Phys. 371 Midterm 1
Solutions

1) a) $\rho(x) = |\psi(x)|^2 = A^2 e^{-\frac{(x-a)^2}{2b^2}}$

$$1 = \int_{-\infty}^{\infty} dx\, \rho(x) = A^2 \sqrt{2\pi b^2}$$

$$\boxed{A = \left(\frac{1}{2\pi b^2}\right)^{1/4}}$$

b) $\rho(x)$ is a Gaussian probability distribution. By inspection,

$$\boxed{\langle x \rangle = a \qquad \Delta x = b}$$

c) $j_x = \mathrm{Re}\left\{\psi^* \frac{p_x}{m} \psi\right\} = \mathrm{Re}\left\{\psi^* \frac{\hbar}{im} \frac{d\psi}{dx}\right\}$

$$\frac{1}{i}\frac{d\psi}{dx} = \left(q + \frac{i(x-a)}{2b^2}\right)\psi$$

$$\boxed{j_x = \frac{\hbar q}{m}|\psi|^2 = \frac{\hbar q}{m}\sqrt{\frac{1}{2\pi b^2}}\, e^{-\frac{(x-a)^2}{2b^2}}}$$

2) $\quad \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

$$\langle p_x \rangle = \int_0^L dx \; \psi^*(x) \frac{\hbar}{i} \frac{d}{dx} \psi(x)$$

$$= \frac{2}{L} \frac{2\pi}{L} \frac{\hbar}{i} \underbrace{\int_0^L dx \; \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi x}{L}\right)}_{= 0}$$

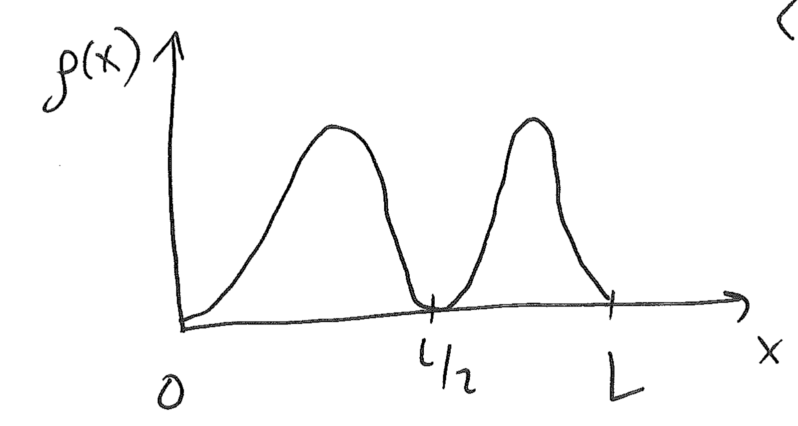
a) $\boxed{\langle p_x \rangle = 0}$

$$\hat{p}_x^2 \psi(x) = \left(\frac{\hbar \, 2\pi}{L}\right)^2 \psi(x) = \frac{h^2}{L^2} \psi(x)$$

$$\langle p_x^2 \rangle = \int_0^L dx \; \psi^*(x) \hat{p}_x^2 \psi(x) = \frac{h^2}{L^2} \int_0^L dx \; \psi^* \psi$$

a) $\boxed{\langle p_x^2 \rangle = \frac{h^2}{L^2}}$

b)



$\langle x \rangle = \frac{L}{2}$

$\Delta x \simeq L/4$

$$\Delta p_x = \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2} = \frac{h}{L}$$

$$\Delta x \, \Delta p_x \simeq \frac{L}{4} \frac{h}{L} = \frac{h}{4} = \frac{2\pi}{4} \hbar = \frac{\pi \hbar}{2} > \frac{\hbar}{2}$$

Q.E.D.