Extra credit problem for Physics 371

Due: 10am Monday, April 25

Open book; open notes; show your work for full credit.

Expectation values for a harmonic oscillator (worth 10 exam points)

Consider a harmonic oscillator with the following wavefunction

$$\psi(x) = \alpha \psi_n(x) + \beta \psi_{n+1}(x),$$

where α and β are complex numbers, and ψ_n is an energy eigenfunction.

(a) Calculate the expectation values

$$\langle x \rangle = \langle \psi | x | \psi \rangle,$$
$$\langle p_x \rangle = \langle \psi | p_x | \psi \rangle.$$

(b) Calculate the uncertainties Δx and Δp_x , and verify that the uncertainty principle is satisfied.

(c) For $\alpha = \beta = 1/\sqrt{2}$, find $\langle x(t) \rangle$ and $\langle p_x(t) \rangle$. Check that the Ehrenfest theorem is satisfied.