Practice Problems for Midterm 3, Physics 371

Show your work for full credit.

Calculator and crib-sheet allowed (8.5”x11”, one side)

1) Uncertainty relations

Consider a particle of mass m moving in a one-dimensional potential $V(x)$.

a) Derive the following uncertainty relation for position and energy:

$$\Delta x \Delta E \geq \frac{\hbar}{2m} \left| \langle p_x \rangle \right| .$$

b) Using the result from part (a), derive the “energy-time uncertainty principle”

$$\Delta E \Delta t \geq \frac{\hbar}{2},$$

where

$$\Delta t \equiv \frac{\Delta x}{|d\langle x \rangle / dt|}$$

is the time for the mean position to change by one standard deviation.

2) Expectation values for the harmonic oscillator

Suppose a harmonic oscillator is in the energy eigenstate ψ_n.

a) Show that $\langle n|x|n \rangle = 0$ and $\langle n|p_x|n \rangle = 0$.

b) Calculate $\langle n|x^2|n \rangle$ and $\langle n|p_x^2|n \rangle$.

c) Show that the uncertainty principle is satisfied. For which state(s), if any, does $\Delta x \Delta p_x$ achieve the minimum allowed value?

Hint: Express x and p_x in terms of a and $a^\dagger$.

3) Angular momentum operators and eigenfunctions

Derive the following commutation relations

a) $[L_x, L_y] = i\hbar L_z$,

b) $[\vec{L}^2, L_z] = 0$,

using the canonical commutation relations $[r_i, p_j] = i\hbar \delta_{ij}$.

c) The result from part (b) implies that the angular momentum eigenfunctions $Y_{\ell m}$ can be chosen as simultaneous eigenfunctions of $\vec{L}^2$ and L_z. Write down (do not derive) the eigenvalue equations for $Y_{\ell m}$, and completely specify the eigenvalues.

4) Spin-1/2

(a) Find the eigenvalues and eigenspinors of

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

(b) If you measured S_y on a particle in the general state

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} \equiv a\chi_\uparrow + b\chi_\downarrow,$$

what values might you get, and what is the probability of each? Check that the probabilities add up to one. *Note:* a and b need not be real!