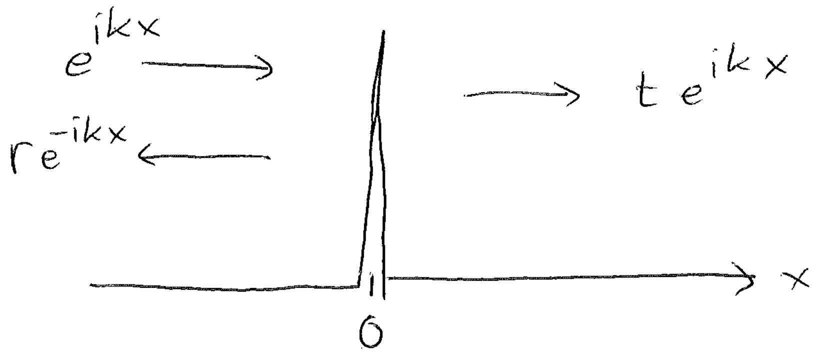
Homework #5 for Physics 371

Due 4pm Friday, February 26

1) Scattering from a delta-function potential

Consider a particle moving in one dimension with a potential $V(x) = \Lambda\delta(x)$.



a) Calculate the transmission amplitude t and the reflection amplitude r.

b) Calculate the transmission probability $T = |t|^2$ and the reflection probability $R = |r|^2$. Simplify your expressions by introducing the "scattering length" $\ell = \hbar^2/m\Lambda$.

c) Show that the reflection and transmission amplitudes may be written

$$r = \sqrt{R}\,e^{i\theta}, \quad t = i\sqrt{T}e^{i\theta},$$

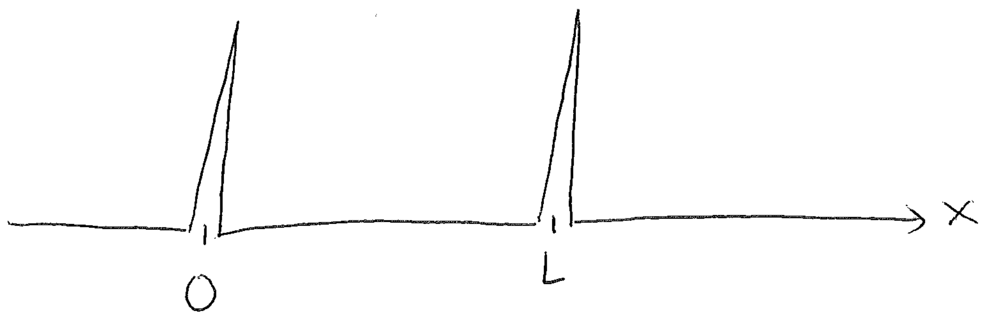
and find the function $\theta(k)$. (Hint: $\tan^{-1}$ is a multi-valued function. You must choose the correct branch.)

2) Resonant tunneling with a double delta-function potential

(This is a computational problem; you may work in groups on it!)

Consider a particle moving in one dimension with a potential

$$V(x) = \Lambda\delta(x) + \Lambda\delta(x - L).$$



In lecture 12, we showed that the transmission probability through a symmetric double barrier is

$$T_{12} = \frac{T^2}{T^2 + 4R\sin^2(kL + \theta)},$$

where T, R, and θ are the transmission probability, reflection probability, and scattering phase shift, respectively, for a single barrier.

Using your results from problem 1, plot T_{12} vs. k for the double delta-function barrier, using *Mathematica*, or some other computer graphic utility. Consider the case $L = 10\ell$. Discuss your result; what happens if you vary L and/or ℓ?