

Practice Problems for Midterm 1, Physics 371

Show your work for full credit.

Calculator and crib-sheet allowed (8.5" x 11", one side)

1) Consider a particle in a one-dimensional box consisting of impenetrable walls located at $x = 0$ and $x = L$. The wave function of the particle is

$$\psi(x) = \begin{cases} Ax(L - x), & 0 < x < L; \\ 0, & \text{otherwise.} \end{cases}$$

- a) Normalize $\psi(x)$ (that is, find A).
- b) Determine the mean position $\langle x \rangle$.
- c) Calculate the expectation value $\langle E \rangle$ of the energy of the particle. (Note that the potential is zero inside the box.) How does $\langle E \rangle$ compare with the ground state energy of a particle in the box?

2) Consider a particle of mass m moving in the one-dimensional potential

$$V(x) = \frac{\alpha}{16}x^4,$$

with $\alpha > 0$.

- a) Show that the energy is bounded by

$$\langle E \rangle \geq \frac{\Delta p_x^2}{2m} + \frac{\alpha \Delta x^4}{16}.$$

Note: You may use the fact that $\langle x^4 \rangle \geq \langle x^2 \rangle^2$.

- b) Use the uncertainty principle to estimate the minimum possible energy of the particle.