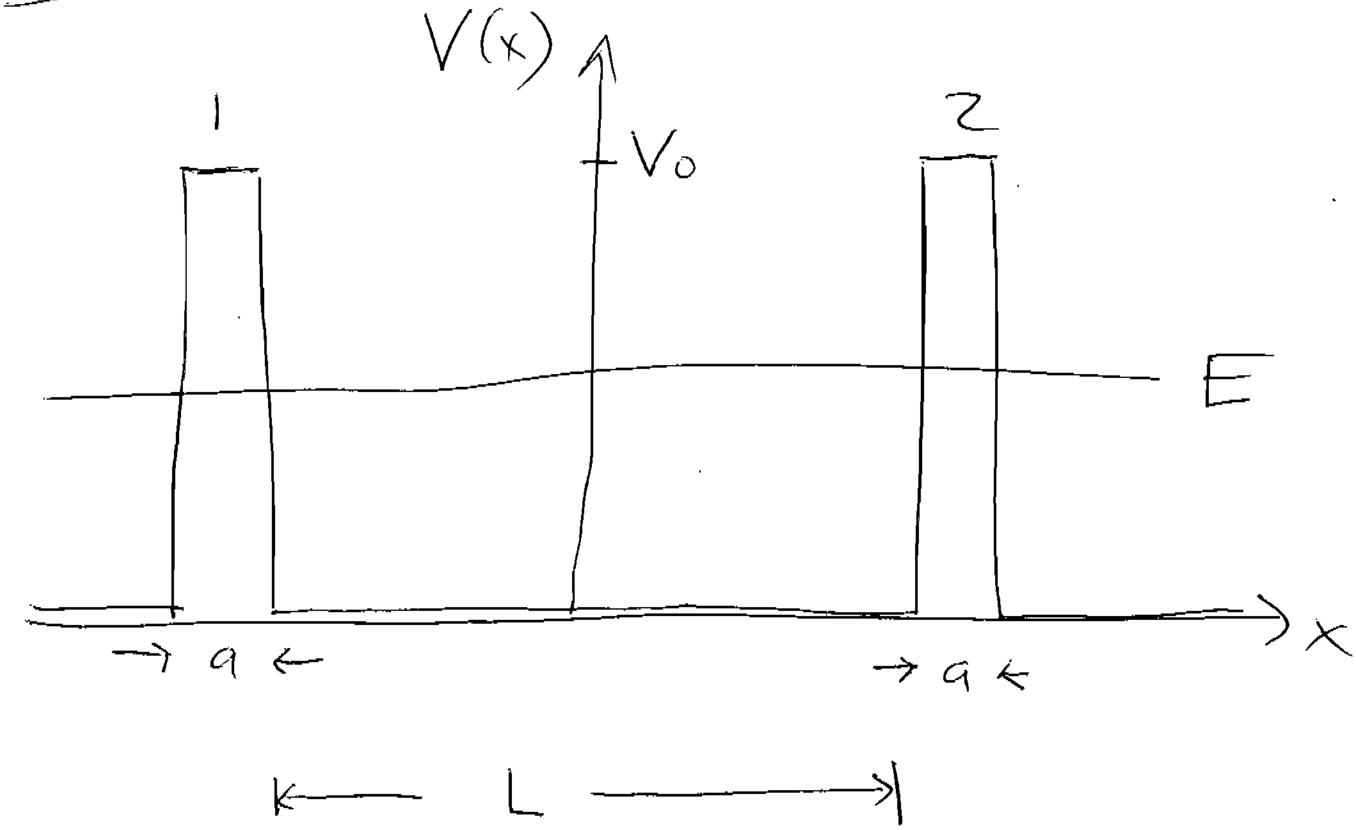
Physics 371 Lecture 12

Resonant tunneling through a double barrier



What is the transmission probability? Classically,

$T_{12} = T_1 T_2$, so the

probability would be much lower than for a single barrier. Quantum mechanically, we need to consider the interference between the transmitted wave and the multiply-reflected waves.

The total transmission amplitude is a geometric series

$$t_{12} = t_1 e^{ikL} t_2 + t_1 e^{ikL} r_2 e^{ikL} r_1 e^{ikL} t_2 + \cdots$$

$$= t_1 t_2 e^{ikL} \left(1 + r_1 r_2 e^{i2kL} + (r_1 r_2 e^{i2k})^2 + \cdots\right)$$

$$= \frac{t_1 t_2 e^{ikL}}{1 - r_1 r_2 e^{i2kL}}$$

For identical barriers,

$$t_1 = t_2 = i T^{1/2} e^{i\theta}$$

$$r_1 = r_2 = R^{1/2} e^{i\theta}$$ (symmetric barriers)

$$t_{12} = - \frac{T e^{i(kL+2\theta)}}{1 - R e^{i(2kL+2\theta)}}$$

Total transmission probability

$$T_{12} = |t_{12}|^2 = \frac{T^2}{(1-Re^{i\alpha})(1-Re^{-i\alpha})}$$

$$= \frac{T^2}{1+R^2-2R\cos\alpha} = \frac{T^2}{1+R^2-2R\cos(2kL+2\theta)}$$

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

- $$T_{12} = \frac{T^2}{1 - 2R + R^2 + 4R\sin^2(kL+\theta)}$$

$$= \frac{T^2}{T^2 + 4R\sin^2(kL+\theta)}$$

$\Rightarrow$ Resonances ($T_{12} = 1$) when

- $kL + \theta = n\pi$. This was precisely the condition for a <u>bound state</u> in the potential well (c.f. lecture 10).

- Q: What is the functional form $T(E)$ near such a resonance?

Let $kL + \theta = n\pi + \varepsilon$

$$\sin^2(kL+\theta) \simeq \varepsilon^2$$

$$T_{12} \simeq \frac{T^2}{T^2 + 4R\varepsilon^2}$$

$$\varepsilon = kL + \theta - n\pi$$

$$k_n L + \theta(k_n) = n\pi \qquad k = k_n + \Delta k$$

$$\Delta k L + \theta'(k_n)\Delta k \simeq \varepsilon$$

$$\varepsilon = \Delta k\left(L + \theta'(k_n)\right) = \frac{\Delta E}{\hbar v_n}\left(L + \theta'(k_n)\right)$$

$$\Delta k = \frac{\Delta E}{\left.\frac{dE}{dk}\right|_{k_n}} = \frac{\Delta E}{\hbar v_n}$$

- $\frac{2(L+\theta'(k_n))}{v_n}$ can be interpreted as the period of the motion within the well for the nth standing wave. This is clear
- if L is sufficiently large ($L \gg \theta'(k_n)$). The additional term is known as the "Wigner delay," and can be thought of as the time the particle dwells
- within the barrier when

it is reflected.

- Q: If we placed a particle in the well with $k \simeq k_n$, how fast would it leak out?

- $$\frac{\Gamma_n}{2} = \frac{\hbar\, T(E_n)\, v_n}{2(L + \theta'(k_n))} = \frac{\hbar}{\tau_1}$$

τ_1 = escape time through a single barrier.

In terms of Γ_n, one

- has

- $$T_{12}(E) \simeq \frac{(\Gamma_n/2)^2}{R_n(E-E_n)^2 + (\Gamma_n/2)^2}$$

This form of resonance is known as a <u>Breit-Wigner</u> resonance. The full

- width at half maximum is

$$\Delta E = \sqrt{R_n}\,\Gamma_n$$

$$\simeq \Gamma_n$$ for nearly opaque barriers.

- Thus the width of the

resonance is related to the escape time of the quasi-bound state

$$\Delta E \, \tau = \Gamma_n \frac{\hbar}{\Gamma_n} = \hbar$$

Via the energy-time uncertainty relation!

Here $\dfrac{1}{\tau} = \dfrac{1}{\tau_1} + \dfrac{1}{\tau_2} = \dfrac{2}{\tau_1}$.

Resonances such as this arise in many fields of physics!