Physics 460/560 Lecture 2

Free Electron Theory of Metals: Transport

I) Classical theory

Newton's 2nd law for an electron of mass m acted on by a force $\vec{F}$ is:

$$m\dot{\vec{v}} = \vec{F} = -e\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$$

In addition to the external force $\vec{F}$ caused by macroscopic fields applied to the metal, an electron is scattered by phonons, defects in the lattice, and other electrons. This scattering has the tendency to return an electron accelerated by an external field toward equilibrium, with a characteristic time

τ. This effect can be included in Newton's 2nd law as follows:

$$m\dot{\vec{v}} = \vec{F} - \frac{m\vec{v}}{\tau}.$$

In the absence of an external force, an electron with initial velocity $\vec{v}(0)$ slows to rest exponentially:

$$\dot{\vec{v}} = -\frac{\vec{v}}{\tau}$$

$$\frac{d\vec{v}}{\vec{v}} = -\frac{dt}{\tau}$$

$$\vec{v} = \vec{v}(0)\, e^{-t/\tau}$$

In the presence of a static electric field $\vec{E}$, we have

$$m\dot{\vec{v}} = -e\vec{E} - \frac{m\vec{v}}{\tau}$$

In steady state $\langle \dot{\vec{v}} \rangle = 0$

$$\boxed{\langle \vec{v} \rangle = -\frac{e\vec{E}\tau}{m}} \quad \text{drift velocity}$$

The electrical current density is $\vec{J}_e = -ne\langle \vec{v} \rangle$:

$$\vec{J}_e = \frac{ne^2\tau}{m}\vec{E} \equiv \sigma\vec{E}$$

$$\sigma = \frac{ne^2\tau}{m}$$

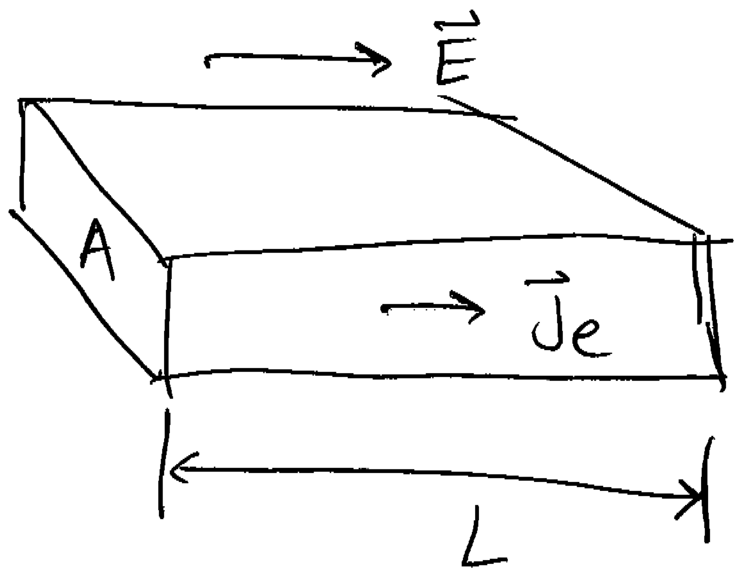
Ohm's law

σ = conductivity

$\rho = \frac{1}{\sigma}$ = resistivity

Consider a specimen of cross section

- A and length L:



- The total current flowing is

$I = A|\vec{J}_e|$. The total voltage

drop is $V = |\vec{E}|L$.

$$I = A\sigma|\vec{E}| = \frac{A\sigma}{L} V$$

The resistance R is defined

- by:

$$\boxed{R = \frac{V}{I} = \frac{L}{\sigma A} = \rho \frac{L}{A}}$$

This is an alternate form of ohm's law, relating the macroscopic, measurable quantities V & I.

Scattering mechanisms

$$\frac{1}{\tau} \simeq \frac{1}{\tau_{e\text{-}ph}} + \frac{1}{\tau_{e\text{-}e}} + \frac{1}{\tau_i}$$

$\tau_{e\text{-}ph}$ = scattering time for electron-phonon scattering

$\tau_{e\text{-}e}$ = scattering time for electron-electron scattering

τ_i = scattering time for electron-impurity scattering

Both $\tau_{e\text{-}ph}$ and $\tau_{e\text{-}e}$ go to infinity as $T \to 0$, since the number of phonons and the number of thermally excited electrons vanish as $T \to 0$ (The filled Fermi sea, being the ground state of the system, does not scatter electrons). However, $\tau_i \approx$ const.

τ_i depends strongly on how pure a specimen is, while $\tau_{e\text{-}ph}$ and $\tau_{e\text{-}e}$ are relatively insensitive to purity, but depend strongly on temperature.

$$\rho = \frac{m}{ne^2}\frac{1}{\tau} = \frac{m}{ne^2}\left(\frac{1}{\tau_i} + \frac{1}{\tau_{e\text{-}ph}} + \frac{1}{\tau_{e\text{-}e}}\right)$$

$$\lim_{T \to 0} \rho(T) = \frac{m}{ne^2}\frac{1}{\tau_i} = \rho_i$$ residual resistivity

The derivation of Ohm's law was one of the first successes of the free electron theory of metals, even before the advent of quantum mechanics.

Hall effect

$$m \frac{d\vec{v}}{dt} = -e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) - \frac{m\vec{v}}{\tau}$$

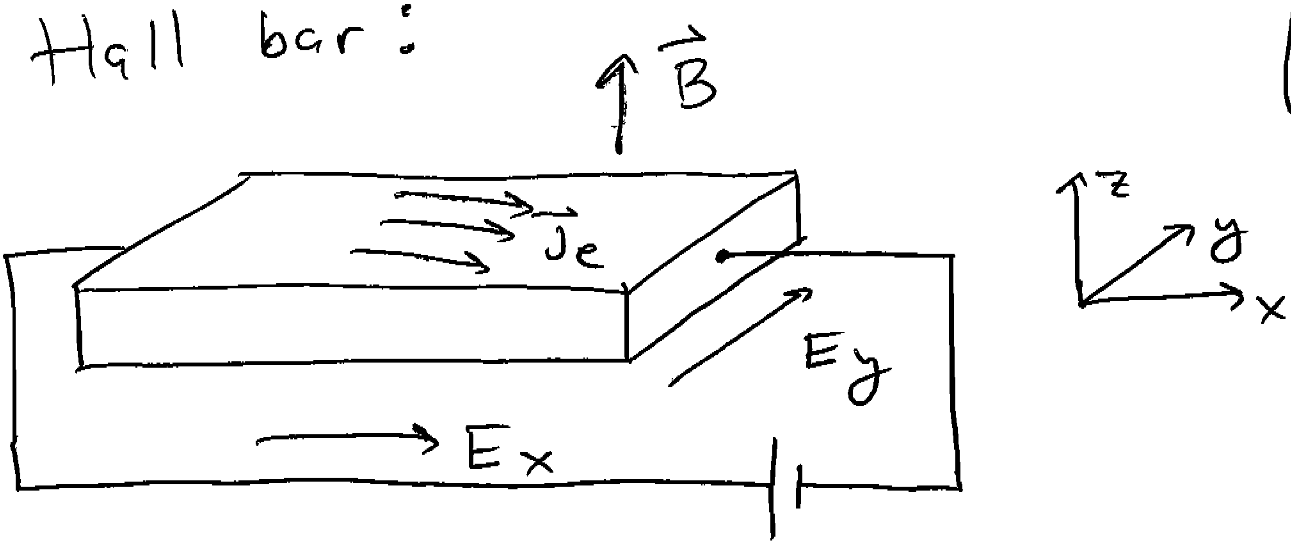
Let $\vec{B} = B\hat{z}$ and consider a steady state current $\langle \dot{\vec{v}} \rangle = 0$,

$$\frac{m}{\tau}\langle v_x \rangle = -e\left(E_x + \frac{B}{c}\langle v_y \rangle\right)$$

$$\frac{m}{\tau}\langle v_y \rangle = -e\left(E_y - \frac{B}{c}\langle v_x \rangle\right)$$

$$\frac{m}{\tau}\langle v_z \rangle = -e\,E_z$$

Hall bar:



In this configuration, $\langle v_y \rangle = \langle v_z \rangle = 0$.

Then, from the 2nd equation,

$$0 = \frac{m}{\tau} \langle v_y \rangle = -e\left(E_y - \frac{B}{c} \langle v_x \rangle \right)$$

$$\Rightarrow \quad E_y = \frac{B}{c} \langle v_x \rangle$$ <u>Hall electric field</u>

But the 1st equation gives

$$\langle v_x \rangle = -\frac{e E_x \tau}{m}$$

so

$$J_x = -ne \langle v_x \rangle = \frac{ne^2 \tau}{m} E_x$$

The Hall resistivity is defined by

$$\rho_H = \frac{E_y}{j_x} = -\frac{B}{nec}.$$

The Hall coefficient is defined by $R_H = \rho_H / B$. One has

$$\boxed{R_H = -\frac{1}{nec} \quad \text{(cgs units)}}$$

$R_H < 0$ because the electron charge is negative. A positive charge carrier would yield a positive Hall coefficient.

Hall coefficients ($\times 10^{-24}$ cgs units)

metal	measured R_H	$-\frac{1}{nec}$
Li	-1.89	-1.48
Na	-2.619	-2.603
K	-4.946	-4.944
Rb	-5.6	-6.04
Cu	-0.6	-0.82
Ag	-1.0	-1.19
Au	-0.8	-1.18
Al	$+1.136$	$+1.135$ $(+\frac{1}{nec})$
In	$+1.774$	$+1.780$ $(+\frac{1}{nec})$
As	$+50$	—
Bi	-6000	—

- In order to explain the positive Hall coefficients of Al and In, we have to go beyond the free electron model. These carriers of apparent positive sign were deemed "holes" by Heisenberg $\Rightarrow$ band theory.

- The good agreement of R_H and $-\frac{1}{nec}$ in monovalent metals is a major success of the free electron model

Thermal Conductivity

- $\vec{J}_Q = -K \nabla T \qquad K = \frac{1}{3} c v \ell$

For a free electron Fermi gas,

- one has $v = v_F$, $\ell = v_F \tau$,

and

$$C = \frac{\pi^2}{2} n \frac{k_B^2 T}{\varepsilon_F} = \frac{\pi^2 n k_B^2 T}{m v_F^2}$$

is the heat capacity per unit volume.

- $$\boxed{K = \frac{\pi^2}{3} \frac{n\tau}{m} k_B^2 T}$$

Wiedemann-Franz law

$$L = \frac{K}{\sigma T} = \frac{\frac{\pi^2}{3} \frac{n\tau}{m} k_B^2 T}{\frac{ne^2\tau}{m} T} = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2$$

$L =$ "Lorentz number"

- $$L = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2 = 2.45 \times 10^{-8} \frac{\text{Watt-ohm}}{K^2}$$

$L \times 10^8 \ \frac{W\Omega}{K^2}$

metal	0°C	100°C
Ag	2.31	2.37
Au	2.35	2.40
Cd	2.42	2.43
Cu	2.23	2.33
Mo	2.61	2.79
Pb	2.47	2.56
Pt	2.51	2.60

The experimatal validity of the Wiedemann-Franz law is an important confirmation of the picture of a metal as a (free) Fermi gas of electrons.