Physics 371 Final Exam

Show your work for full credit!

Calculator and crib-sheet allowed (8.5"x11", two sides)

1) 1D potential step

Consider a particle of energy E incident from the left on the potential step

$$V(x) = V_0\,\theta(x), \quad \text{where} \quad \theta(x) = \begin{cases} 0 & x < 0, \\ 1 & x \geq 0, \end{cases}$$

and $V_0 > 0$.

a) Calculate the transmission probability for $E = 25V_0/16$. *Note: You must derive your result by solving Schrödinger's equation.*

b) Calculate the transmission probability for $E = 16V_0/25$. *Again, you must justify your result with a detailed calculation.*

2) Harmonic oscillator

Consider a harmonic oscillator with the following initial wavefunction

$$\psi(x, t = 0) = \sqrt{\tfrac{2}{3}}\psi_n(x) + e^{i\theta}\sqrt{\tfrac{1}{3}}\psi_{n+1}(x),$$

where ψ_n is the nth energy eigenfunction.

a) If the energy of the system is measured, what are the possible outcomes, and with what probabilities do they occur? What is the expectation value of the energy of the system?

b) Calculate the mean position of the particle $\langle x(t) \rangle$ as a function of time.

c) Calculate the average momentum of the particle $\langle p_x(t) \rangle$ as a function of time.

3) Orbital angular momentum

Consider a particle in an eigenstate of orbital angular momentum: $\Psi(r, \theta, \phi) = \psi(r)\, Y_{\ell m}(\theta, \phi)$.

a) Determine $\langle L_x \rangle$, $\langle L_y \rangle$, and $\langle L_z \rangle$.

b) Determine $\langle L_x^2 \rangle$ and $\langle L_y^2 \rangle$.

c) Calculate $\Delta L_x \Delta L_y$ and verify that the *generalized uncertainty principle* for angular momentum is satisfied.

4) Spin-1/2: Measurement of S_z or S_y

Consider a spin-1/2 particle in the general spin state

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix} \equiv a\psi_\uparrow + b\psi_\downarrow,$$

where a and b are complex numbers, and $\psi_\uparrow$ and $\psi_\downarrow$ are eigenstates of S_z.

a) If a measurement of the z-component of the particle's spin, S_z, is performed, what are the possible outcomes, and with what probabilities do they occur? What is the expectation value $\langle S_z \rangle$?

b) If, instead, a measurement of the y-component of the particle's spin, S_y, is performed, what are the possible outcomes, and with what probabilities do they occur? What is the expectation value $\langle S_y \rangle$?

5) Uncertainty principle: momentum and angular momentum

a) Derive inequalities for the products

$$\Delta p_x \, \Delta L_z \geq ? \quad \text{and} \quad \Delta p_y \, \Delta L_z \geq ?$$

b) Derive inequalities for the products

$$\Delta p_z \, \Delta L_z \geq ? \quad \text{and} \quad \Delta (p_x^2 + p_y^2) \, \Delta L_z \geq ?$$