1) $I = j_x A$

$$j_x = e \int_{\varepsilon_F^0}^{\varepsilon_F^0 + eV} dE \int_0^{\pi/2} d\theta \; v_F \cos\theta \; 2\pi \sin\theta \; \frac{\partial^2 n}{\partial E \partial \Omega}$$

Ω = solid angle

$$\frac{\partial^2 n}{\partial E \partial \Omega} = \frac{1}{4\pi} \frac{\partial n}{\partial E} = \frac{1}{4\pi} D(E) \qquad \text{(isotropic)}$$

$$j_x = e v_F \frac{D(E)}{2} \int_{\varepsilon_F^0}^{\varepsilon_F^0 + eV} dE \underbrace{\int_0^{\pi/2} d\theta \cos\theta \sin\theta}_{1/2}$$

$$= e^2 V \, v_F \frac{D(E)}{2} \times \frac{1}{2}$$

$$= e^2 V \, v_F \frac{3n}{2\varepsilon_F} \times \frac{1}{4} \qquad n = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F = \frac{m v_F^2}{2}$$

$$= \frac{e^2 V}{4} \frac{3 k_F^3}{3\pi^2} \frac{1}{m v_F} = \frac{e^2 V}{4\pi^2} \frac{k_F^3}{\hbar k_F}$$

$$= \frac{2e^2}{h} \frac{k_F^2}{4\pi} V$$

$$I = J \times A = \frac{2e^2}{h} \frac{k_F{}^2 A}{4\pi} V$$

$$G = \frac{I}{V} = \frac{2e^2}{h} \frac{k_F{}^2 A}{4\pi}$$

2) $$I_\alpha^{(\nu)} = \sum_\beta \frac{1}{h} \int dE \, (E-\mu_\alpha)^\nu \, T_{\alpha\beta}(E) \left[f_\beta - f_\alpha \right]$$

$$f_\alpha(E) = f_0(E) + \frac{\partial f_0}{\partial \mu}(\mu_\alpha - \mu_0) + \frac{\partial f_0}{\partial T}(T_\alpha - T_0)$$

+ non-linear terms

$$f_0 = \frac{1}{e^{\beta_0(E-\mu_0)}+1} \qquad \frac{\partial f_0}{\partial \mu} = -\frac{\partial f_0}{\partial E}$$

$$\frac{\partial f_0}{\partial T} = \frac{E-\mu_0}{T_0}\left(-\frac{\partial f_0}{\partial E}\right)$$

$$f_\beta - f_\alpha = \left(-\frac{\partial f_0}{\partial E}\right)\left[\mu_\beta - \mu_\alpha + \frac{E-\mu_0}{T_0}\left(T_\beta - T_\alpha\right)\right]$$

+ nonlinear terms

Since $f_\beta - f_\alpha$ is linear in the bias, we can set $(E-\mu_\alpha)^\nu$ in integrand for $I_\alpha^{(\nu)}$ to be $(E-\mu_0)^\nu$. The error is higher-order in the bias.

Then

$$I_\alpha^{(\nu)} = \sum_\beta \frac{1}{h} \int dE \; T_{\alpha\beta}(E) \left(-\frac{\partial f_0}{\partial E}\right)$$

$$\times \left[(E-\mu_0)^\nu (\mu_\beta - \mu_\alpha) + (E-\mu_0)^{\nu+1} \frac{(T_\beta - T_\alpha)}{T_0} \right]$$

$$= \sum_\beta \left[L_{\alpha\beta}^{(\nu)} (\mu_\beta - \mu_\alpha) + L_{\alpha\beta}^{(\nu+1)} \frac{(T_\beta - T_\alpha)}{T_0} \right]$$

Q.E.D.

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3) <u>Electrical conductance</u>:

$$\mu_1 - \mu_2 = -eV, \quad \Delta T = 0$$

$$I_1^{(0)} = L_{12}^{(0)} (\mu_2 - \mu_1) = -L_{12}^{(0)} eV$$

$$I_1^{\text{electric}} = -e\, I_1^{(0)} = e^2 L_{12}^{(0)} V = GV$$

<u>Thermopower</u>: $\Delta T \neq 0$

$$0 = I_1^{(0)} = L_{12}^{(0)} (\mu_2 - \mu_1) + L_{12}^{(1)} \frac{(T_2 - T_1)}{T_0}$$

$$\mu_2 - \mu_1 = -eV = -\frac{L_{12}^{(1)}}{L_{12}^{(0)}} \frac{T_2 - T_1}{T_0}$$

$$V = +\frac{1}{eT_0} \frac{L_{12}^{(1)}}{L_{12}^{(0)}} \Delta T = -S \Delta T$$

(signs correspond to $\vec{E} = S \nabla T$
with $\vec{E} = -\nabla V$)

Thermal conductance

$$I_1^{(1)} = L_{12}^{(1)} (\mu_2 - \mu_1) + L_{12}^{(2)} \frac{(T_2 - T_1)}{T_0}$$

but $$\mu_2 - \mu_1 = -\frac{L_{12}^{(1)}}{L_{12}^{(0)}} \frac{T_2 - T_1}{T_0}$$ (open electric circuit)

$$I_1^{(1)} = \frac{1}{T_0} \left[L_{12}^{(2)} - \frac{\left(L_{12}^{(1)}\right)^2}{L_{12}^{(0)}} \right] (T_2 - T_1)$$

$$= K (T_2 - T_1)$$

4) Perfect 1D wire: $T_{12}(E) = 1$

$\Rightarrow \quad L_{12}^{(1)} = 0$ by symmetry

$$K_0 = \frac{1}{T_0} L_{12}^{(2)}$$

$$L_{12}^{(2)} = \frac{1}{h} \int dE \, (E-\mu_0)^2 \left(-\frac{\partial f_0}{\partial E}\right)$$

$$= \frac{1}{h} \int_0^\infty dE \, (E-\mu_0)^2 \frac{\beta e^{\beta(E-\mu_0)}}{\left(e^{\beta(E-\mu_0)} + 1\right)^2}$$

let $x = \beta(E-\mu_0)$

$$L_{12}^{(2)} = \frac{(k_B T)^2}{h} \int_{-\beta\mu_0}^\infty dx \; x^2 \frac{e^x}{(e^x+1)^2}$$

$$\simeq \frac{(k_B T)^2}{h} \underbrace{\int_{-\infty}^\infty dx \, \frac{x^2 e^x}{(e^x+1)^2}}_{\pi^2/3} \qquad (\mu_0 \gg k_B T)$$

$$K = \frac{L_{12}^{(2)}}{T_0} = \frac{\pi^2}{3} \frac{k_B^2 T}{h}$$

$$= 2.837 \times 10^{-10} \ \frac{W}{K} \quad @ \ 300\,K$$

$$= K_0$$ thermal conductance quantum