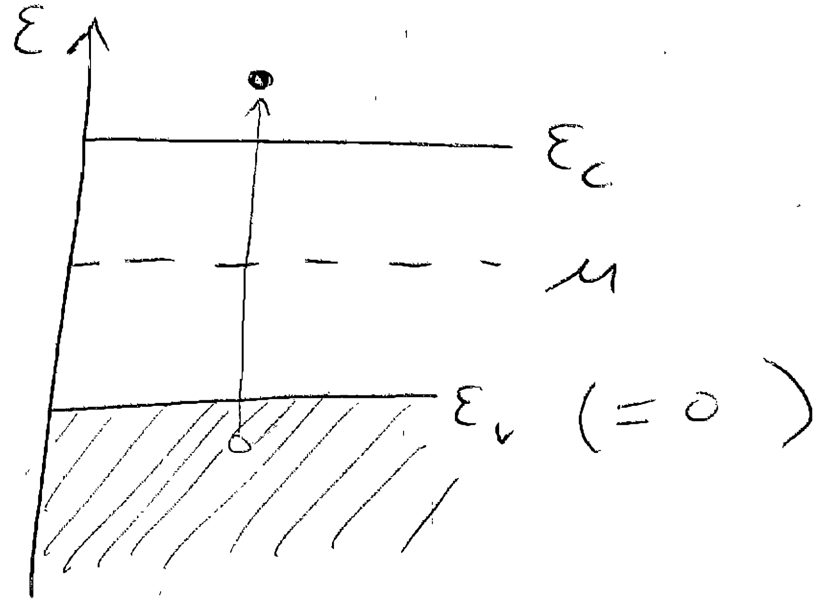
Physics 460/560 Lecture 14

Semiconductors II

Intrinsic carrier concentration

Semiconductors are often doped with special impurity atoms to alter their electrical properties. A pure semiconductor crystal, by contrast, is said to be intrinsic. An intrinsic semiconductor is an insulator at $T=0$, but some electrons are excited from the filled valence band into the empty conduction band at finite temperatures.

- In an intrinsic semiconductor the chemical potential μ lies in the forbidden gap:



energy gap
$\varepsilon_c - \varepsilon_v = \Delta$

- The probability to find an electron of energy ε in the conduction band is

$$f_e(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1} \simeq e^{\beta(\mu-\varepsilon)}$$

(assuming $\varepsilon - \mu \gg k_B T$).

- The conduction band energies are

$$\varepsilon(k) = \varepsilon + \frac{\hbar^2 k^2}{}$$

where m_e is the effective mass in the conduction band. The density of states in the conduction band is (per unit volume)

$$D_e(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(\varepsilon - \varepsilon_c)^{1/2}$$

The concentration of thermally excited electrons in the conduction band is

$$n = \int_{\varepsilon_c}^{\infty} D_e(\varepsilon)\, f_e(\varepsilon)\, d\varepsilon$$

$$\simeq \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{\beta(\mu-\varepsilon_c)} \int_{\varepsilon_c}^{\infty} (\varepsilon-\varepsilon_c)^{1/2} e^{\beta(\varepsilon_c -}$$

$$\boxed{n \simeq 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(\mu-\varepsilon_c)/k_B T}}$$

Note, however that $\mu = \mu(T)$.

The holes near the top of the valence band behave like particles with effective mass m_h:

$$\varepsilon_h(k) = -\varepsilon_e(k) = -\left[\varepsilon_v - \frac{\hbar^2 k^2}{2m_h}\right)$$

$$= -\varepsilon_v + \frac{\hbar^2 k^2}{2m_h}$$

$$\Rightarrow D_h(\varepsilon) = \frac{1}{2\pi}2\left(\frac{2m_h}{\hbar^2}\right)^{3/2}\left(\varepsilon_v - \underset{\substack{\uparrow \\ \text{energy of missing} \\ \text{electron}}}{\varepsilon}\right)^{1/2}$$

The density of holes thermally excited in the valence band is thus

$$p = \int_{-\infty}^{\varepsilon_v} D_h(\varepsilon) f_h(\varepsilon)\, d\varepsilon \simeq 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{3/2} e^{\beta(\varepsilon_v -}$$

The product np does not involve μ:

$$np \simeq 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-\beta\Delta},$$

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-\beta\Delta}$$

Intrinsic semiconductor:

$$n = p \equiv n_i$$

$$m_e^{3/2} e^{\beta(\mu-\varepsilon_c)} = m_h^{3/2} e^{\beta(\varepsilon_v-\mu)}$$

$$e^{2\beta\mu} = \left(\frac{m_h}{m_e}\right)^{3/2} e^{\beta(\varepsilon_v+\varepsilon_c)}$$

$$\frac{2\mu}{k_B T} = \frac{\varepsilon_v+\varepsilon_c}{k_B T} + \ln\left(\frac{m_h}{m_e}\right)^{3/2}$$

$$\mu = \frac{\varepsilon_v+\varepsilon_c}{2} + \frac{3}{4}k_B T\ln\left(\frac{m_h}{m_e}\right)$$

$$\mu \underset{T\to 0}{=} \frac{\varepsilon_v+\varepsilon_c}{2}$$ (middle of the gap)

where $\Delta = \varepsilon_c - \varepsilon_v$.

Actually, we have nowhere assumed the material is intrinsic in this derivation. All that we have assumed is that

$$\varepsilon_c - \mu, \quad \mu - \varepsilon_v \gg k_B T.$$

At 300K one has

material	np
Si	$2.1 \times 10^{19}\ cm^{-6}$
Ge	$2.89 \times 10^{26}\ cm^{-6}$
GaAs	$6.55 \times 10^{12}\ cm^{-6}$

Intrinsic Conductivity

$$\sigma = \frac{n e^2 \tau_e}{m_e} + \frac{p e^2 \tau_h}{m_h}$$

Although τ_e and τ_h are dependent on temperature due to scattering from thermally excited phonons, the main temperature dependence comes from the exponential dependence $e^{-\Delta/2k_BT}$ of the carrier concentration.

Impurities

Donor states

(Go over problem set)

Pentavalent atom in Si, Ge

$$E_d = \frac{m_e e^4}{2\epsilon^2 \hbar^2}$$

ϵ = static dielectric constant

$$a_d = \frac{\epsilon \hbar^2}{m_e e^2}$$

m_e = electron effective mass

In general $m_e < m$ and $\epsilon > 1$, so the binding energy of a donor is << Rydberg and a_d >> Bohr radius. For Ge, for example, $\frac{m_e}{m} \frac{1}{\epsilon^2} = 4 \times 10^{-4}$.

Crystal	ϵ	m_e/m
diamond	5.5	—
Si	11.7	0.2
Ge	15.8	0.1
GaAs	13.13	.066

For a density n_d of donor impurities, the donor electrons will be bound to their parent impurities provided

$n_d^{-1/3} \gg a_d$.

When $n_d^{-1/3} \sim a_d$, the wavefunctions of neighboring donor electrons begin to overlap, and an *impurity band* of delocalized electrons forms. This phenomenon was first desribed by Sir Neville Mott (c.f. lecture on tightbinding approx.) Such a material is known as an n-type semiconductor, and behaves as a semimetal.

Acceptor states

Similarly, a trivalent impurity in Si or Ge will form 4 covalent bonds with its 4 Si (Ge) neighbors, becoming a negative ion. A hole in the valence band will be bound to this negative ion:

$$E_a = \frac{m_h e^4}{2\epsilon^2 \hbar^2}$$

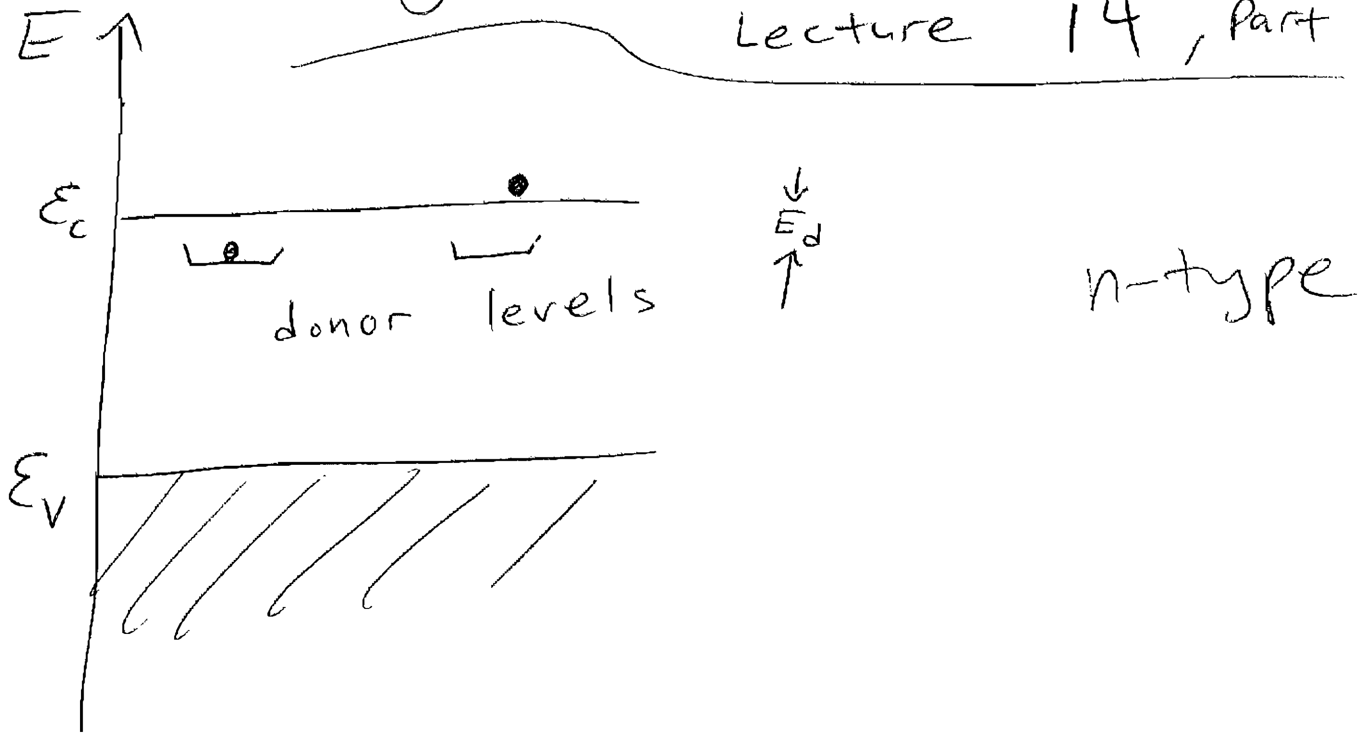
m_h = hole effecti
mass

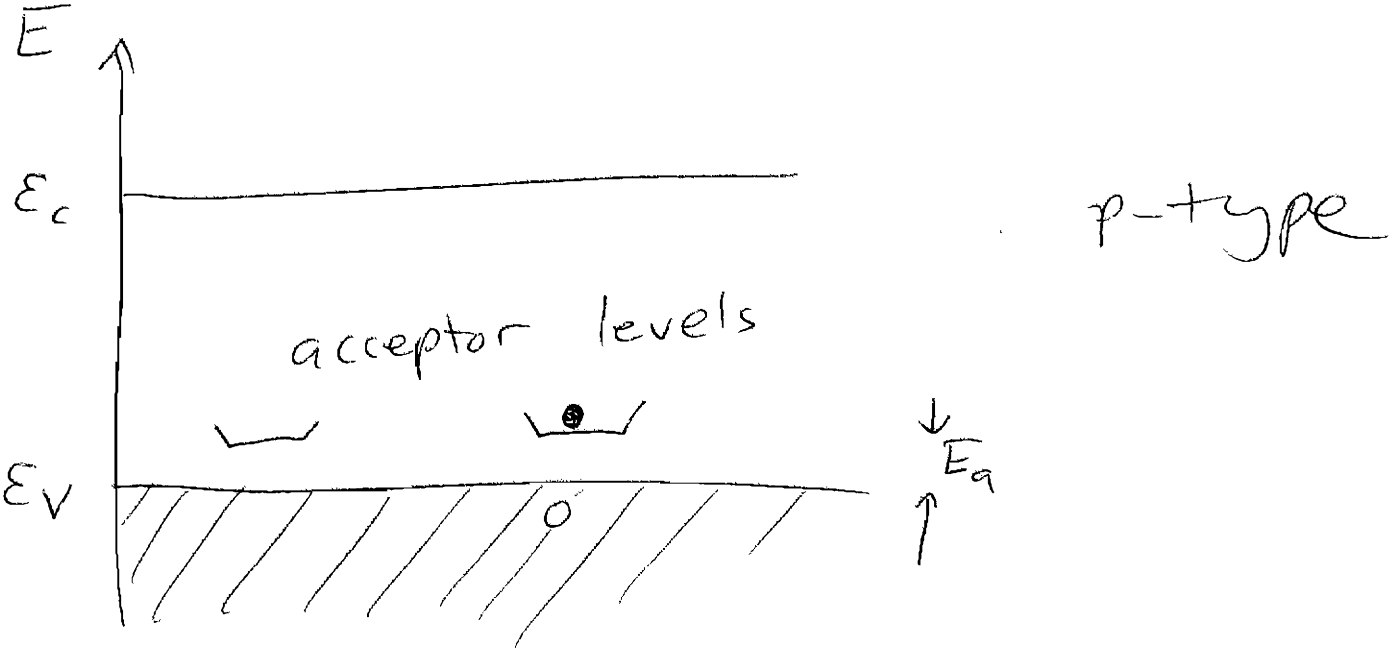
$$a_a = \frac{\epsilon \hbar^2}{m_h e^2}$$

For $n_a^{-1/3} \sim a_a$ an impurity band of holes forms — a p-type semiconductor.

Physics 460/560

Lecture 14, Part 2





n_d = concentration of donor atoms

$n_d = n_d^+ + n_d^0$

n_d^+ = concentration of ionized donors

n_d^0 = concentration of neutral donors

n_d^+ = # of electrons contributed, some of which populate the conduction band and some of which fill holes in the valence band.

Typically $n_d^+ \gg n_i = p_i$,

So $n \simeq n_d^+$.

Ex. Ge at 300K

$$n_i = 5 \times 10^{13} \text{ cm}^{-3}$$

$$\rho = 4.4 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$$

1 ppm impurity concentration

$$= 4.4 \times 10^{16} \text{ cm}^{-3}$$

So even if only 1 % of the impurities are ionized,

$n_d^+ \sim 10\, n_i$.

$$n_d^0 = \frac{n_d}{e^{\beta(\varepsilon_c - E_d - \mu)} + 1}$$

(Fermi-Dirac
distributio

$$n \simeq n_d^+ = n_d - n_d^0$$

$$n \simeq \frac{n_d}{e^{\beta(\mu - \varepsilon_c + E_d)} + 1} \approx n_d e^{-\beta(\mu - \varepsilon_c + E_d)}$$

But we know

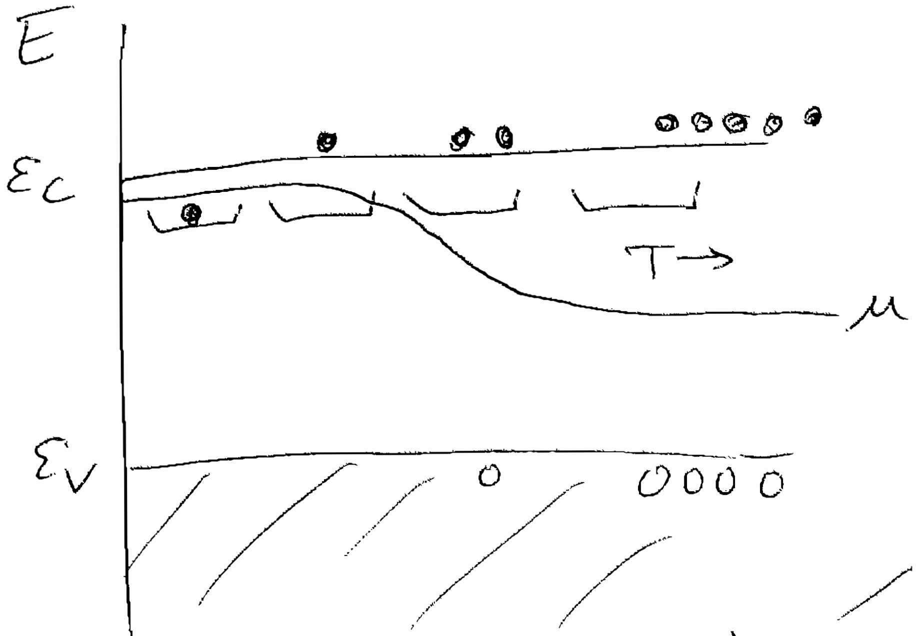
$$n = n_{Qe}\, e^{\beta(\mu - \varepsilon_c)}$$

$$\Rightarrow \quad n_d\, e^{\beta(\varepsilon_c - E_d - \mu)} \simeq n_{Qe}\, e^{\beta(\mu - \varepsilon_c)}$$

$$\Rightarrow \boxed{\mu = \varepsilon_c - \frac{E_d}{2} + \frac{k_B T}{2} \ln\left(\frac{n_d}{n_{Qe}(T)}\right)}$$

Finally, $n = n_{Qe}\, e^{\beta(\mu - \varepsilon_c)}$

$$\cong (n_{Qe} n_d)^{1/2}\, e^{-\beta E_d/2}$$



(similarly for acceptors)