Physics 371 Midterm 2

Calculator and crib-sheet allowed (8.5"x11", one side)

1) Time evolution and energy measurement

Consider a free particle of mass m moving around a one-dimensional ring of circumference L. The initial wavefunction of the particle is

$$\Psi(x, t = 0) = \sqrt{\tfrac{1}{3}}\psi_0(x) + \sqrt{\tfrac{2}{3}}\psi_2(x),$$

where

$$\psi_0(x) = \sqrt{\frac{1}{L}},$$

$$\psi_2(x) = \sqrt{\frac{1}{L}}\exp(i4\pi x/L).$$

Note: The potential $V(x) = 0$ in this problem! Recall that this problem is equivalent to a free particle in one dimension with periodic boundary conditions.

a) Find $\Psi(x, t)$ and $\rho(x, t) = |\Psi(x, t)|^2$. (Express the later in terms of sinusoidal functions of time, eliminating the exponentials with the help of Euler's formula: $e^{iz} = \cos z + i \sin z$.) *Full credit will only be given if your result for ρ is explicitly real and non-negative!*

b) If the energy of the particle is measured, what are the possible outcomes? With what probability does each result occur? What is the wavefunction after the measurement in each case? Do your answers depend on the time of measurement, t?

2) 1D Scattering

Calculate the transmission probability for a particle of energy $E = V_0/99$ incident from the left on the negative potential step

$$V(x) = -V_0\,\theta(x), \quad \text{where} \quad \theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

and $V_0 > 0$. *Note: You must derive your result by solving Schrödinger's equation.*

3) Sequential measurements

An operator $\hat{A}$, representing observable a, has two normalized eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable b, has two normalized eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1 = \sqrt{\frac{1}{2}}(\phi_1 + \phi_2), \qquad \psi_2 = \sqrt{\frac{1}{2}}(\phi_1 - \phi_2).$$

a) Observable a is measured, and the value a_1 is obtained. What is the state of the system (immediately) after the measurement?

b) If b is now measured, what are the possible results, and with what probabilities do they occur?

c) Right after the measurement of b, a is measured again. What is the probability of getting a_1 again?