Physics 460/560 Lecture 5

Quantum Transport

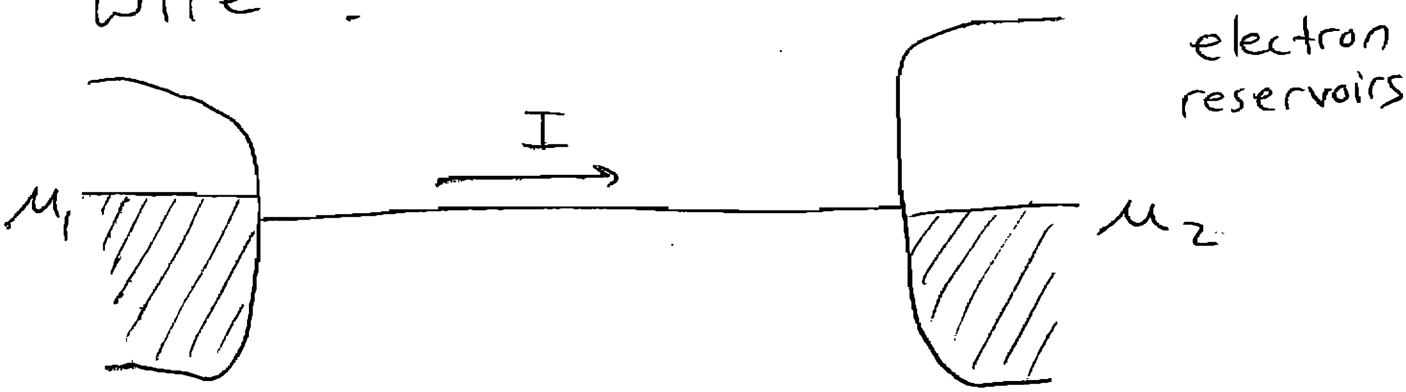
We have seen that the conditions for the validity of Ohm's law $R = \rho \frac{L_x}{A}$ are that the dimensions of the specimen $L_x, L_y, L_z \gg \ell, \lambda$.

$L_i \gg \ell$ is necessary in order to be able to define a coarse grained distribution $f(\vec{r}, \vec{p}, t)$ which varies across the specimen.

$L_i \gg \lambda$ is necessary because the Boltzmann equation neglects wave-mechanical effects. The opposite limit $L_i \ll \ell$, $L_i \sim \lambda$ is the extreme quantum limit of <u>ballistic</u>

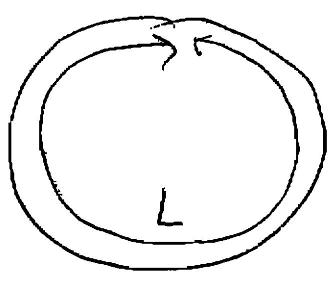
transport.

Q: What is the resistance of a perfect one-dimensional wire?



$$I = e\, v_F \frac{\partial n}{\partial E} (\mu_1 - \mu_2)$$

$\frac{\partial n}{\partial E}$ = density of states per unit length (unidirectional)



$\psi(x) = A e^{ikx}$

$\psi(x+L) = \psi(x) \Rightarrow e^{ikL} = 1$

$k = \frac{2\pi N}{L}, \quad N \in \mathbb{Z}$

$$\frac{\partial n}{\partial E} = \frac{\partial (N/L)}{\partial k} \frac{1}{\frac{\partial E}{\partial k}}$$

$$= \frac{1}{2\pi} \frac{1}{\hbar v_F} = \frac{1}{h v_F}$$

$$I = \frac{e}{h}(\mu_1 - \mu_2) = \frac{e^2}{h} V$$

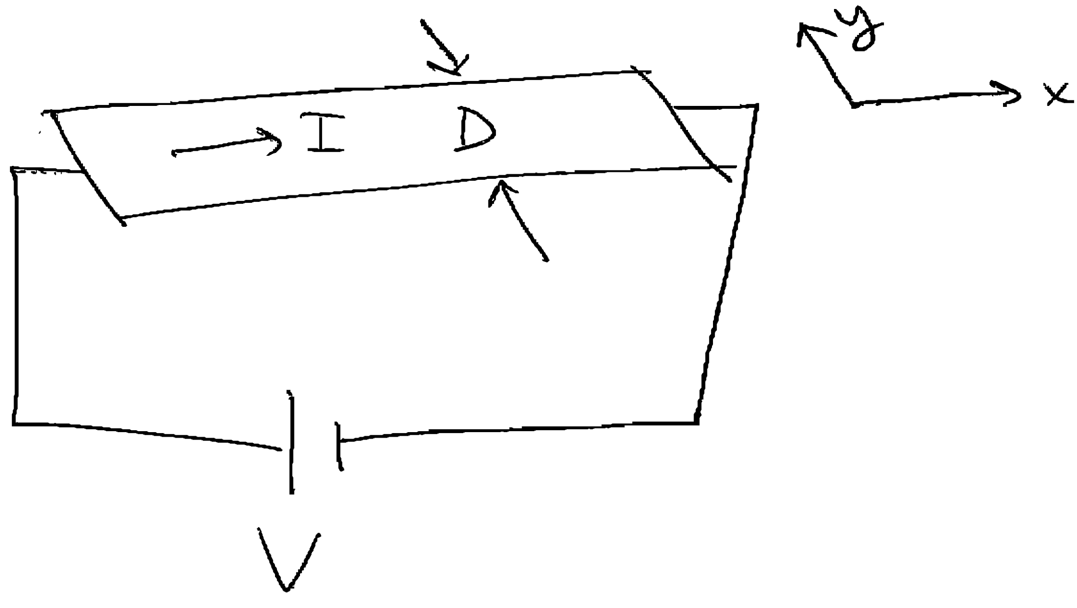
$$V = IR \qquad \boxed{R = \frac{h}{e^2}}$$

$$\boxed{\frac{h}{e^2} = 25,812.8\,\Omega}$$

sometimes this unit of resistance is known as the "von Klitzing," after the Nobel laureate who first measured it accurately in ca. 1980.

Contrary to the expectation of ohm's law, R is independent of the length of the wire!

Q: What is the resistance of a perfect two-dimensional wire of width D?



Schrödinger's equation:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi(x,y) = E\,\Psi(x,y)$$

Boundary conditions:

$$\Psi(x,0) = \Psi(x,D) = 0$$

Separation of variables:

$$\Psi(x,y) = \psi(x)\,\phi(y)$$

$$\frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{\phi}\frac{d^2\phi}{dy^2} = \frac{-2mE}{\hbar^2}$$

$$\psi(x) = e^{ikx}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dy^2} = \left(E - \frac{\hbar^2k^2}{2m}\right)\phi$$

Solution:

$$\phi(y) = A \sin(py)$$

$$p^2 = \frac{2mE}{\hbar^2} - k^2$$

B.C.: $\sin pD = 0$

$$pD = n\pi \qquad p = \frac{n\pi}{D}$$

$$\boxed{\begin{aligned} E &= \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 \pi^2 n^2}{2mD^2} \\ E &= \frac{\hbar^2 k^2}{2m} + \varepsilon_n \end{aligned}}$$

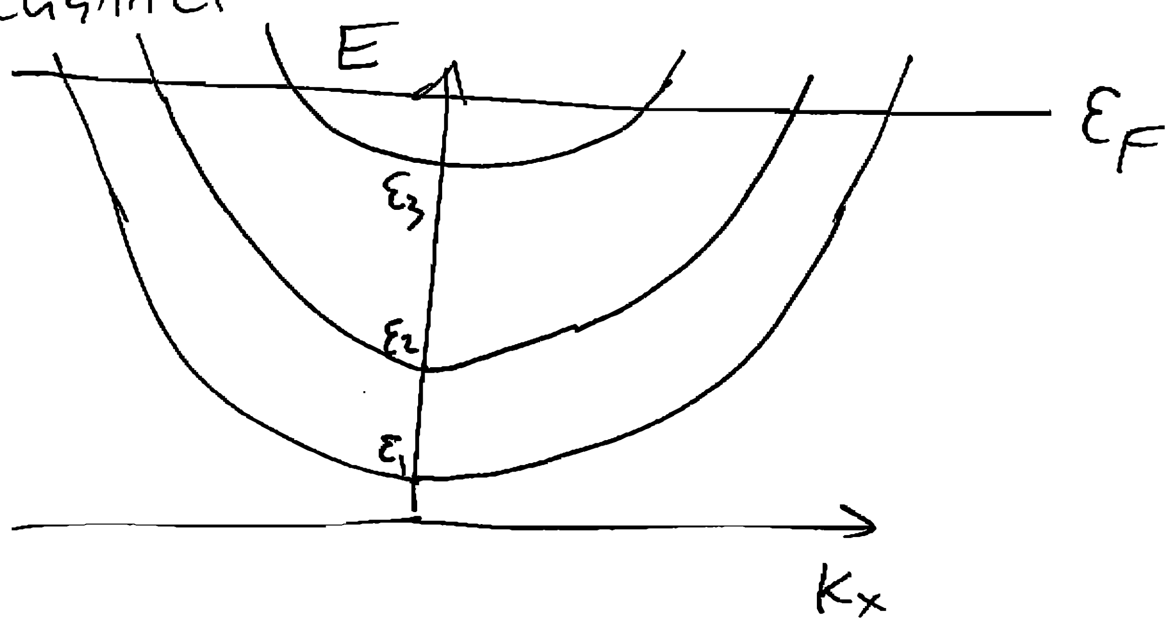
How many transverse states are there with $\varepsilon_n < \varepsilon_F$?

$$\frac{\hbar^2 \pi^2 N^2}{2mD^2} \le \frac{\hbar^2 k_F^2}{2m}$$

$$n \le \frac{k_F D}{\pi}$$

$$N = \max\{n\} = \mathrm{Int}\left\{\frac{k_F D}{\pi}\right\}$$

# of modes grows roughly proportional to D. Each mode n acts as a one-dimensional channel:



$$I = N\frac{e^2}{h}V$$

$$R = \frac{h}{Ne^2} \qquad G = R^{-1} = N\frac{e^2}{h}$$

Landauer formula

In general, if the quantum mechanical probability for an electron in channel n to traverse the wire is T_n, the conductance is

$$G = \frac{e^2}{h}\sum_n T_n .$$

Conductance Quantization

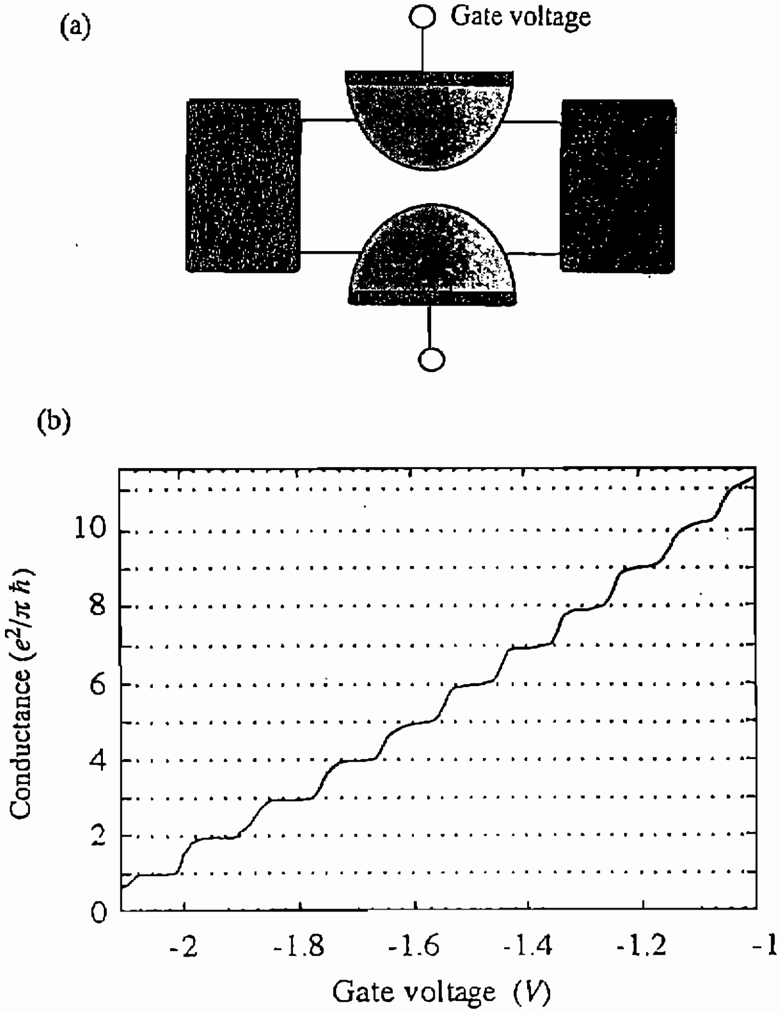


Fig. 2.1.2. Quantized conductance of a ballistic waveguide. (*a*) A negative voltage on a pair of metallic gates (called the split-gate configuration) is used to deplete and narrow down the constriction progressively. (*b*) Measured conductance vs. gate voltage. The measured resistance also includes a series resistance due to the wide regions connecting the constriction to the contacts. This series resistance is measured separately by removing the negative voltage on the gates and is subtracted off before plotting. Reproduced with permission from B. J. van Wees *et al.* (1988), *Phys. Rev. Lett.*, 60, 848. Similar results were reported simultaneously by D. Wharam *et al.* (1988), *J. Phys. C*, 21, L209.

Q: What is the resistance of a perfect 3-D wire with a square cross section $D^2 = A$?

$$E = \frac{\hbar k^2}{2m} + \frac{\hbar^2 \pi^2}{2mA}(n^2+m^2)$$

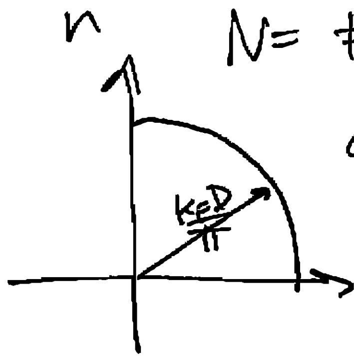
# of modes with $\varepsilon_{nm} < \varepsilon_F$?

$$\varepsilon_{nm} = \frac{\hbar^2 \pi^2}{2mA}(n^2+m^2)$$

$$\frac{\hbar^2 \pi^2}{2mA}(n^2+m^2) < \frac{\hbar^2 k_F^2}{2m}$$

$$n^2+m^2 < \frac{k_F^2 A}{\pi^2}$$

$$\sqrt{n^2+m^2} < \frac{k_F D}{\pi}$$



$N = \#$ of modes $\approx$

$$\text{area} = \frac{\pi}{4} r^2$$

$$= \frac{\pi}{4}\left(\frac{k_F D}{\pi}\right)^2 = \frac{k_F^2 A}{4\pi}$$

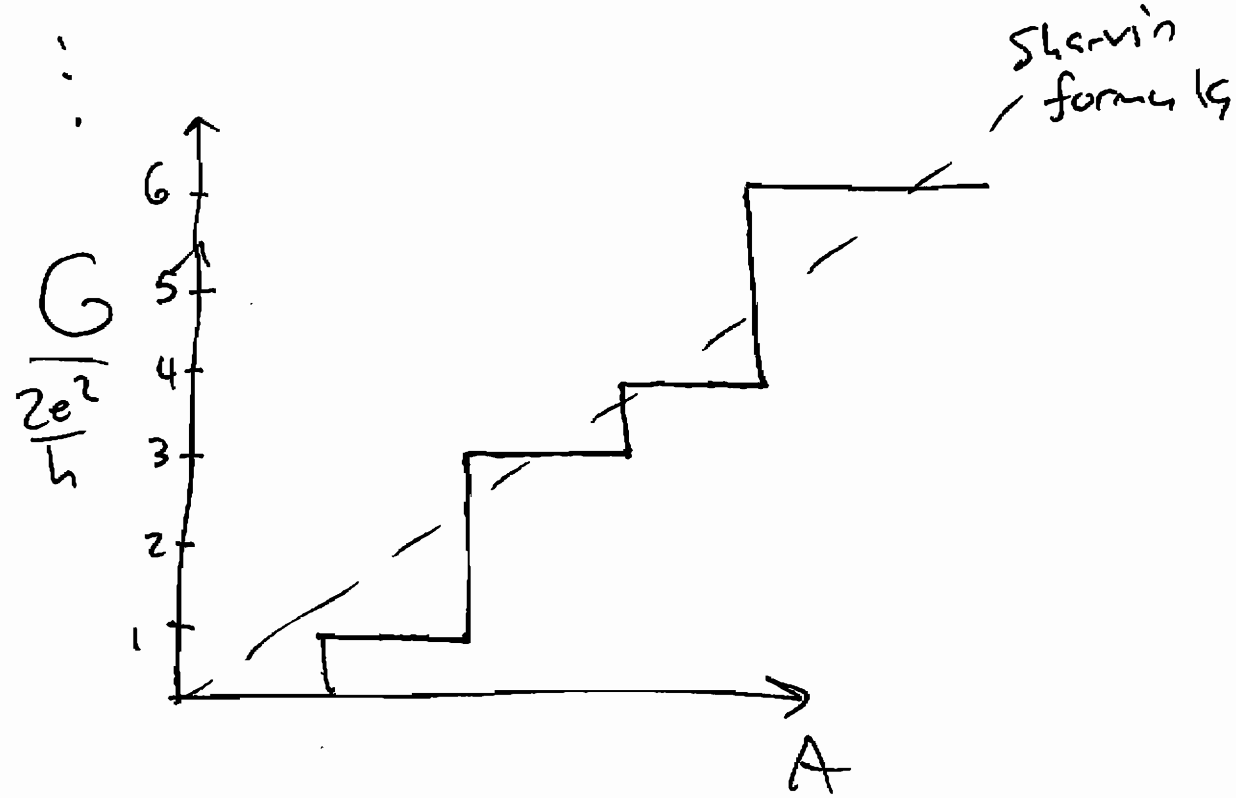
$$G = N\frac{e^2}{h} \approx \frac{k_F^2 A}{4\pi}\frac{e^2}{h}$$

If $\vec{B}=0$, and charge carriers have spin-$1/2$, we must multiply G by 2:

$$G \simeq \frac{2e^2}{h}\frac{k_F^2 A}{4\pi} \quad \left(\text{Sharvin formula}\right)$$

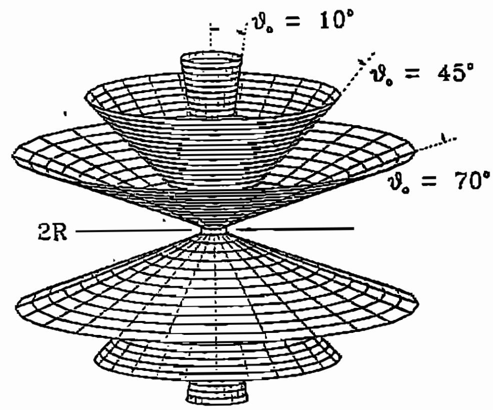
Like Ohm's law, $G \propto A$; unlike Ohm's law, G indep. of L!

mode	$\varepsilon \frac{2mA}{\pi^2\hbar^2}$
1 1	2
1 2	5
2 1	5
2 2	8
1 3	10
3 1	10
$\vdots$	



Calculation for hyperbolic constriction

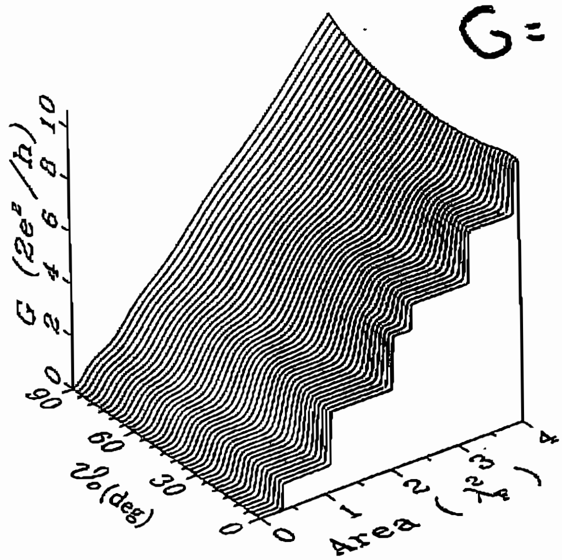
[Torres, Pascual, and Sáenz, PRB **49**, 16581 (1994)]



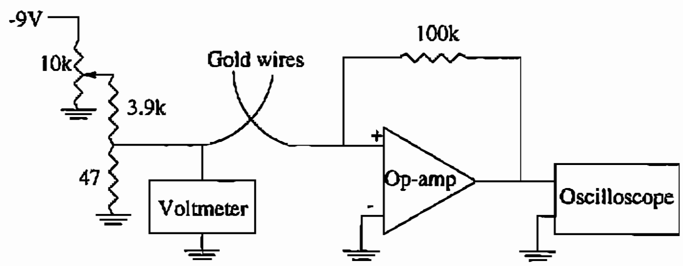
$f = k_F R$

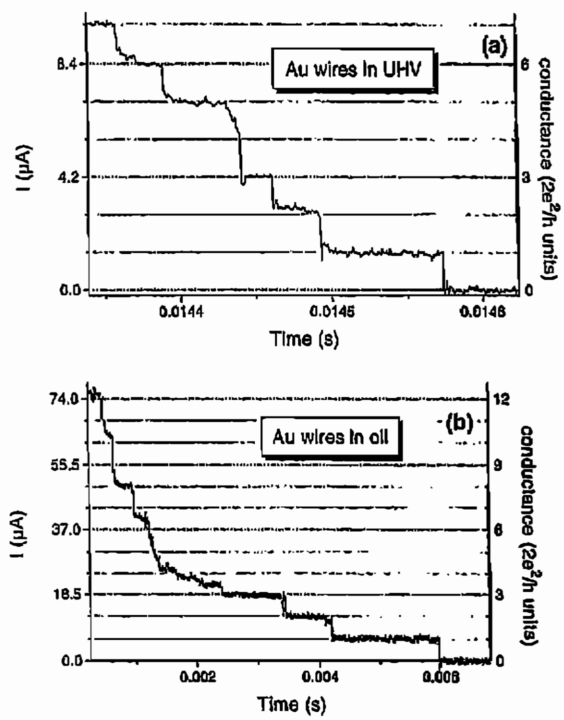
$$T_{\nu\nu'} = \frac{\delta_{\nu\nu'}}{1 + \exp\left(\frac{\pi}{f}\left[\varepsilon_\nu - m^2 - f^2 + 1/4\right]\right)}$$

$$G = \frac{2e^2}{h} \sum_{\nu\nu'} T_{\nu\nu'}$$



Spontaneously occuring nanocontacts

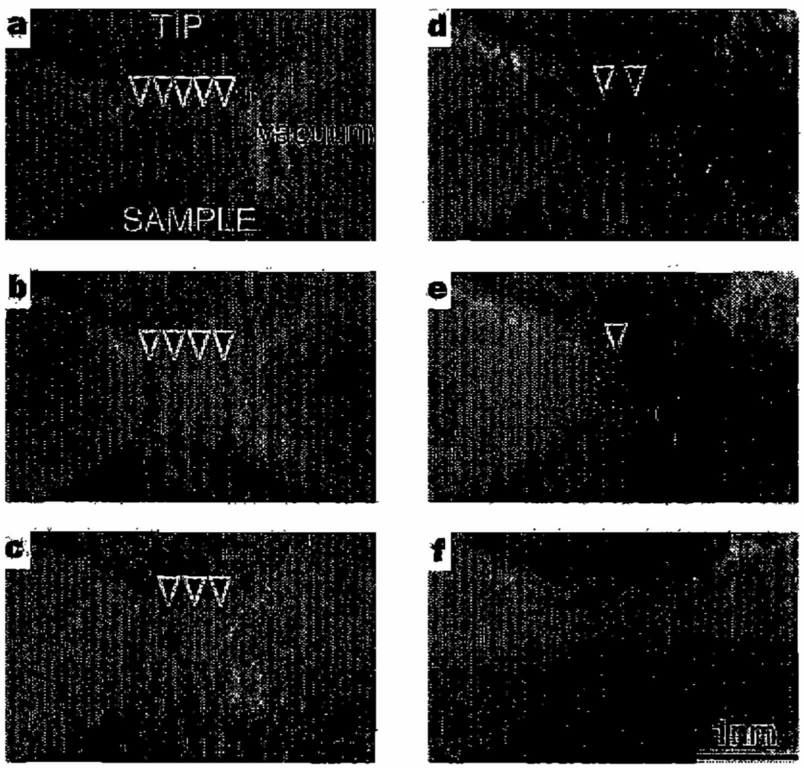




J. L. Costa-Krämer *et al.*, Surf. Sci. **342**, L1144 (1995)

E. L. Foley *et al.*, Am. J. Phys. **67**, 389 (1999)

Electron microscope video of an atomic-scale gold contact breaking

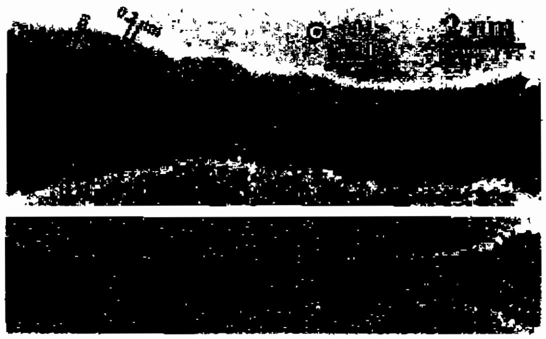


$G = 2G_0$

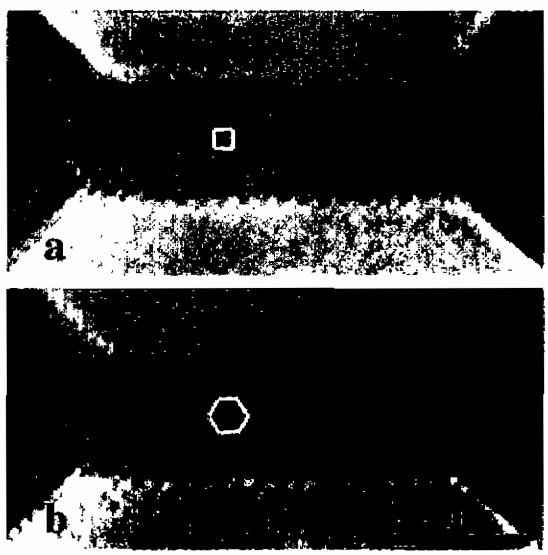
$G = G_0$

H. Ohnishi *et al.*, Nature **395**, 780 (1998)

Gold nanobridges



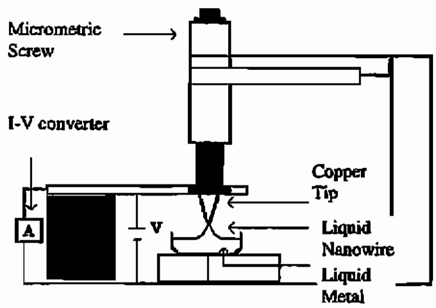
Amorphous

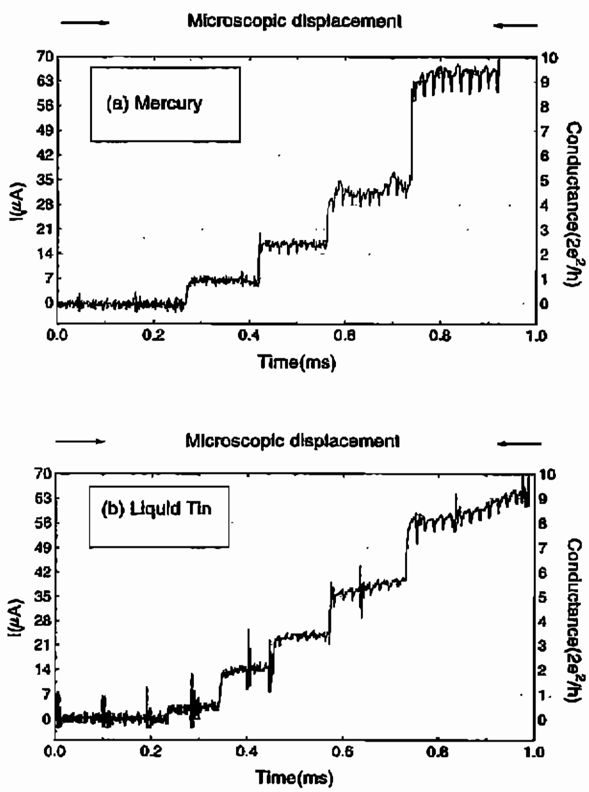


Crystalline

Y. Kondo & K. Takayanagi, PRL **79**, 3455 (1997)

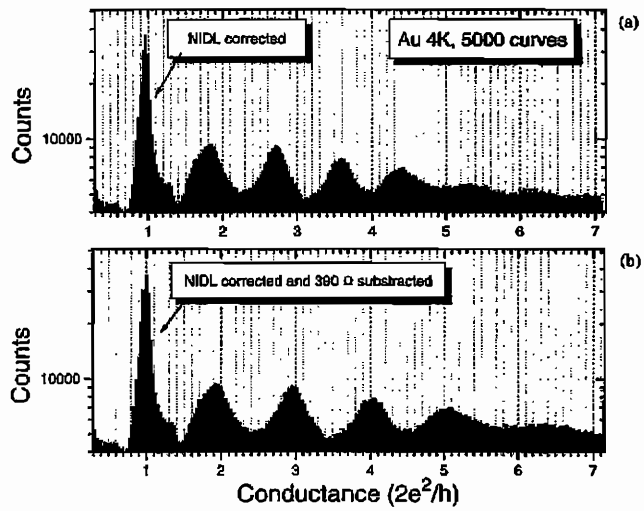
Liquid metal nanowires



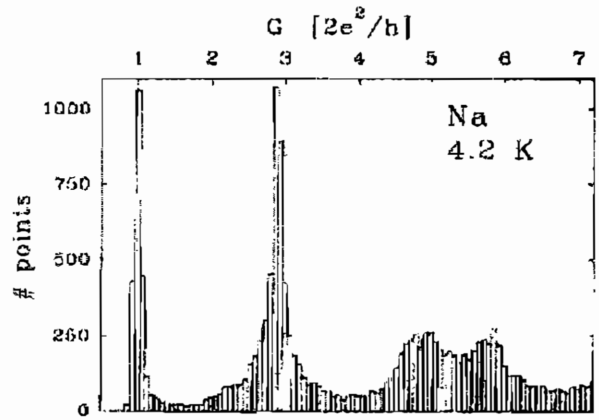


J. L. Costa-Krämer *et al.*, PRB **55**, 5416 (1997)

Conductance histograms



J. L. Costa-Krämer *et al.*, PRB **55**, 12910 (1997)

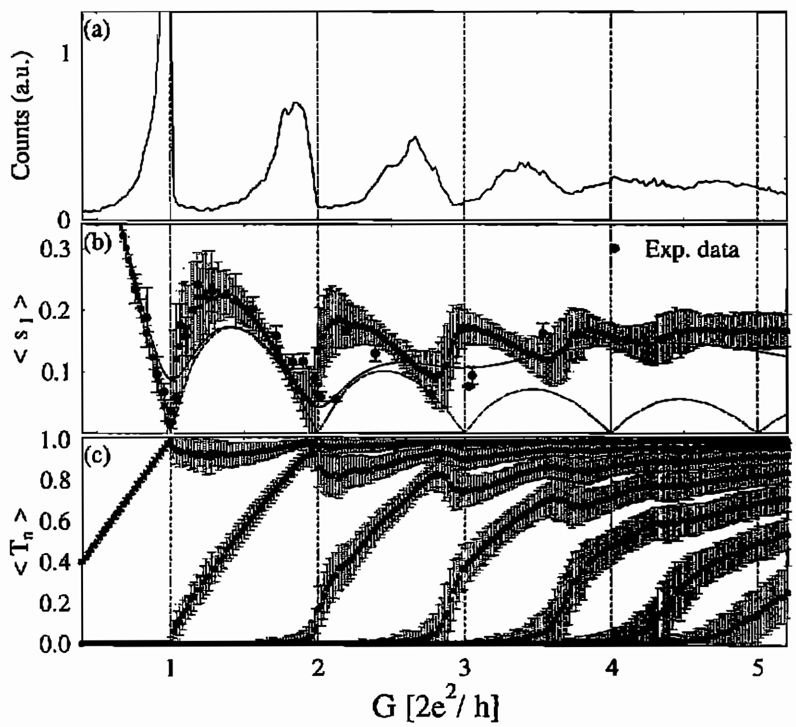


axial symmetry

J. M. Krans *et al.*, Nature **375**, 767 (1995)

Quantum reduction of shot noise

$$s_I = \frac{P_I}{2eI} = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$



Exp: van den Brom & van Ruitenbeek, PRL **82**, 1526 (1999)

Theory: J. Bürki & CAS, PRL **83**, 3342 (1999)