Phys 371 Midterm 2 solutions

1) $\psi_n(x) = \sqrt{\frac{1}{L}}\, e^{i\frac{2\pi n x}{L}}$ is an energy eigenstate:

$$\hat{H}\psi_n = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_n = \frac{\hbar^2}{2m}\frac{(2\pi)^2 n^2}{L^2}\psi_n$$

$$E_n = \frac{h^2 n^2}{2mL^2}, \quad n = 0, \pm 1, \pm 2, \text{ etc.}$$

$\psi_n(x)$ satisfies B.C. $\psi_n(x+L) = \psi_n(x)$.

a) $$\Psi(x,t) = \sqrt{\frac{1}{3}}\,\psi_0 + \sqrt{\frac{2}{3}}\,\psi_2(x)\, e^{-i\frac{E_2 t}{\hbar}}$$

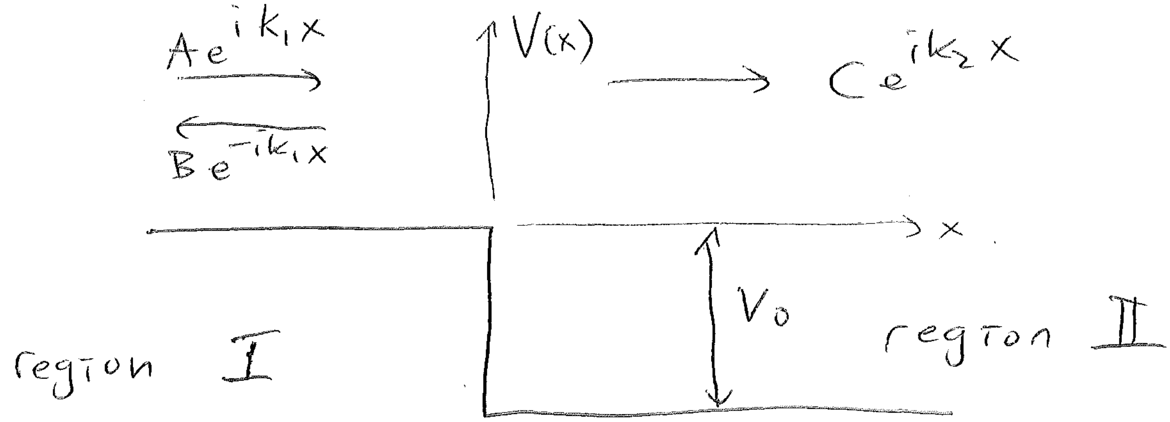
$$= \sqrt{\frac{1}{3L}} + \sqrt{\frac{2}{3L}}\, e^{i\left(\frac{4\pi x}{L} - \frac{E_2 t}{\hbar}\right)}$$

$$\rho(x,t) = |\Psi(x,t)|^2 = \frac{1}{3L} + \frac{2}{3L} + \frac{2\sqrt{2}}{3L}\cos\left(\frac{4\pi x}{L} - \frac{E_2 t}{\hbar}\right)$$

b) $E = 0$ or $E = E_2 = \frac{2h^2}{mL^2}$

$P(E=0) = \frac{1}{3}$ $\quad P(E=E_2) = \frac{2}{3}$ $\quad$ indep. of t

2)



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\psi_{II}(x) = Ce^{ik_2x}$$

$$E = \frac{\hbar^2 k_1^2}{2m} = \frac{\hbar^2 k_2^2}{2m} - V_0$$

i) $\psi_I(0) = \psi_{II}(0) \Rightarrow A + B = C$

ii) $\psi_I'(0) = \psi_{II}'(0) \Rightarrow ik_1(A-B) = ik_2 C$

$$A - B = \frac{k_2}{k_1} C$$

$$2A = \left(1 + \frac{k_2}{k_1}\right) C \qquad C = \frac{2k_1}{k_1+k_2} A$$

$$T = \frac{|j_{tr}|}{|j_{in}|} = \frac{k_2}{k_1}\frac{|C|^2}{|A|^2} = \frac{4k_1k_2}{(k_1+k_2)^2}$$

$$T(E) = \frac{4\sqrt{E}\sqrt{E+V_0}}{(\sqrt{E}+\sqrt{E+V_0})^2} = \frac{40}{121} = 0.33$$

3) a) $\psi = e^{i\theta}\psi_1 \quad , \quad \theta \in \mathbb{R}$

b) b_1 or b_2

$P(b_1) = |\langle \phi_1 | \psi_1 \rangle|^2 = \frac{1}{2}$

$P(b_2) = |\langle \phi_2 | \psi_1 \rangle|^2 = \frac{1}{2}$

c) $P(a_1) = P(b_1)\,|\langle \psi_1 | \phi_1 \rangle|^2 + P(b_2)\,|\langle \psi_1 | \phi_2 \rangle|^2$

$= \frac{1}{2} P(b_1) + \frac{1}{2} P(b_2)$

$= \frac{1}{2}$