

### 3) Generalized Uncertainty Principle

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$$\Delta A \Delta B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\Delta x \Delta E \geq \frac{1}{2} \left| \langle [\hat{E}, \hat{x}] \rangle \right|$$

$$= \frac{1}{2} \left| \langle [x, \frac{p_x^2}{2m}] \rangle \right|$$

$$= \frac{1}{4m} \left| \langle p_x [x, p_x] + [x, p_x] p_x \rangle \right|$$

$$= \frac{\hbar}{2m} \left| \langle p_x \rangle \right| = \frac{\hbar}{2} \left| \langle \frac{p_x}{m} \rangle \right|$$

## §) Ehrenfest theorem

$$\frac{d}{dt} \langle V(x) \rangle = ?$$

$$= \frac{1}{i\hbar} \langle [\hat{V}(x), \hat{H}] \rangle$$

$$= \frac{1}{2i\hbar m} \langle [V(x), p_x^2] \rangle$$

$$= \frac{1}{2i\hbar m} \langle p_x [V(x), p_x] + [V(x), p_x] p_x \rangle$$

$$\begin{aligned} [V(x), p_x] \psi(x) &= V \frac{\hbar}{i} \psi'(x) - \frac{\hbar}{i} (V' \psi + V \psi') \\ &= \left( -\frac{\hbar}{i} V'(x) \right) \psi(x) \end{aligned}$$

$$\frac{d}{dt} \langle V(x) \rangle = \frac{1}{2m} \langle p_x V'(x) + V'(x) p_x \rangle$$

$$\sim \langle -\vec{F} \cdot \vec{v} \rangle$$