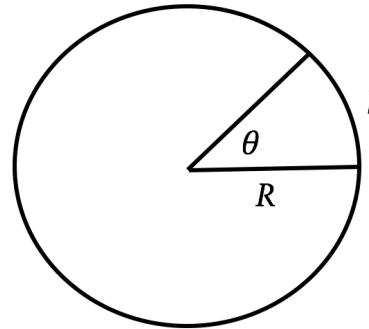


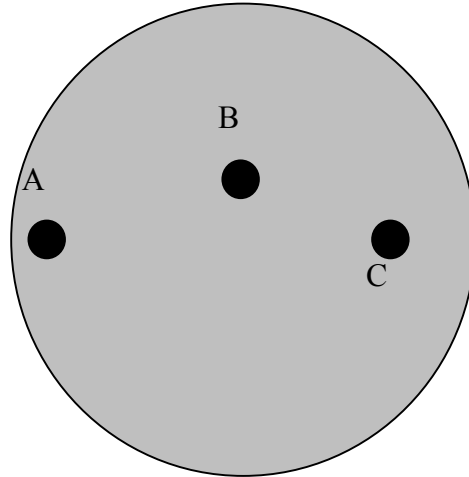
First Name: _____ Last Name: _____

Consider a circle of radius R as shown below.



1. What is the circumference of the circle in terms of R ?
2. Consider the arc corresponding to the angle θ shown in the Figure. Find the length of the arc l in terms of R , **assuming that θ is measured in degrees.**
3. Find the length of the arc l in terms of R , **assuming that θ is measured in radians.**
4. Use the above result to explain why radians are an awesome way to measure angles.

A wheel is spinning counterclockwise at a constant angular speed about an axis that passes through its center and is perpendicular to the page. This type of motion is referred to as rigid body motion since no point on the wheel moves relative to any other point. The diagram below represents a snapshot of the wheel at one instant in time (t_1). Three points on the wheel are labeled (A-C).



5. Rank points A-C based on the total linear distance each point travels during one complete rotation of the wheel, from the greatest to the least linear distance.

Greatest

Least

Explain your reasoning.

6. Rank points A-C based on the total amount of time each point needs to complete one rotation about the axis, from the greatest to the least amount of time.

Greatest

Least

Explain your reasoning.

7. Rank points A-C based on the linear speed of each point, from the fastest to the slowest.

Fastest

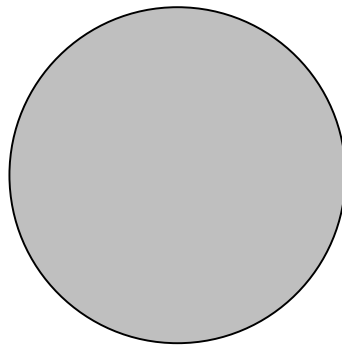
Slowest

Explain your reasoning.

8. Draw vectors on the diagram for Question #5 to represent the linear velocity of each point (A-C) at that particular instant. The relative lengths and directions of your vectors matter.

At a later time (t_2), the wheel has completed one-fourth of a rotation.

9. On the diagram below, sketch points A-C at time t_2 . Draw vectors on this diagram to represent the linear velocity of each point (A-C) at that instant. The relative lengths and directions of your vectors matter.



10. Is there one single linear velocity vector \vec{v} that applies to every point on the wheel at all times? Explain your reasoning.

11. Rank, from greatest to least, the magnitude of the angular displacement of points A-C between times t_1 and t_2 .

Greatest

Least

Explain your reasoning.

12. Two students are discussing their answers to Question #11.

Student 1: *Clearly all points move the same angular distance because the object is a rigid body, so the angle swept in all cases is the same.*

Student 2: *I disagree. Precisely because the wheel is a rigid body, point A must travel a larger distance. Since angular displacement is proportional to distance, then point A must have a larger angular displacement than B and C.*

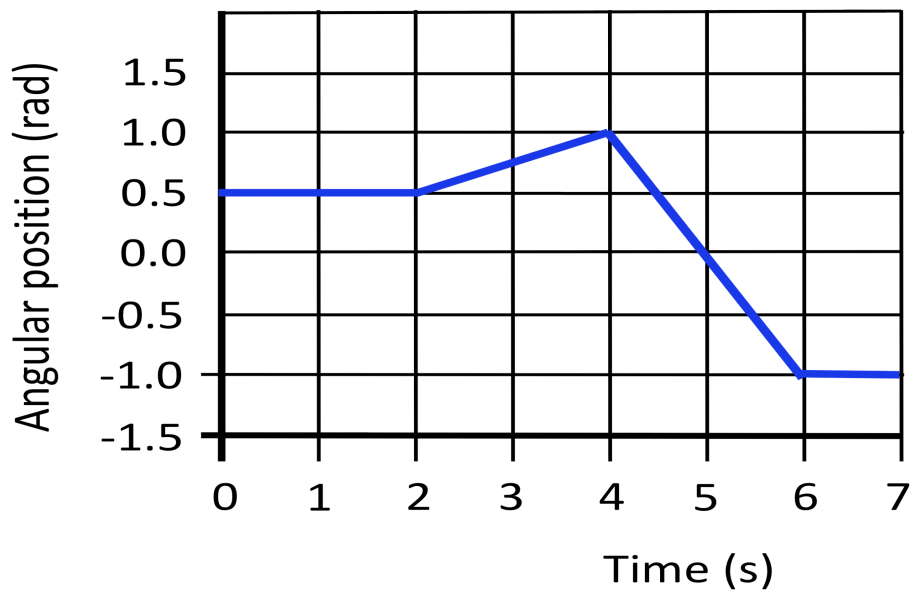
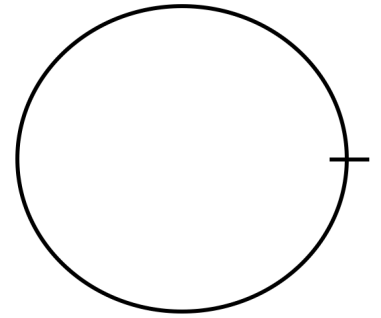
Do you agree or disagree with either or both of the students? Explain your reasoning, and help explain the problem with their reasoning to the student that you disagree with.

Angular Kinematics

The graph below shows the angular position $\theta(t)$ of a wheel which lies in the xy plane for $0 \text{ s} \leq t \leq 7 \text{ s}$. This graph is used for Questions #9-13.

Note that by convention, then angle θ is **always** measured starting from the x -axis in a counter-clockwise direction when θ is positive, or clock-wise if θ is negative.

Tip: an easy way to visualize rotation is to imagine there is a little notch on the wheel as shown in the picture at right. The rotation angle θ is the angle that this notch makes after you rotate the circle by an angle θ .

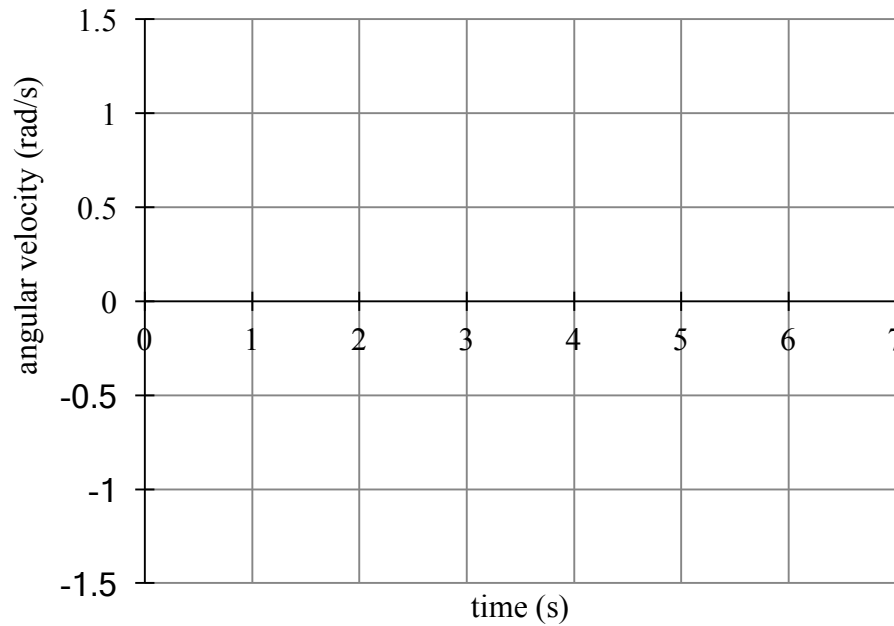


13. Determine if the wheel is rotating clockwise, counterclockwise, or not rotating at all.

- $0 \text{ s} < t < 2 \text{ s}$:
- $2 \text{ s} < t < 4 \text{ s}$:
- $4 \text{ s} < t < 5 \text{ s}$:
- $5 \text{ s} < t < 6 \text{ s}$:
- $6 \text{ s} < t < 7 \text{ s}$:

Explain your reasoning.

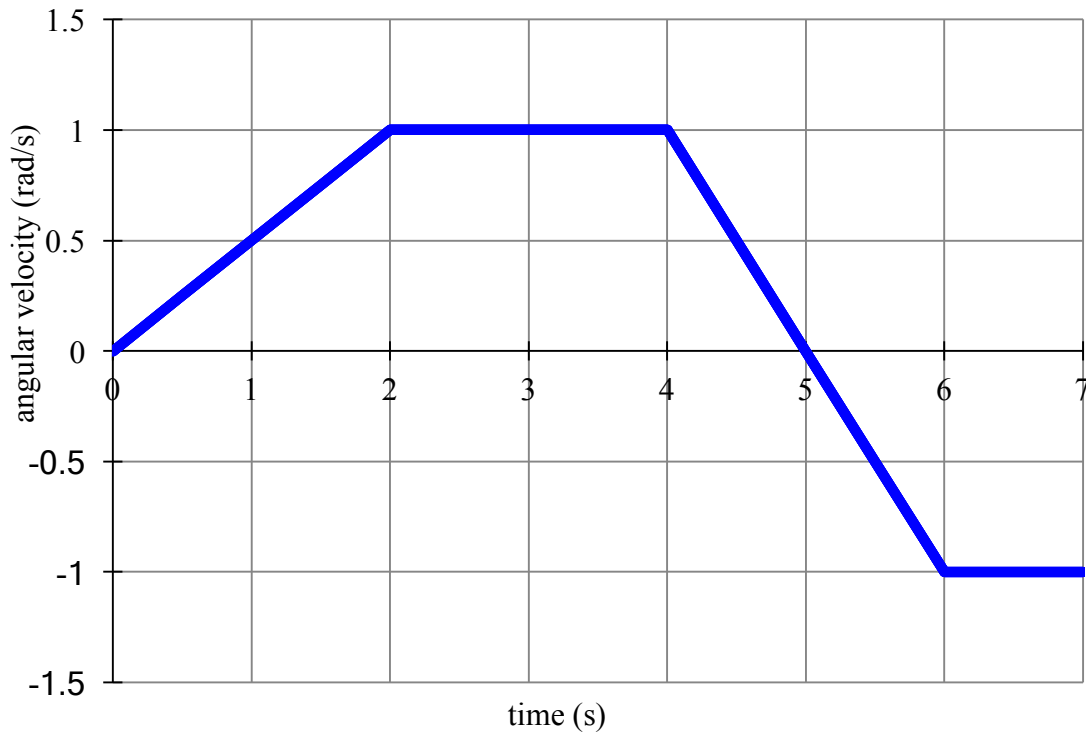
14. Just like velocity $v = dx/dt$, the **ANGULAR VELOCITY** ω is $\omega = d\theta/dt$. Determine $\omega(t)$ and graph it below.



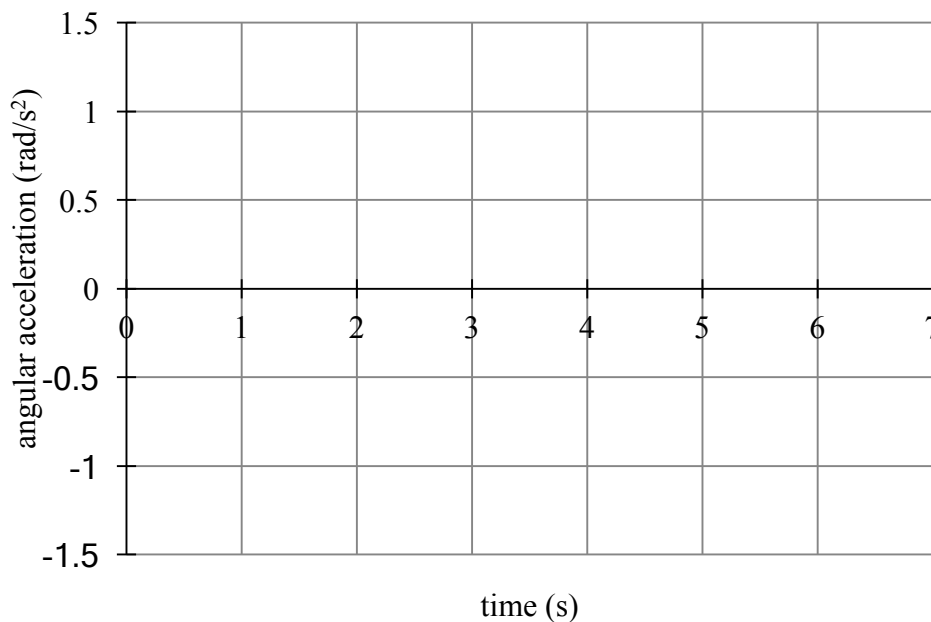
15. If ω is positive, is the wheel rotating clock-wise or counter clock-wise? Explain. **Hint:** how is the angle θ changing in an amount of time dt ?

Angular Kinematics

The graph below shows the angular velocity $\omega(t)$ of a wheel which lies in the xy plane for $0 \text{ s} \leq t \leq 7 \text{ s}$. This graph is used for Questions #16-20.



16. Just as the acceleration $a = dv/dt$, the **ANGULAR ACCELERATION** $\alpha = d\omega/dt$. Determine $\alpha(t)$ for the situation described in the graph above, and plot it in the graph below.



17. Determine if the wheel is rotating clockwise, counterclockwise, or not rotating at all.

- $0 \text{ s} < t < 2 \text{ s}$:
- $2 \text{ s} < t < 4 \text{ s}$:
- $4 \text{ s} < t < 5 \text{ s}$:
- $5 \text{ s} < t < 6 \text{ s}$:
- $6 \text{ s} < t < 7 \text{ s}$:

Explain your reasoning.

18. Determine if the wheel's angular speed is increasing, decreasing, or constant for:

- $0 \text{ s} < t < 2 \text{ s}$:
- $2 \text{ s} < t < 4 \text{ s}$:
- $4 \text{ s} < t < 5 \text{ s}$:
- $5 \text{ s} < t < 6 \text{ s}$:
- $6 \text{ s} < t < 7 \text{ s}$:

Explain your reasoning.

19. Determine the time between $t = 0$ s and $t = 7$ s when the wheel has its largest angular displacement relative to its starting position. (Hint: you should be able to determine this just by looking at the graph.) Explain your reasoning.
20. Angular kinematics work exactly the same way as linear motion kinematics. Below, write the equation of motion for $x(t)$ and $v(t)$ for objects moving with constant acceleration (Hint: think back to the beginning of class!).
21. The angular analog of position x is angle θ . The angular analog of velocity is the angular velocity ω , and the angular analog of acceleration is the angular acceleration α . Use these analogs to write the equations of motion for angular motion with constant angular acceleration.

22. Use the above formula to write $\theta(t)$ and $\omega(t)$ for the graph above in each of the intervals. Assume that $\theta(t) = -2$ at $t=0$.

- $0 \text{ s} < t < 2 \text{ s}$:

- $2 \text{ s} < t < 4 \text{ s}$:

- $4 \text{ s} < t < 5 \text{ s}$:

- $5 \text{ s} < t < 6 \text{ s}$:

- $6 \text{ s} < t < 7 \text{ s}$:

23. Determine $\theta(t)$ for the situation described in Question #16 and graph it below. Your graph should be qualitatively and quantitatively correct. Once again, assume $\theta(t) = -2$ at $t=0$.

