1) If a particle is in an angular momentum eigenstate $\psi_{\ell m}$, show that

$$\langle L_x \rangle = \langle L_y \rangle = 0.$$ 

Also show that

$$\langle L_x^2 \rangle = \langle L_y^2 \rangle = \frac{\hbar^2}{2} [\ell (\ell + 1) - m^2].$$

2) Using the results of problem 1, show that

$$\Delta L_x \Delta L_y \geq \frac{\hbar}{2} \langle L_z \rangle$$

in the state $\psi_{\ell m}$. For which state(s) $\psi_{\ell m}$, if any, does the equality in the above expression hold?

3) Suppose $\vec{L}^2$ is measured for a system and the value $6\hbar^2$ is obtained. If $L_y$ is measured immediately thereafter, what possible values can result?

4) Show that if a state exists which is a simultaneous eigenstate of $\hat{L}_x$ and $\hat{L}_y$, this state has eigenvalues $L_x = L_y = L_z = 0$.

5) Show that for a state with angular momentum quantum number $\ell$, the angle $\theta$ between the angular momentum vector and the $z$ axis obeys

$$|\tan \theta| \geq \ell^{-1/2}.$$ 

6–8) Griffiths 4.1, 4.19, and 4.21.