Phys 460 HW 2
Solutions

1) $$\vec{J}_0 = L^{(0)}(-\nabla\mu) + L^{(1)}\left(-\frac{\nabla T}{T}\right) = 0$$ (open electric circuit)

$$\Rightarrow \nabla\mu = -\left(L^{(0)}\right)^{-1} L^{(1)} \frac{\nabla T}{T}$$

$$\mu(\vec{r}) = \mu_{int} - e\,U_{el}(\vec{r}) \simeq \varepsilon_F - e\,U_{el}(\vec{r})$$

$$\nabla\mu \simeq -e\nabla U_{el} = e\vec{E}$$

$$\vec{E} = -\frac{1}{eT}\left(L^{(0)}\right)^{-1} L^{(1)} \nabla T \equiv S\,\nabla T$$

Q.E.D.

2) $$\vec{J}_1 = L^{(1)}(-\nabla\mu) + L^{(2)}\left(\frac{-\nabla T}{T}\right)$$

But $\nabla\mu = -\left(L^{(0)}\right)^{-1} L^{(1)} \dfrac{\nabla T}{T}$,

so $$\vec{J}_1 = -\frac{1}{T}\left[L^{(2)} - L^{(1)}\left(L^{(0)}\right)^{-1} L^{(1)}\right]\nabla T$$

$$\equiv -K\,\nabla T$$

Q.E.D.

3) $L_{ij}^{(\nu)} = \delta_{ij} L^{(\nu)} = \delta_{ij} \int \frac{d^3p}{h^3} \frac{\vec{v}^2}{3} \left(-\frac{\partial f_0}{\partial \varepsilon}\right)(\varepsilon-\mu)^\nu \tau$

$$L^{(\nu)} = \frac{2\tau}{3m} \int \frac{d^3p}{h^3}\, \varepsilon\, (\varepsilon-\mu)^\nu \left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$= \frac{2\tau}{3m} \int d\varepsilon\, D(\varepsilon)\, \varepsilon\, (\varepsilon-\mu)^\nu \left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$= \frac{2\tau}{3m} \int d\varepsilon\, D(\varepsilon)\, (\mu + \varepsilon - \mu)(\varepsilon-\mu)^\nu \left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$L^{(0)} \cong \frac{2\tau}{3m} \int d\varepsilon\, D(\varepsilon)\, \mu \left(-\frac{\partial f_0}{\partial \varepsilon}\right) = \frac{2\tau}{3m} \mu D(\mu)$$

$$D(\mu) = \frac{3n}{2\mu} \Rightarrow \boxed{L^{(0)} = \frac{n\tau}{m}}$$

$$L^{(1)} \cong \frac{2\tau}{3m} \int d\varepsilon \left[D(\mu) + \mu D'(\mu)\right] (\varepsilon-\mu)^2 \left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$= \frac{2\tau}{3m} \left[D(\mu) + \mu D'(\mu)\right] \frac{\pi^2}{3} (k_B T)^2$$

But $D(\mu) = \frac{1}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \mu^{1/2}$

$$\mu D'(\mu) = \frac{1}{2} D(\mu)$$

$$D(\mu) + \mu D'(\mu) = \frac{3}{2} D(\mu) = \frac{9}{4}\frac{n}{\mu}$$

$$\boxed{L^{(1)} \simeq \frac{\pi^2}{2} \frac{n\tau}{m} \frac{(k_B T)^2}{\mu}}$$

Note: different from form given! (sorry)

$$L^{(2)} \simeq \frac{2\tau}{3m} \mu D(\mu) \int d\varepsilon \, (\varepsilon-\mu)^2 \left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$\boxed{L^{(2)} = \frac{\pi^2}{3} \frac{n\tau}{m} (k_B T)^2}$$

4) $$S = -\frac{1}{eT} \frac{L^{(1)}}{L^{(0)}} = -\frac{\pi^2}{2} \frac{k_B}{e} \frac{k_B T}{\mu}$$

Note that $\sigma(\mu) = \dfrac{n(\mu) e^2 \tau}{m}$

$$n(\mu) = \frac{1}{3\pi^2} \left(\frac{2m\mu}{\hbar^2}\right)^{3/2}$$

$$\frac{dn}{d\mu} = \frac{3n}{2\mu} \qquad \frac{\sigma'(\mu)}{\sigma} = \frac{3}{2\mu}$$

$$\boxed{S = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{d\ln\sigma(\mu)}{d\mu}}$$

Mott formula

$$K = \frac{1}{T}\left[L^{(2)} - \frac{\left(L^{(1)}\right)^2}{L^{(0)}}\right]$$

$$= \frac{\pi^2}{3}\frac{n\tau}{m}k_B^2 T - \frac{1}{T}\left[\frac{n\tau}{m}\left(\frac{\pi^2}{2}\right)^2\frac{(k_B T)^4}{\mu^2}\right]$$

$$K = \frac{\pi^2}{3}\frac{n\tau}{m}k_B^2 T - \frac{\pi^4}{4}\frac{n\tau}{m}\frac{k_B^4 T^3}{\mu^2}$$

$$K = \frac{\pi^2}{3}\frac{n\tau}{m}k_B^2 T\left[1 - \frac{3\pi^2}{4}\left(\frac{k_B T}{\mu}\right)^2\right]$$

$$\frac{K}{\sigma T} = \frac{\pi^2}{3}\frac{k_B^2}{e^2}\left[1 - \cancel{\frac{3\pi^2}{4}\left(\frac{k_B T}{\mu}\right)^2}\right]$$

very small correction