Postulate 4

$q \longleftrightarrow \hat{Q}$ s.t. $\hat{Q} = \hat{Q}^+$

An ideal meas. of q must yield an eigenvalue of $\hat{Q}$:

$$\hat{Q}\psi_n = q_n \psi_n$$

After the meas.,

$$\psi \longrightarrow \psi_n$$

$$P(q_n) = |c_n|^2 \qquad \text{(postulate 5)}$$

$$\psi = \sum_n c_n \psi_n$$

$$c_n = \langle n | \psi \rangle$$

b) $\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{2\pi x}{L}\right)$

Hint

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

i) x

ii) p_x

iii) E

iv) $\psi(x,t) = ?$

c) $\psi(x) = \sqrt{\frac{2}{L}}\, e^{i\frac{2\pi x}{L}} \cos\left(\frac{2\pi x}{L}\right)$

i) x

ii) p_x

iii) E

iv) $\psi(x,t)$

$$\hat{p}_x = \hat{p}_x^{\dagger} \qquad \hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\hat{p}_x \psi_k(x) = \hbar k \, \psi_k(x)$$

$$\frac{\hbar}{i}\frac{\partial \psi_k}{\partial x} = \hbar k \, \psi_k(x)$$

$$\frac{\partial \psi_k}{\partial x} = ik \, \psi_k(x)$$

$$\Rightarrow \psi_k(x) = A e^{ikx}$$

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b) $\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{2\pi x}{L}\right) = \frac{1}{\sqrt{2}}\left(\psi_{\frac{h}{L}} + \psi_{-\frac{h}{L}}\right)$

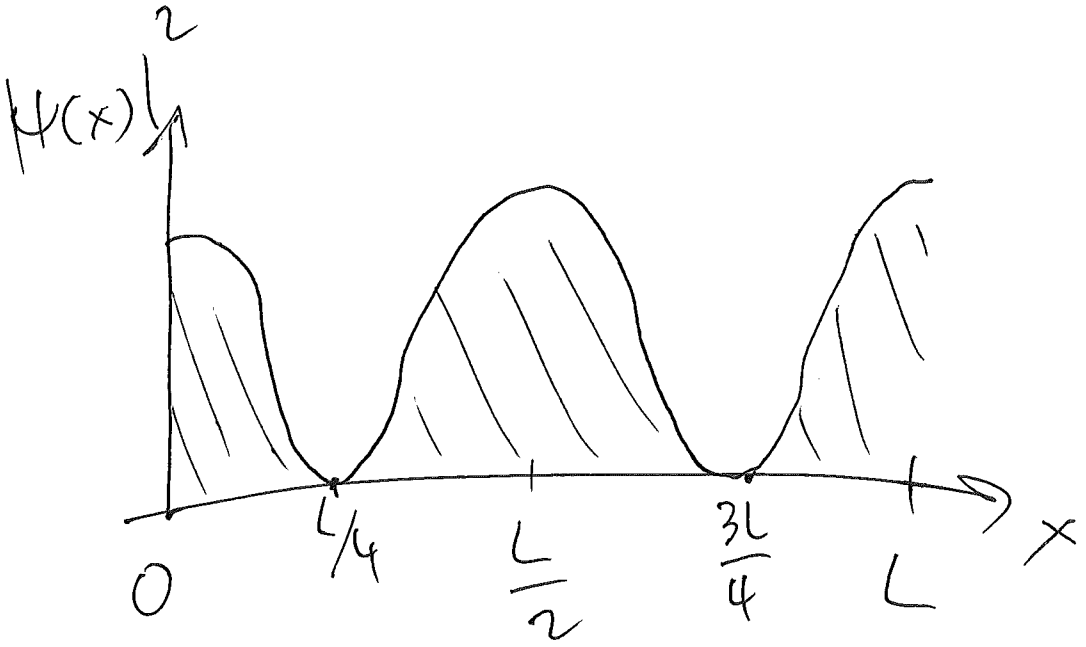
ii)

$P(\pm\frac{h}{L}) = \frac{1}{2}$, $\psi_{\pm\frac{h}{L}}(x) = \sqrt{\frac{1}{L}} e^{\pm i \frac{2\pi x}{L}}$

$p_x = \pm \frac{h}{L}$

$p_x = \pm \frac{h}{L} \rightarrow \psi = \sqrt{\frac{1}{L}} e^{\pm \frac{2\pi x}{L}}$

i)



$$\rho(x) = |\psi(x)|^2$$

$$P(x_0 - \epsilon/2 < x < x_0 + \frac{\epsilon}{2}) = \rho(x_0)\,\epsilon$$

iii) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ $\qquad \hat{H}\psi = \frac{h^2}{2mL^2}\psi$

$$E = \frac{h^2}{2mL^2} \qquad P = 100\%$$

iv) $\Psi(x,t) = \psi(x)\, e^{-i\frac{Et}{\hbar}}$

c) ii) $\psi(x) = \sqrt{\frac{1}{2}}\left(\sqrt{\frac{1}{L}} + \sqrt{\frac{1}{L}} e^{i\frac{4\pi x}{L}}\right)$

$\psi_k(x) = \sqrt{\frac{1}{L}} e^{ikx}$, $k = 0, \frac{4\pi}{L}$

$P(p_x = 0) = \frac{1}{2}$ $\quad P(p_x = \frac{2h}{L}) = \frac{1}{2}$

after measurement,

$\psi \to \psi_{k=0}$ or $\psi_{k=\frac{4\pi}{L}}$

i) If x is measured,

$\rho(x) = |\psi(x)|^2 = \frac{2}{L}\cos^2\left(\frac{2\pi x}{L}\right)$

same as (b).

iii) If E is measured,

$E = 0$ or $E = \frac{2h^2}{mL^2}$, each with 50% probability

iv) $\psi(x,t) = \sqrt{\frac{1}{2}}\left(\sqrt{\frac{1}{L}} + \sqrt{\frac{1}{L}} e^{i\frac{4\pi x}{L} - i\frac{4\pi h}{mL^2}t}\right)$