Phys. 371 Example Topics

1) Hermitian operators

Def. $\hat{Q} = \hat{Q}^{\dagger}$ if

$$\langle \phi | \hat{Q} \psi \rangle = \langle \hat{Q} \phi | \psi \rangle \quad \forall\ \phi(x), \psi(x) \in \mathcal{H}$$

$$\langle \phi | \hat{Q} \psi \rangle = \int dx\ \phi^*(x)\, \hat{Q}\, \psi(x)$$

$$\langle \hat{Q} \phi | \psi \rangle = \int dx\ \left(\hat{Q}^* \phi^*(x)\right) \psi(x)$$

which operators are Hermitian?

a) $b x^3$

b) $\frac{\partial}{\partial x}$

c) $\frac{\partial^2}{\partial x^2}$

d) $i \frac{\partial}{\partial x}$

$$\langle \phi | \tfrac{\partial}{\partial x}\psi \rangle = \int_{-\infty}^{\infty} dx \;\; \phi^*(x)\,\frac{\partial \psi}{\partial x}$$

$$= \cancel{\phi^*_{(x)}\psi(x)\Big|_{-\infty}^{\infty}} \to 0 \; - \int dx \;\; \frac{\partial \phi^*}{\partial x}\,\psi(x)$$

$$\frac{\partial}{\partial x}\left(\phi^*\psi\right) = \phi^*\frac{\partial \psi}{\partial x} + \frac{\partial \phi^*}{\partial x}\,\psi$$

$$\langle \phi | \tfrac{\partial \psi}{\partial x} \rangle = - \langle \tfrac{\partial \phi}{\partial x} | \psi \rangle$$

2) Postulates of QM

Example: 1D ring

$$\psi_n(x) = \sqrt{\frac{1}{L}}\, e^{i k_n x} \quad , \quad k_n = \frac{2\pi n}{L}$$

$$n \in \mathbb{Z}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

a) $\Psi(x) = \sqrt{\frac{1}{L}}$

i) If x is measured ...

ii) If p_x is measured

iii) If E is measured ...

iv) What is $\Psi(x,t)$?

What are the possible outcomes? With what probabilities do they occur? What is ψ after meas.?

b) $\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{2\pi x}{L}\right)$

i) x

ii) p_x

iii) E

iv) $\psi(x,t) = ?$

c) $\psi(x) = \sqrt{\frac{2}{L}}\, e^{i\frac{2\pi x}{L}} \cos\left(\frac{2\pi x}{L}\right)$

i) x

ii) p_x

iii) E

iv) $\psi(x,t)$

3) Generalized Uncertainty Principle

$$\Delta A \, \Delta B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\Delta x \, \Delta E \geq \frac{1}{2} \left| \langle [\hat{x}, \hat{H}] \rangle \right|$$

$$\hat{H} = \frac{\hat{p}_x^{\,2}}{2m} + V(x)$$

$$\Delta x \, \Delta E \geq \, ?$$

4) Ehrenfest theorem

$$\frac{d}{dt} \langle \hat{Q} \rangle = \frac{1}{i\hbar} \langle [\hat{Q}, \hat{H}] \rangle$$

$$\frac{d}{dt} \langle V(x) \rangle = \, ?$$