## **Rotational Dynamics**

First Name: \_\_\_\_\_ Last Name:

A uniform rod (of length L and mass M) is initially standing perfectly vertically. It is given an extremely tiny bump and begins to rotate clockwise about a frictionless pin as shown. The moment of inertia of the rod about its pivot point is  $I_{rod} = \frac{1}{3}ML^2$ . The three labeled positions refer to it when it is still perfectly vertical, when it has rotated through some non-specific angle and when it is perfectly horizontal and has rotated through an angle of  $\frac{\pi}{2}$  radians. Let the time interval between positions A and C be referred to as  $\Delta t_{rod}$ . Assume that is +x axis is to the right and the +y axis is up.

1. Is there a torque about the pivot due to the weight of the rod when it is at position A? If there is, determine its direction in an xyz coordinate system. If there is not, explain why not. Don't forget that  $\vec{\tau} = \vec{r} \times \vec{F}$ . You should make a drawing illustrating the position and force vectors to help you.

2. Is there a torque about the pivot due to the weight of the rod when it is at position B? If there is, determine its direction in an xyz coordinate system. If there is not, explain why not.

3. Is there a torque about the pivot due to the weight of the rod when it is at position C? If there is, determine its direction in an xyz coordinate system. If there is not, explain why not.

1

С

4. Does the magnitude of the torque about the pivot due to the weight of the rod increase, decrease, or remain the same during  $\Delta t_{rod}$ ? Explain your reasoning.

- 5. Two students are discussing their answers to Question #4.
  - **Student 1:** The magnitude of the gravitational torque should stay the same because the radius of the rod, the mass of the rod, and the gravitational force exerted by the Earth on the rod all stay the same.
  - **Student 2:** I disagree. The magnitude of the torque should increase because the angle between the radius vector and the gravitational force gets closer and closer to 90° as the rod rotates. The torque actually reaches its maximum value at the instant when the angular displacement is  $\frac{\pi}{2}$  radians.

Do you agree or disagree with either or both of the students? Explain your reasoning.

6. Does the magnitude of the angular acceleration of the rod increase, decrease, or remain the same during  $\Delta t_{rod}$ ? Explain your reasoning.

7. Use conservation of energy to find the angular velocity of the bar when it is horizontal (point C).

Each figure below shows a different object (A-C) and its rotation axis (dashed line). The moments of inertia for these objects about their rotation axes are provided. All objects have the same mass *m* and the same radius *R*. Starting from rest, a force F is applied to each object to make the rotate counterclockwise (as viewed from above) about its rotation axis. The force F need not be the same for all 3 objects. You do know, however, that the amount of time  $\Delta t$  that it takes for each of the 3 objects to reach an angular velocity  $\omega_f$  is the same.



Rotational Dynamics II Tutorial, University of Arizona Contact: (Drew) <u>milsom@email.arizona.edu</u> 8. Rank the objects based on the magnitude of their angular accelerations during  $\Delta t$ , from largest to smallest. Remember all objects end up with the same angular velocity in the time  $\Delta t$ .

Largest

Explain your reasoning.

9. Rank the objects based on the magnitude of the torque each one experiences, from largest to smallest.

Largest

Explain your reasoning.

10. Rank the objects based on the magnitude of the force acting on each one, from largest to smallest.

11. Rank the objects based on their final rotational kinetic energies, from largest to smallest.

Largest

Largest

Explain your reasoning.

Smallest

Smallest

Smallest

Smallest

Let's revisit the problem from questions 1 through 5, but now using a solid disc. A solid disc (of diameter L and mass M) is initially standing perfectly vertically. It is given an extremely tiny bump and begins to rotate clockwise about a frictionless pin as shown. The moment of inertia of the disc about its pivot point is  $I_{\text{disc}} = \frac{3}{8} \text{ML}^2$ .

Label the time interval between the instant when the disc starts to move and the instant when its angular displacement is  $\frac{\pi}{2}$  radians (as shown in the figure) as  $\Delta t_{\text{disc}}$ .

- 12. Is the gravitational torque on the disc at the end of  $\Delta t_{\text{disc}}$  greater than, less than, or the same size as the gravitational torque on the rod at the end of  $\Delta t_{\text{rod}}$ ? Explain your reasoning.

13. Is the angular acceleration of the disc at the end of  $\Delta t_{\text{disc}}$  greater than, less than, or the same size as the angular acceleration of the rod at the end of  $\Delta t_{\text{rod}}$ ? Explain your reasoning.

14. Is  $\Delta t_{\text{disc}}$  greater than, less than, or the same size as  $\Delta t_{\text{rod}}$ ? Explain your reasoning.