Pion Leptonic Decays and Supersymmetry

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We compute supersymmetric contributions to pion leptonic ($\pi \ell$) decays in the Minimal Super-
symmetric Standard Model (MSSM). When R-parity is conserved, the largest contributions to the
ratio $R_{e/\mu} \equiv \Gamma(\pi^+ \rightarrow e^+ \nu_e (\gamma)) / \Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu (\gamma))$ arise from one-loop ($V - A$) \times ($V - A$) corrections.
These contributions may be comparable to sensitivities at future experiments if the mass of the
lightest chargino is $\lesssim 250$ GeV and if the masses of the first and second generation left-handed
lepton superpartners (sleptons) are such that (1) the lighter slepton has mass $\lesssim 300$ GeV and (2)
the heavier slepton mass is greater by a factor of two or more. We also analyze R-parity violating
interactions, which may produce a detectable deviation in $R_{e/\mu}$, while remaining consistent with all
other precision observables.

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I. INTRODUCTION

Low-energy precision tests provide important probes of new physics which are complementary to collider
experiments[1–3]. In particular, effects of weak-scale supersymmetry (SUSY) — one of the most popular extensions
of the Standard Model (SM) — can be searched for in a wide variety of low-energy tests: muon $(g - 2)$ [4], $\beta$
and $\mu$-decay [5, 6], parity-violating electron scattering [7],
electric dipole moment searches [8], and SM-forbidden transitions like $\mu \rightarrow e\gamma$ [9], etc (for a recent review, see Ref. [10]). In this paper, we compute the SUSY contributions to pion leptonic ($\pi \ell$) decays and analyze the conditions under which they can be produced observable effects in the next generation of experiments.

In particular, we consider the ratio $R_{e/\mu}$, defined by

$$R_{e/\mu} \equiv \frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e + e^+ \nu_e \gamma)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma)}.$$

The key advantage of $R_{e/\mu}$ is that a variety of QCD effects — such as the pion decay constant $F_\pi$ and lepton flavor
independent QCD radiative corrections, which bring large theoretical uncertainties — cancel from this ratio.
Indeed, $R_{e/\mu}$ is one of few electroweak observables which involve hadrons and yet are precisely calculable (see [11]
for discussion and Refs. [12, 13] for explicit computations). Moreover, measurements of this quantity provide
unique probes of deviations from lepton universality of the charged current (CC) weak interaction in the SM that
are induced by loop effects and possible extensions of the SM. In the present case, we focus on effects in SUSY that
can lead to deviations from lepton universality.

Currently, the two best theoretical estimates of $R_{e/\mu}$ in the SM are

$$R_{e/\mu}^{SM} = (1.2352 \pm 0.0005) \times 10^{-4} \quad [12]$$
$$= (1.2356 \pm 0.0001) \times 10^{-4} \quad [13]$$

where the theoretical uncertainty comes from pion structure effects. By utilizing chiral perturbation theory, it may be possible to reduce this uncertainty even further [14]. Experimentally, the most precise measurements of $R_{e/\mu}$ have been obtained at TRIUMF [15] and PSI [16], taking the average of which gives [17]

$$R_{e/\mu}^{EXPT} = (1.230 \pm 0.004) \times 10^{-4},$$

in agreement with the SM. Future experiments at these facilities will reeterminate $R_{e/\mu}$, aiming for precision at the level of $< 1 \times 10^{-3}$ (TRIUMF [18]) and $5 \times 10^{-4}$ (PSI [19]). These projected uncertainties are close to the conservative estimate of theoretical uncertainties given in Ref. [12].

Deviations $\Delta R_{e/\mu}$ from the SM predictions in Eq. (2) would signal the presence of new, lepton flavor-dependent
physics. In the Minimal Supersymmetric Standard Model (MSSM), a non-vanishing $\Delta R_{e/\mu}^{SUSY}$ may arise from either tree-level or one-loop corrections. In section II, we consider contributions to $\Delta R_{e/\mu}^{SUSY}$ arising from R-parity conserving interactions (Fig. 1). Although tree-level charged Higgs exchange can contribute to the rate $\Gamma(\pi^+ \rightarrow \ell^+ \nu_\ell (\gamma))$, these effects are flavor-independent and cancel from $R_{e/\mu}$. One-loop corrections induce both scalar and vector semi-leptonic dimension six operators.

Such interactions contribute via pseudoscalar and axial vector pion decay matrix elements, respectively. We show that the pseudoscalar contributions are negligible unless the ratio of the up- and down-type Higgs vacuum expectation values (vevs) is huge $(\nu_u/\nu_d \approx \tan \beta \gtrsim 10^3)$. For smaller $\tan \beta$ the most important effects arise from one-loop contributions to the axial vector amplitude, which we analyze in detail by performing a numerical scan over MSSM parameter space. We find that experimental observation of SUSY loop-induced deviations at a significant level would require further reductions in both the experimental error and theoretical SM uncertainty. Such improvements could lead to stringent tests of "slepton