Physics 371 HW #5

Solutions

$V(x)=\Lambda\,\delta(x)$

1)

i) $\psi_{I}(0)=\psi_{II}(0) \Rightarrow 1+r=t$

ii) $\psi_{II}'(0)-\psi_{I}'(0)=\dfrac{2m\Lambda}{\hbar^2}\psi_{II}(0)$

$$ikt-ik(1-r)=\frac{2m\Lambda}{\hbar^2}t$$

$$t+r-1=\frac{2m\Lambda}{i\hbar^2 k}t=\frac{2t}{ik\ell}$$

Use $r=t-1$

$$\left(2-\frac{2}{ik\ell}\right)t=2$$

$$t = \frac{1}{1 - \frac{1}{ik\ell}} = \frac{ik\ell}{ik\ell - 1}$$

a)

$$r = t - 1 = \frac{1}{ik\ell - 1}$$

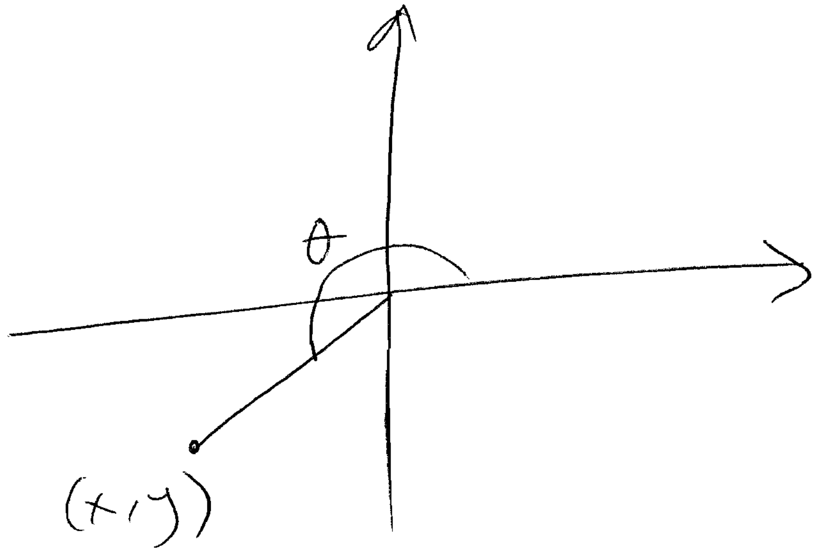
b) $$T = |t|^2 = \frac{(k\ell)^2}{1 + (k\ell)^2}$$

$$R = |r|^2 = \frac{1}{1 + (k\ell)^2}$$

c) $$r = \sqrt{R}\, e^{i\theta} = \frac{-ik\ell - 1}{1 + (k\ell)^2} = x + iy$$

$$x = -\frac{1}{1 + (k\ell)^2} \qquad y = \frac{-k\ell}{1 + (k\ell)^2}$$

$$\tan\theta = \frac{y}{x}$$ θ in 3rd quadrant.



$$\theta(k) = \pi + \tan^{-1}(k\ell)$$

$$t = ik\ell r = i k\ell \sqrt{R}\, e^{i\theta}$$

$$T = (k\ell)^2 R \qquad \sqrt{T} = k\ell\sqrt{R}$$

$$\therefore \quad t = i\sqrt{T}\, e^{i\theta}$$

$$S = e^{i\theta}\begin{pmatrix} \sqrt{R} & i\sqrt{T} \\ i\sqrt{T} & \sqrt{R} \end{pmatrix}$$

2) $$T_{12} = \frac{1}{1 + \frac{4R}{T^2}\sin^2(kL + \theta(k))}$$

$$\frac{4R}{T^2} = \frac{4(1+(k\ell)^2)}{(k\ell)^4}$$

$$\theta(k) = \pi + \tan^{-1}(k\ell)$$

$\Rightarrow$ Plot T_{12} vs. $k\ell$

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In[1]:= teta[x_] := -Pi + ArcTan[x]

In[25]:= a := 3 Pi

In[3]:= T[x_] := 1/ (1 + 4 (1 + x^2) (Sin[a x + teta[x]])^2 /x^4)

In[27]:= Plot[T[x], {x, 0.1, 1}]
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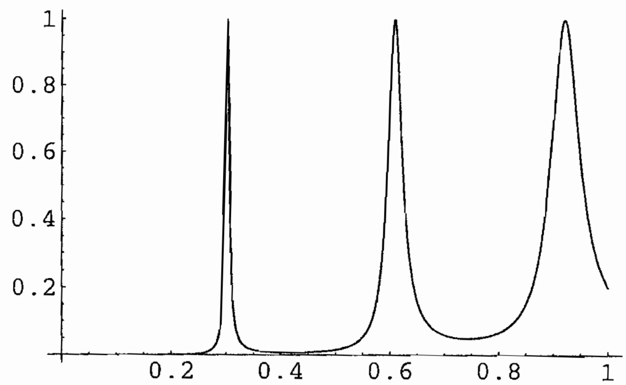


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Out[27]= - Graphics -
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Discussion:

c.f. Lecture 12