

$$4) \quad a) \quad \Delta x \Delta(p_x^2) \geq \frac{1}{2} |\langle [\hat{x}, \hat{p}_x^2] \rangle| \quad \boxed{3}$$

$$\begin{aligned} [\hat{x}, \hat{p}_x^2] &= \hat{p}_x [\hat{x}, \hat{p}_x] + [\hat{x}, \hat{p}_x] \hat{p}_x \\ &= 2i\hbar \hat{p}_x \end{aligned}$$

$$\Delta x \Delta(p_x^2) \geq \hbar |\langle p_x \rangle|$$

b) An energy eigenstate of a particle in a box is an eigenstate of \hat{p}_x^2 , so $\Delta(p_x^2) = 0$. Also, $\Delta x < L$. However $\langle p_x \rangle = 0$, so the inequality is satisfied.