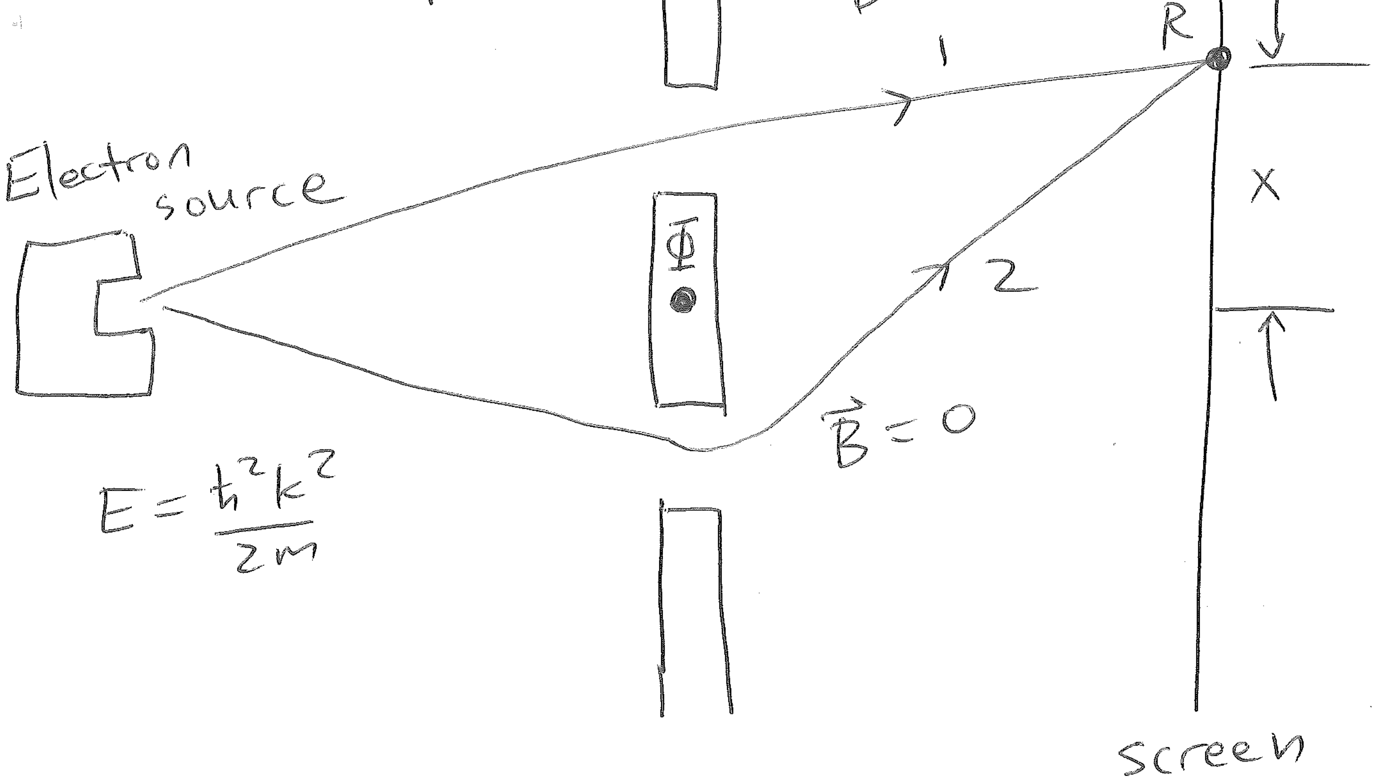
Physics 371 Lecture 33

The Aharonov-Bohm effect

In addition to the Meissner effect, another interesting consequence of the way the vector potential enters the Schrödinger equation is the Aharonov-Bohm effect, which describes the QM effect of the vector potential, even when $\vec{B} = 0$.

See Griffiths 10.2.3.

Modified 2-slit exp.



$$E = \frac{\hbar^2 k^2}{2m}$$

Magnetic flux Φ due to a solenoid within central barrier. $\vec{B} = 0$ everywhere the electrons can propagate.

<u>Time-indep. Sch. eq.</u>

$$E\,\Psi = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\vec{A}\right)^2 \Psi \qquad (V=0)$$

Outside barrier, $\vec{B} = 0$, so

$$\vec{A} = \nabla f(\vec{r}, t) = \frac{\hbar c}{q} \nabla \theta$$

$$E \psi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \hbar \nabla \theta \right)^2 \psi$$

Let $\psi = e^{i\theta(\vec{r})} \psi'$

ψ' satisfies free Sch. eq.

$$E \psi' = -\frac{\hbar^2}{2m} \nabla^2 \psi'$$

<u>Interference pattern</u>

$$\rho(\vec{R}) = |\psi_1(\vec{R}) + \psi_2(\vec{R})|^2$$

$$= |\psi_1'(\vec{R}) e^{i\theta_1} + \psi_2'(\vec{R}) e^{i\theta_2}|^2$$

$$\psi_1'(\vec{R}) = \sqrt{\rho_1}\, e^{ikL_1}, \quad \psi_2'(\vec{R}) = \sqrt{\rho_2}\, e^{ikL_2}$$

$$\rho(\vec{R}) = \rho_1 + \rho_2 + 2\sqrt{\rho_1 \rho_2}\cos\left(k\Delta L + \theta_2 - \theta_1\right)$$

$\Delta L = L_2 - L_1$ = difference in path lengths

$$\theta_1 = \int_1 \frac{q}{\hbar c}\vec{A}\cdot d\vec{l}\,, \qquad \theta_2 = \int_2 \frac{q}{\hbar c}\vec{A}\cdot d\vec{l}$$

$$\theta_2 - \theta_1 = \oint \frac{q}{\hbar c}\vec{A}\cdot d\vec{l}$$

$$= \frac{q}{\hbar c}\Phi = \frac{2\pi\,\Phi}{\Phi_0}$$

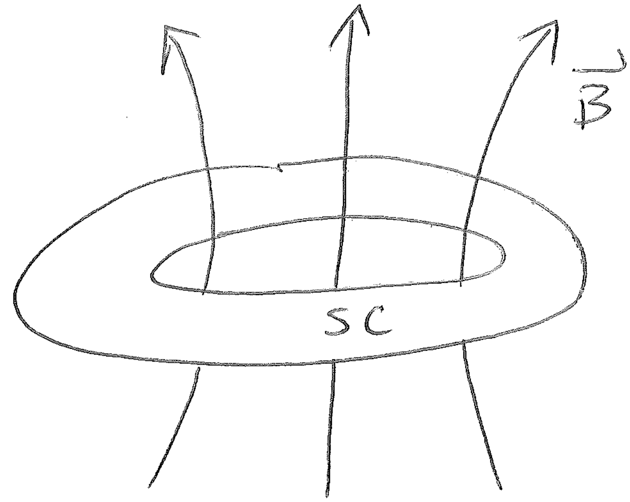
$\Phi_0 = \dfrac{hc}{q}$ = "flux quantum"

Interference pattern is shifted by magnetic flux in solenoid, even though $\vec{B}=0$ everywhere particle can propagate! Shift is periodic in Φ with period Φ_0.

Flux quantization

One consequence of the AB effect is the quantization of the magnetic flux piercing a SC ring.

The Meissner effect implies that $\vec{B} = 0$ and $\vec{J}_e = 0$ in the interior of the superconductor:



$$0 = \vec{J}_e = \frac{n_s q}{m}\left(\hbar \nabla \theta - \frac{q}{c}\vec{A}\right)$$

$$\Rightarrow \quad \nabla \theta = \frac{q}{\hbar c}\vec{A} \qquad \left(\psi_s(\vec{r}) = \sqrt{n_s(\vec{r})}\, e^{i\theta}\right)$$

The wavefunction of the SC is single-valued, which means $\theta(\vec{r})$ must return to its original value on traversing the ring, or change by an integer multiple of 2π:

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi s, \qquad s \in \mathbb{Z}$$

$$\parallel$$

$$\oint \frac{q}{\hbar c} \vec{A} \cdot d\vec{l} = \frac{q}{\hbar c} \Phi = 2\pi s$$

$$\boxed{\begin{aligned} \Phi &= \frac{hc}{q} s = s\,\Phi_0, \\ \Phi_0 &= \frac{hc}{2e} \quad \text{superconducting flux quantum} \end{aligned}}$$

The superconducting surface currents adjust to fix the total magnetic flux through the ring to an integer times Φ_0. Such persistent currents have been observed to flow, undiminished, for several years.