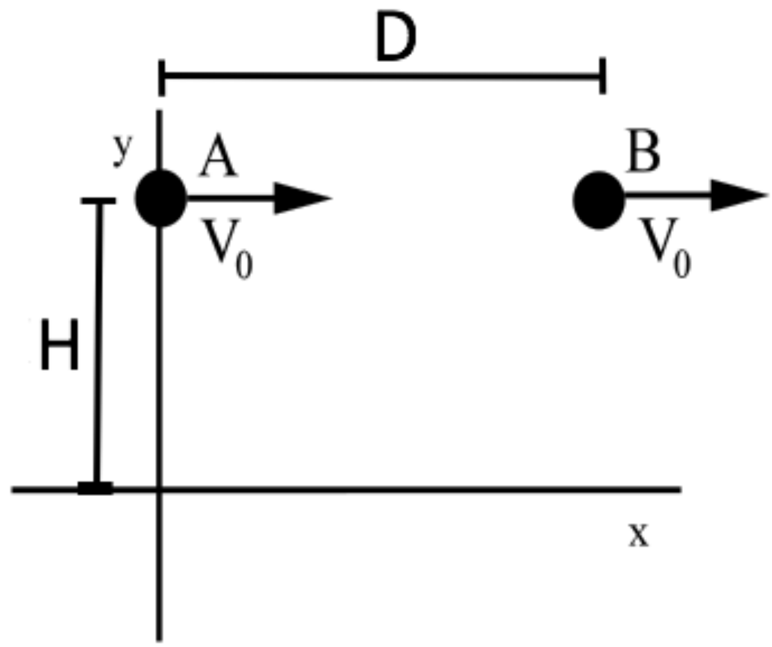
First Name:____________________ Last Name:________________________________

A particle of mass *m* is sliding across a frictionless horizontal surface. Its position is shown in a top view at two different times (A and B).

1. Draw the position vector for the particle on the diagram at the times corresponding to positions A and B.



2. Determine the direction of the particle's angular momentum when it is at positions A and B relative to the origin shown. Your answer should be stated in an *xyz* coordinate system.

3. Given that $\vec{L} = \vec{r} \times \vec{p}$ would you expect the magnitude of the angular momentum at points A and B to be the same? Or is one larger than the other? Explain your reasoning.

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4. Two students are discussing their answers to Question #3.

Student 1: *I think the angular momentum is larger at point B. The equation used to calculate it is* $\vec{L} = \vec{r} \times \vec{p}$. *The magnitude of the position vector is definitely larger at point B and the magnitude of the linear momentum vector is the same size at each point, so the angular momentum should be larger at point B.*

Student 2: *I disagree. I think this is a bit tricky. You are forgetting about the cross product there. At point A, the position vector and the linear momentum vector are perpendicular so the cross product will be large. However, at point B while* $|\vec{r}|$ *is larger the vectors are no longer perpendicular so I am not sure which angular momentum is larger. In fact, I think it is possible that they are the exact same size.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

5. In terms of the quantities given in Question #1, determine the magnitude of the angular momentum at points A and B.

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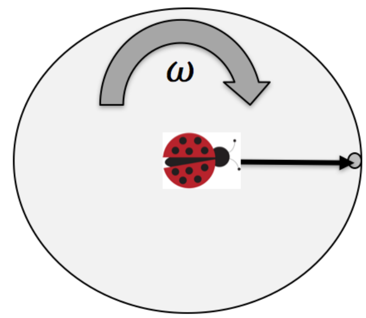
6. In Question #5 you should have found that the angular momentum is exactly the same at points A and B. Is the particle experiencing a net torque? Explain why or why not.



Consider a spinning disc rotating around a frictionless axis that goes through center of the disc. The disc has a mass *M* and a radius *R*, and is rotating with an angular velocity ω as shown in the figure along. A ladybug of mass m is sitting at the middle. It decides to walk along a radial path to the edge of the record. Once it gets there, it just rotates with the disc.

7. When is the angular momentum of the ladybug+disc system higher: when the ladybug is at the middle, or when it is at the rim? Or is the angular momentum the same in both instances? **Justify your answer.**

8. When is the angular velocity of the record + lady bug larger: when the lady bug is at the center of the record, or when the lady bug is at the rim? **Justify your answer.**

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9. Assuming the angular velocity of the record when the lady bug is at the middle is ω_i, find the angular velocity of the record ω_f when the ladybug is at the edge of the record.

10. Describe what would happen if the ladybug jumped straight up from its point of view. Would the record spin with a higher angular velocity, a lower angular velocity, or would the angular velocity of the record remain unchanged? Would the ladybug land back on the record, or off the record? If off, where would the lady bug land?

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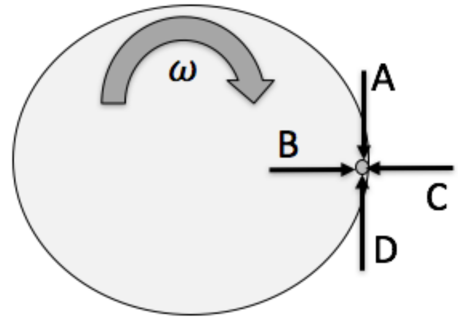
Consider now a rotating disc that has no ladybug on it. Fear not! We shall remedy the situation forthwith. A ladybug lands on the record at the point shown. The ladybug can land along any of the 4 directions shown at right. Just before landing, it has a speed v. As soon as it lands, the ladybug rotates along with the record. Weeeeeeeeeeee!



11. Rank order the paths A through D according to the final angular velocity of the record after the ladybug lands on the record.

Smallest angular velocity   Largest angular velocity

12. Of the 4 possible paths, is it possible for the ladybug to choose a path and a speed v such that, when it lands, it does not change the angular velocity of the record? **Justify your answer**.

13. Suppose the ladybug flies in along the path identified in question 12. How fast must it be flying in order for the angular velocity of the disc to not change when it lands on the rim of the disc?

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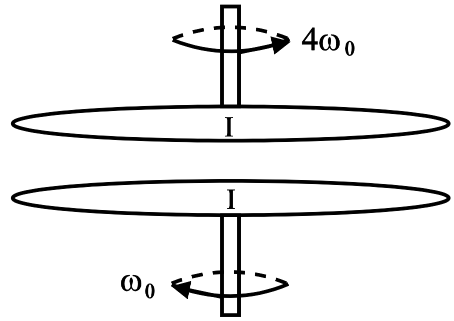
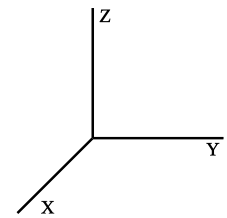
The figure shows two identical clutch plates (so they have the same moment of inertia *I*) rotating about frictionless axles with different angular velocities. The upper plate will be lowered onto the lower plate ("engaging" the clutch) and after a short time they will rotate together with the same final angular velocity.



14. Before the plates touch, what is the direction of the angular momentum of the top plate? Use the coordinate system defined in this diagram and explain your reasoning.



15. Before the plates touch, what is the direction of the angular momentum of the bottom plate? Use the coordinate system defined in this diagram and explain your reasoning.

16. Determine the total angular momentum of the system consisting of both plates before they are brought together.

17. Does the total angular momentum of the system of two plates change after the plates touch, or does it stay the same? Explain your reasoning.

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18. Use the conservation of angular momentum to find the magnitude and direction of the final angular velocity of the top and bottom plates after they come together (in terms of ω_0).

19. Determine the ratio of the final kinetic energy to the initial kinetic energy $\left(\frac{K_f}{K_i}\right)$. If your answer is > 1, explain where the additional energy came from. If your answer is < 1, explain why energy was lost and what happened to it. Lastly, if energy was conserved, explain why it was conserved.

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