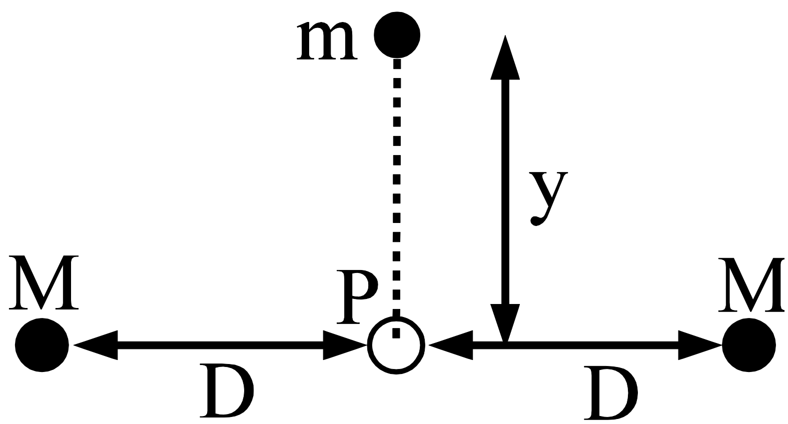
First Name:____________________ Last Name:________________________________

Three masses are arranged as shown in an otherwise empty region of space. The two masses *M* are fixed in place, but *m* is released from rest and allowed to move. The label P refers to the point in space along the line connecting the two *M's* which is equidistant from each *M*.

1. On the diagram, draw vectors representing the individual gravitational forces that mass *m* feels and then graphically add them together to obtain the net gravitational force on *m*.



2. When *m* is directly between the two masses *M*, what is the magnitude of its acceleration? Explain your reasoning.

3. Imagine that *y* is initially very large so $y_i \cong \infty$. As mass *m* moves straight downward and approaches point P, would you expect the net force on it to increase in magnitude, decrease in magnitude, increase then decrease, decrease then increase or stay the same? Explain your reasoning.

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4. Two students are discussing their answers to Question #3.

Student 1: *I think the net force will increase that entire time. As m gets closer to the two M's, the magnitude of each gravitational force on m must increase rapidly since the gravitational force law has radius squared in the denominator. This means that the magnitude of the net gravitational force must also increase.*

Student 2: *I disagree. I think there is a maximum net force somewhere between the initial position and point P. The force does increase initially, but it has to decrease at some point since the net force must be zero at point P. When m gets close to that point, the individual forces are almost completely horizontal so the vertical components which add together are very small.*

Do you agree or disagree with either or both of the students? Explain your reasoning.

5. Determine an expression for the net gravitational force in terms of G, D, M, m and y. Don't forget that forces are *vectors*.

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6. Use your answer to Question #5 to determine the value of y for which the net force is a maximum.

7. When *m* reaches point P, will it keep moving or will it come to rest and remain at that point? Explain your reasoning.

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8. Two students are discussing their answers to Question #7.

   Student 1: *It will definitely keep moving past P. The entire time it moved towards P it was speeding up. So when it gets to P it will have the highest speed it has had up to that point. Since there is no acceleration at point P, its velocity cannot change and it keeps moving and reaches negative values of y.*

   Student 2: *I disagree. The acceleration is zero at point P so the velocity must be zero as well. It will stop at point P and remain there indefinitely.*

   Do you agree or disagree with either or both of the students? Explain your reasoning.

9. Use the conservation of mechanical energy to determine the speed of *m* at the instant it passes through point P.

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10. After *m* passes point P, will it speed up, slow down, or continue moving at the same speed? Explain your reasoning.

11. Describe the motion of *m* over a very long time.

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12. Imagine that you start this "experiment" over. In this case, you release the mass from rest at a distance y where $y \ll D$. Analyze your answer to Question #5 in that limit and explain why this mass will undergo simple harmonic motion.

13. Determine the period for that simple harmonic motion.

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Comments about this tutorial:

This is definitely intended for calculus-based courses. It could be used in algebra-based courses if question 6 is skipped but it would be quite challenging.

This is meant to allow them to combine lots of material they have covered throughout the semester (kinematics, vectors, forces, energy conservation and simple harmonic motion) in one cohesive problem. The mass configuration is certainly frequently seen in textbooks with either point masses or point charges.

The sentence “Don’t forget that forces are *vectors*” was added to Question #5 since probably 2/3 of the class were adding them like scalars (even though their graphical addition diagrams were correct).

In 50 minutes, most groups are somewhere between Questions #7 and 9. Question #5 creates most of the delays. They have performed pretty well on the minimization in Question #6.

Questions #12 and 13 certainly connect this to simple harmonic motion. Since some faculty may cover gravity first, they can clearly eliminate these questions.

Tutorial Source(s):

All questions were written by Drew Milsom.

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