\rangle \neq 0 . \tag{5.26}$$

This condensate violates electric charge but it is $SU(2)_{LEFT}$ invariant so the electroweak gauge group breaks from $SU(2)_{LEFT} \times U(1)_{HYP} \to SU(2)_{LEFT}$ which is an unacceptable breaking pattern. Which vacuum is preferred depends on the relative strength of the $SU(2)_{LEFT}$ coupling constant g and the $U(1)_{HYP}$ coupling constant g' as well as the mean charge q of the doublet. If $|q| < \frac{1}{4}$ the preferred vacuum is the second one leaving $SU(2)_{LEFT}$ unbroken. If $|q| > \frac{1}{4}$ and

$$\cot^2 \theta_w = g_2^2/g_1^2 < \tfrac{1}{3}[16q^2 - 1] \tag{5.27}$$

then the first vacuum leaving $U(1)_{EM}$ as a good symmetry is preferred. For one doublet and the observed weak mixing angle this requires $|q| > 0.85$ which is not satisfied by quarks ($|q| = \frac{1}{6}$) or leptons ($|q| = \frac{1}{2}$).

In models with real representations of the technicolour group the correct breaking of the weak interaction gauge group may be hard to implement. (For a discussion of models where the symmetric product of two representations forms a singlet see ref. [58].) Technicolour groups of this kind often arise in models where technicolour is unified with other forces and again these results can place severe constraints on model building (see section 5.2). However, the true vacuum must also be aligned with respect to the interactions which give light fermions their masses and these interactions might cause the vacuum to align differently.

6. Composite fermions

Physics at energy scales up to the weak interaction scale may well be described by quarks, leptons and technifermions viewed as pointlike fermions interacting through gauge boson exchange. The successes and difficulties of this point of view have been reviewed in the previous sections. However, there may be advantages in imagining that the observed fermions and the technifermions are not pointlike but rather composite [62] just like the proton is a composite state of three quarks. This opens the door onto a new set of interactions between ordinary fermions and technifermions which may help cure some of our previous problems.

6.1. Light composite fermions

If quarks and leptons are composite, the forces which bind their constituents must be acting on a scale much higher than the ordinary particle mass scale of ~ 1 GeV. This must be true because the electron looks pointlike down to distances below 10^{-16} cm while its inverse Compton wavelength is $m_e^{-1} \approx 4 \times 10^{-11}$ cm. How can a particle have a mass so much smaller than the scale of interactions experienced by its constituents? As was emphasized in section 1, the mass must be protected by a symmetry and for fermions this will be a chiral symmetry.

Suppose that the leptons are composite but their masses are protected by a chiral symmetry. There are still limits which can be imposed on the scale B of the binding forces from low energy experiments. The gyromagnetic ratios g_e and g_μ of the electron and muon are very close to their Dirac values of two with the deviations from two being accounted for beautifully by QED. For the electron the agreement is good up to a fractional error of $\sim 5 \times 10^{-10}$ and for the muon $\sim 10^{-8}$. If a fermion is composite you expect g to differ from two by factors on the order of m_f/B. However, if the mass m_f is much less than B because of a chiral symmetry, the same chiral symmetry will guarantee [63] that the deviations from two are of the order of $(m_f/B)^2$. For the electron this gives $B \gtrsim 25$ GeV and for the muon $B \gtrsim 1$ TeV. It is possible that the observed fermions are bound by forces which are strong at or above the technicolour scale.

6.2. 't Hooft's conditions

Suppose a set of fermions interact through a strong force associated with a confining gauge theory and the force gets strong at the scale B. The fermions will form a variety of bound states which are singlets under the strong gauge group and some of these bound states will also be fermions. Which of these composite fermions are likely to be massless on the scale B? 't Hooft has provided a partial answer to this question [64].

We will call the fermions interacting through the strong confining force *preons*. The preons transform non-trivially under the strong group G_S and there will also be some global chiral flavour group G_F. (If the strong force is QCD and the preons are like the up and down quarks then G_S is SU(3) and G_F is $SU(2)_{LEFT} \times SU(2)_{RIGHT} \times$ Baryon Number.) In general if the chiral flavour group G_F is not broken, either explicitly or spontaneously, it will protect the preons from getting a mass. (If $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ is unbroken the up and down quarks are massless.) Now the theory will have anomalies associated with massless preons running through triangle diagrams where there is a flavour current at each corner of the triangle. Suppose you wanted to gauge the flavour group G_F. For the gauge theory of the flavour group to be sensible it must be anomaly free [50] so these flavour anomalies must be cancelled by other fermions. 't Hooft has called these other fermions *spectators* because they do not feel the strong force but they do have non-trivial transformation properties under G_F. (The flavour anomalies of quarks are exactly cancelled by the flavour anomalies of leptons which are spectators here because they do not feel the strong force.)

The preons and spectators together make a sensible anomaly free theory. However, the strong force is confining and the physical states will be spectators and bound states of preons all of which are singlets under G_S. It should be possible to write down a sensible low energy anomaly free theory in terms of physical states. This can be used as a guide in deciding which of the possible composite fermions should be massless on the scale B.

All of the possible fermions which can be made of preons in a G_S singlet will have different transformation properties under the flavour group G_F and each will contribute differently to the anomaly diagrams associated with G_F. 't Hooft argues that the set of massless composite fermions which exists in the spectrum is the set whose G_F anomaly contributions exactly cancel those of the spectators. In other words, the massless composite fermions will have the same total G_F anomalies as the preons.

't Hooft only considered the case where the strong force does not cause a spontaneous breakdown of G_F and he found only one example to illustrate these ideas. Again it is QCD with two quarks u and d. If QCD does *not* break $SU(2)_{LEFT} \times SU(2)_{RIGHT}$ then you expect the spin $\frac{1}{2}$ colour singlet states, the proton and the neutron, to be massless. And in fact the G_F anomalies of the proton and the neutron exactly cancel those of the leptons whereas before it was the quark anomalies that did the job.

More generally the strong force will cause a condensation that breaks G_F down to another flavour group G'_F. Usually G'_F is contained in G_F but it may also be equivalent to G_F. The anomaly cancellation should be enforced only with respect to G'_F currents. Examples of this kind have been constructed.

A useful aid in finding examples is the *complementarity principle* [65]. This states that in certain cases a gauge theory which undergoes spontaneous symmetry breakdown can equivalently be viewed as a totally confining theory without symmetry breakdown. This principle is very elegant and can be useful but we will not elaborate on it here.

The point we want to emphasize is that there is a natural framework in which you would expect fermionic preons binding at the scale B to produce massless fermions or at least fermions much lighter than B. This will occur in a theory with a chiral global symmetry group if you require anomaly cancellations of the effective low energy theory. The symmetry group can then protect the masses of certain composite fermions.

6.3. Technifermions as preons

The observed quarks and leptons could be bound states of fermionic preons whose binding scale is experimentally restricted to be near or above the technicolour scale Λ_{TC}. As we saw in the last section it is even possible to imagine that there could be good theoretical reasons to expect these particles to be light on the scale Λ_{TC}. It is then possible that these preons are the technifermions or that the technifermions are made of the same preons which make quarks and leptons [66].

Imagine a theory of preons which feel a strong force associated with a gauge group G_S. There is a global chiral symmetry group G_F which protects the preon masses. Also imagine that a subgroup $G_w \subset G_F$ is gauged and is associated with the weak interactions. G_w should at least contain $SU(2)_{LEFT} \times U(1)_{HYP}$ and at the Lagrangian level all of G_F is unbroken. There are no Higgs bosons in the theory.

At the scale Λ_{TC} the strong force causes a spontaneous symmetry breakdown and the global symmetry group is reduced from G_F to G_{UB}. Certain preons will have masses protected by G_{UB} and will remain massless while the others will get mass of the order of Λ_{TC}. We can also look at the fermions which can be made of confined preons in G_S singlets. We argued in the last section that a set of these will remain massless also being protected by G_{UB}. The others will get mass on the order of Λ_{TC}. (Those fermions made of massive preons get mass.) We will identify the massless composite fermions as ordinary quarks and leptons.

The spontaneous symmetry breaking also affects the weak gauge boson sector. The weak bosons associated with generators of G_w which are also in G_{UB} will remain massless. The others will get a mass on the order of $g_w\Lambda_{TC}$ where g_w is the weak interaction coupling constant.

The weak interactions can also give small masses to the massless preons. If some of the broken generators of G_w connect a massless preon to a massive one then we expect the massless preon to pick up a mass of the order of $\alpha_w\Lambda_{TC}$ ($\alpha_w = g_w^2/4\pi$). This was the scheme Weinberg [61] originally had in mind for giving light fermions mass in dynamical theories. The same effect then gives a mass of the order of $\alpha_w\Lambda_{TC}$ to some of the massless composites by connecting then through weak boson exchange to massive composites. If we imagine this to be the mass of a real quark or lepton it is about right for the heavy generation. Perhaps the other generations only get a mass of order $\alpha_w^2\Lambda_{TC}$ or $\alpha_w^3\Lambda_{TC}$.

Many of these points can be illustrated with an example [67]. Imagine the strong force is SU(3) just like QCD and there are three flavours of preons p_A, p_B and p_C each transforming as a left-handed and a right-handed $\underline{3}$ of SU(3). The flavour symmetry G_F is $SU(3)_{LEFT} \times SU(3)_{RIGHT} \times$ Preon Number and all of the preons are massless. Suppose the strong force causes the unusual condensation

$$\langle \bar{p}_C p_C \rangle \neq 0 . \tag{6.1}$$

This will break the G_F symmetry group down to $SU(2)_{LEFT} \times SU(2)_{RIGHT} \times$ Preon Number $\times$ Preon C Number. The preon p_C becomes massive with a mass on the order of Λ_{TC} while p_A and p_B are still massless.

The physical states should be singlets under the strong group SU(3). With three types of preons there is an octet of possible spin $\frac{1}{2}$ singlets under SU(3). This octet is analogous to the baryon octet in QCD with three flavours. The two states which do not contain p_C should be massless and in fact their masses are protected by the unbroken flavour group. In fact these two massless composites have the same flavour anomaly as the preons, with respect to the unbroken flavour group. We will think of these two massless composites as ordinary fermions.

Now imagine gauging a subgroup G_w of G_F which will be the weak group. Suppose it is the SU(3) formed as the diagonal sum of $SU(3)_{LEFT}$ and $SU(3)_{RIGHT}$. The condensate of eq. (6.1) breaks $SU(3)_{WEAK} \to SU(2)_{WEAK} \times U(1)$ where the U(1) is associated with the eight generator λ_8 of $SU(3)_{WEAK}$. The unbroken weak gauge group has four generators and the four generators of the broken weak group connect p_A and p_B to p_C in a left-right symmetric manner. The bosons corresponding to these generators will give p_A and p_B a mass on the order of $\alpha_w \Lambda_{TC}$. At the same time the two massless composite states will also get a mass on the order of $\alpha_w \Lambda_{TC}$. This can be thought of in two ways. The massless composite picks up a small mass because one of its constituent preons gets a mass. Alternatively you can imagine the massless composite emits a heavy boson and becomes a massive composite which reabsorbs the boson. These different views are shown in fig. 22 where the massless states pick up small masses from massive states.

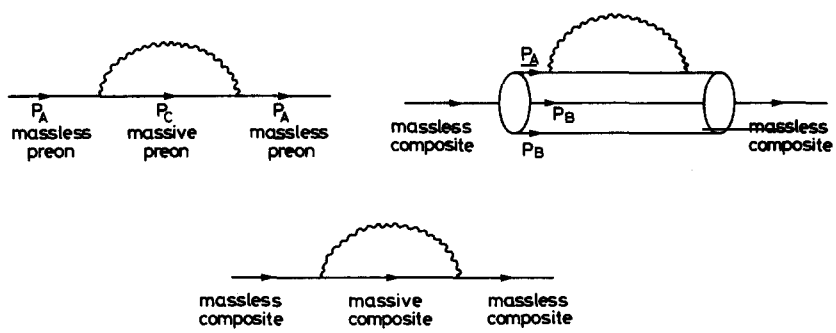

Fig. 22.

Of course this example is totally unrealistic but it has many interesting features. In particular it is possible to give mass to light fermions without introducing a new scale of interactions Λ_E. Ordinary fermion masses might be α_w or α_w^2 effects on the technicolour scale. The search for a truly realistic model of this sort is underway.

It is not clear that this type of model will avoid the difficulties which exist in extended technicolour models. The simplest technicolour model works beautifully in helping us understand weak interaction dynamics but it does not provide fermion masses. The complications required to give fermion masses lead to severe problems. In the original weak interaction theory with fundamental scalars, masses arise from arbitrary Yukawa couplings. The theory can fit the data but no insight is gained. Understanding the origin of fermion masses is one of the great challenges facing particle physicists today.

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