Physics 371
HW # 7 Solutions

6) Given $\Psi(x) = \sum_n C_n \psi_n(x)$, let

$$\Delta\hat{Q} = \hat{Q} - \langle\Psi|\hat{Q}|\Psi\rangle = \hat{Q} - \langle\hat{Q}\rangle.$$

$$\Delta\hat{Q}\,\Psi = \sum_n \left(q_n - \langle\hat{Q}\rangle\right) C_n \psi_n$$

$$(\Delta\hat{Q})^2\Psi = \sum_n \left(q_n - \langle\hat{Q}\rangle\right)^2 C_n \psi_n$$

$$\langle(\Delta\hat{Q})^2\rangle = (\Delta Q)^2 = \int dx\, \Psi^*(x) \sum_n \left(q_n - \langle\hat{Q}\rangle\right)^2 C_n \psi_n$$

$$= \sum_{n'} C_{n'}^* \sum_n \left(q_n - \langle\hat{Q}\rangle\right)^2 C_n \underbrace{\int dx\, \psi_{n'}^*(x)\psi_n(x)}_{\delta_{nn'}}$$

$$= \sum_n |C_n|^2 \left(q_n - \langle\hat{Q}\rangle\right)^2$$

$(\Delta Q)^2 = 0$ only if every term in sum is zero, which requires $|C_n|^2 = 0$ or $q_n = \langle\hat{Q}\rangle$ (which must occur at least once). Q.E.D.

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Solutions

2) Griffiths 3.31

$$\frac{d}{dt}\langle xp\rangle = \frac{1}{i\hbar}\langle [\hat{x}\hat{p}, \hat{H}]\rangle$$

$$[\hat{x}\,\hat{p}, \hat{H}] = \hat{x}[\hat{p}, \hat{H}] + [\hat{x}, \hat{H}]\hat{p}$$

$$= \hat{x}[\hat{p}, V(x)] + [\hat{x}, \frac{\hat{p}^2}{2m}]\hat{p}$$

$$= \hat{x}\frac{\hbar}{i}\frac{\partial V}{\partial x} + \frac{i\hbar}{m}\hat{p}^2$$

(using results of 2a–c).

$$\frac{d}{dt}\langle xp\rangle = \langle \frac{p^2}{m}\rangle - \langle x\frac{\partial V}{\partial x}\rangle$$ ✓

$0 = \frac{d}{dt}\langle Q\rangle$ in a stationary state since

$$\langle Q(t)\rangle = \int dx\, \Psi^*(x,t)\,\hat{Q}\,\Psi(x,t) = \int dx\, \psi^*(x)\,\hat{Q}\,\psi(x)$$

if $\Psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$.

3: 2.15) $E = \frac{\hbar\omega}{2}$

$$x_\pm = \pm\sqrt{\frac{2E}{m\omega^2}} = \pm\sqrt{\frac{\hbar}{m\omega}}$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x \quad , \quad \xi_\pm = \pm 1$$

$$P\left(|x| > \sqrt{\frac{2E_0}{m\omega^2}}\right) = 2\int_1^\infty d\xi \,\frac{e^{-\xi^2}}{\sqrt{\pi}} = 0.157$$

because $\rho_0(\xi) = |\psi_0(\xi)|^2 = \frac{e^{-\xi^2}}{\sqrt{\pi}}$

4: 2.16) result: $H_5 = 120\xi - 160\xi^3 + 32\xi^5$

$$H_6 = -120 + 720\xi^2 - 480\xi^4 + 64\xi^6$$

5: 2.42) $\psi(x)$ is a sol'n of schrödinger equation for harmonic oscillator for $x > 0$.

$\psi(x) = 0$ for $x \leq 0$. $\Rightarrow$ odd solutions!

$$E_n = \hbar\omega(n + 1/2), \quad n = 1, 3, 5, 7, \ldots$$

6: 3.4) a) Let $\hat{Q} = \hat{A} + \hat{B}$

$$\hat{Q}^{+} = \hat{A}^{+} + \hat{B}^{+} = \hat{A} + \hat{B} = \hat{Q} \quad \checkmark$$

b) $(\alpha\hat{Q})^{+} = \alpha^{*}\hat{Q}^{+} = \alpha^{*}\hat{Q} = \alpha\hat{Q}$

if $\alpha \in \mathbb{R}$

c) $(AB)^{+} = B^{+}A^{+} = BA = AB$

if $[A, B] = 0$

d) $\langle\psi|x\phi\rangle = \int dx\ \psi^{*}(x)\ x\ \phi(x)$

$$= \int dx\,(x\psi(x))^{*}\,\phi(x) = \langle x\psi|\phi\rangle \quad \checkmark$$

$$\langle\psi|\hat{H}\phi\rangle = \int dx\ \psi^{*}(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\phi(x)$$

$$= \int dx\,\psi^{*}(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\phi(x) + \int dx\,(V(x)\psi(x))^{*}\phi(x)$$

$$= \int dx\left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\right)^{*}\phi(x) + \langle V\psi|\phi\rangle$$

(integrating by parts twice and dropping boundary terms, which are zero)