Homework #8 for Physics 371

Due 4pm Friday March 25

1) Find the eigenvalues of the Hamiltonian

$$\hat{H} = E_0\, \hat{a}^\dagger \hat{a} + E_1(\hat{a}^\dagger + \hat{a}),$$

where $[\hat{a}, \hat{a}^\dagger] = 1$. Hint: Define new operators $\hat{b} = \hat{a} + \beta$ and $\hat{b}^\dagger = \hat{a}^\dagger + \beta$, where β is a real number to be determined.

2) The operator $\hat{c}$ is defined by the following relations:

$$\hat{c}^{\,2} = 0,$$

$$\hat{c}\,\hat{c}^\dagger + \hat{c}^\dagger \hat{c} = 1,$$

where $\hat{c}^\dagger$ is the hermitian conjugate of $\hat{c}$. Show that
(a) $\hat{N} = \hat{c}^\dagger \hat{c}$ is hermitian;
(b) $\hat{N}^2 = \hat{N}$;
(c) The eigenvalues of $\hat{N}$ are 0 and 1;
(d) If ψ_0 and ψ_1 denote the two eigenfunctions of $\hat{N}$ corresponding to the eigenvalues 0 and 1, respectively, show that

$$\hat{c}^\dagger \psi_0 = \psi_1,$$

$$\hat{c}\,\psi_0 = 0,$$

$$\hat{c}^\dagger \psi_1 = 0.$$

3–5) Griffiths 2.12, 3.1, 3.6