Homework #6 for Physics 371

Due 4pm Friday, March 4

1–6) Griffiths, 2.8, 2.38, 2.39, 3.7, 3.26, 3.27

7) For linear operators $\hat{A}$, $\hat{B}$, and $\hat{C}$, show that

a) $[\hat{A}, \hat{B} + \hat{C}] = [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}]$,

b) $[\hat{A}, \hat{B}\hat{C}] = \hat{B}[\hat{A}, \hat{C}] + [\hat{A}, \hat{B}]\hat{C}$.

c) Show that $[f(x), \hat{p}_x] = i\hbar \dfrac{df}{dx}$.

8) A particle moves in one dimension with Hamiltonian $\hat{H} = \hat{p}_x^2/2m + V(\hat{x})$. Show that the uncertainties Δp_x and ΔE obey the inequality

$$\Delta p_x \, \Delta E \geq \frac{\hbar}{2} \left| \left\langle \frac{\partial V}{\partial x} \right\rangle \right|.$$

What does this imply for stationary states?

9) Show that for a particle moving in one dimension,

$$m \frac{d^2}{dt^2} \langle x \rangle = \left\langle -\frac{\partial V}{\partial x} \right\rangle.$$

Hint: Use the identity

$$\frac{d}{dt} \langle Q \rangle = \left\langle \frac{\partial Q}{\partial t} \right\rangle + \frac{1}{i\hbar} \langle [\hat{Q}, \hat{H}] \rangle$$

one or more times.