Physics 371 Homework # 1

Solutions

1) a) $\underline{1} = \int_{-\infty}^{\infty} \rho(x)\,dx$ let $y = x - a$

$$\underline{1} = A\int_{-\infty}^{\infty} e^{-\lambda y^2}\,dy = A\sqrt{\frac{\pi}{\lambda}}$$

$A = \sqrt{\frac{\lambda}{\pi}}$ (see mathematical necessities, section 2).

b) $\langle y\rangle = \langle x\rangle - a = \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} y\, e^{-\lambda y^2}\,dy \rightarrow 0$

since integrand is odd

$\Rightarrow \langle x\rangle = a$

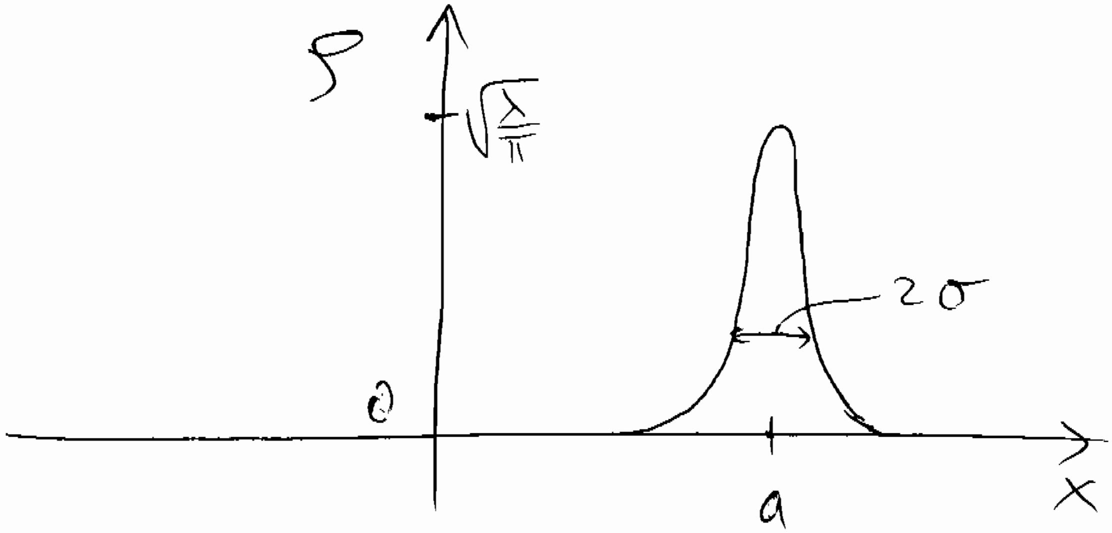
$$\langle x^2\rangle = \langle (y+a)^2\rangle = \langle y^2\rangle + 2a\underbrace{\langle y\rangle}_{0} + a^2$$

$$= a^2 + \langle y^2\rangle = a^2 + \frac{1}{2\lambda}$$

$$\langle y^2\rangle = \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} y^2 e^{-\lambda y^2}\,dy = \sqrt{\frac{\lambda}{\pi}}\;\frac{1}{2}\frac{\sqrt{\pi}}{\lambda^{3/2}} = \frac{1}{2\lambda}$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2\lambda}$$

c)



2) $$\langle E \rangle = \frac{1}{2m} \langle p_x^2 \rangle + \frac{m\omega^2}{2} \langle x^2 \rangle$$

$$\langle x \rangle = 0 = \langle p_x \rangle \Rightarrow \langle E \rangle = \frac{1}{2m} \Delta p_x^2 + \frac{m\omega^2}{2} \Delta x^2$$

Minimum $\langle E \rangle$ for minimum Δp_x + Δx, but $\Delta p_x \, \Delta x \geq \hbar/2$. Best we can do is $\Delta x \, \Delta p_x = \hbar/2$

$$\langle E \rangle \geq \frac{1}{2m} \Delta p_x^2 + \frac{m\omega^2}{2} \frac{\hbar^2}{4 \Delta p_x^2}$$

Find minimum:

$$0 = \frac{\partial \langle E \rangle}{\partial \Delta p_x} = \frac{\Delta p_x}{m} - \frac{m\omega^2 \hbar^2}{4 \Delta p_x^3}$$

$$\Delta p_x^4 = \frac{m^2 \omega^2 \hbar^2}{4} \Rightarrow \Delta p_x^2 = \frac{m\omega\hbar}{2}$$

$$\Delta x^2 = \left(\frac{\hbar}{2\Delta p_x}\right)^2 = \frac{\hbar^2}{4} \frac{2}{m\omega\hbar} = \frac{\hbar}{2m\omega}$$

$$\min \langle E \rangle = \frac{1}{2m} \frac{m\hbar\omega}{2} + \frac{m\omega^2}{2} \frac{\hbar}{2m\omega}$$

$$= \frac{\hbar\omega}{2}$$

Note: This is the correct quantum ground-state energy of a harmonic oscillator.