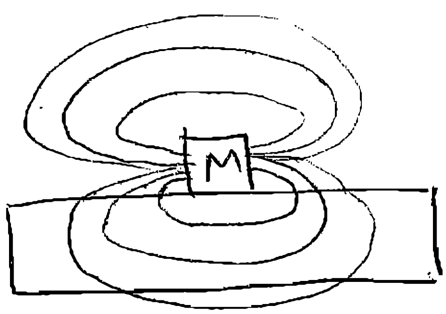
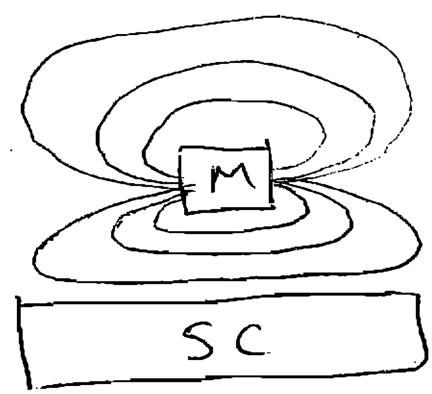
Physics 460 Lecture 18

Superconductivity I

Meissner effect

$T > T_c$ $T < T_c$





magnetic field penetrates normal metal

magnetic field expelled from superconductor

Can we understand the Meissner effect in terms of perfect conductivity?

$$m \frac{d\vec{v}}{dt} = q\vec{E}$$

$$\vec{J} = n_s q \langle \vec{v} \rangle$$

$$n_s q \frac{d\langle \vec{v} \rangle}{dt} = \frac{n_s q^2}{m} \vec{E}$$

$$\frac{d\vec{J}}{dt} = \frac{n_s q^2}{m} \vec{E}$$

$$\frac{d}{dt} \nabla \times \vec{J} = \frac{n_s q^2}{m} \nabla \times \vec{E}$$

$$\frac{d}{dt} \nabla \times \vec{J} = -\frac{n_s q^2}{mc} \frac{\partial \vec{B}}{\partial t}$$

$$\frac{d}{dt} \left[\nabla \times \vec{J} + \frac{n_s q^2}{mc} \vec{B} \right] = 0$$

$$\Rightarrow \quad \nabla \times \vec{J} + \frac{n_s q^2}{mc} \vec{B} = \text{const.}$$

Currents flow only on the surface of a perfect conductor. In order to force $\vec{B} = 0$ in the interior, we must have the integration constant in the above equation $= 0$.

$$\boxed{\nabla \times \vec{J} = -\frac{n_s q^2}{mc} \vec{B}}$$

This equation explains the Meissner effect. Perfect conductivity <u>does not</u> imply the Meissner effect, since a perfect conductor maintains a <u>constant field</u> in its interior, but this constant field need not be zero!

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Superconductivity (cont.)

- London equation

recall: perfect conductivity implies

$$\frac{d}{dt}\left[\nabla \times \vec{J} + \frac{n_s q^2}{mc}\vec{B}\right] = 0$$

or $$\nabla \times \vec{J} + \frac{n_s q^2}{mc}\vec{B} = \text{const.}$$

But the <u>Meissner effect</u> (expulsion of magnetic field by a superconductor) implies

const. = 0.

$$\boxed{\nabla \times \vec{J} + \frac{n_s q^2}{mc} \vec{B} = 0}$$

London equation

sometimes this is rewritten:

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \left[\vec{J} + \frac{n_s q^2}{mc} \vec{A} \right] = 0$$

using the maxwell equation

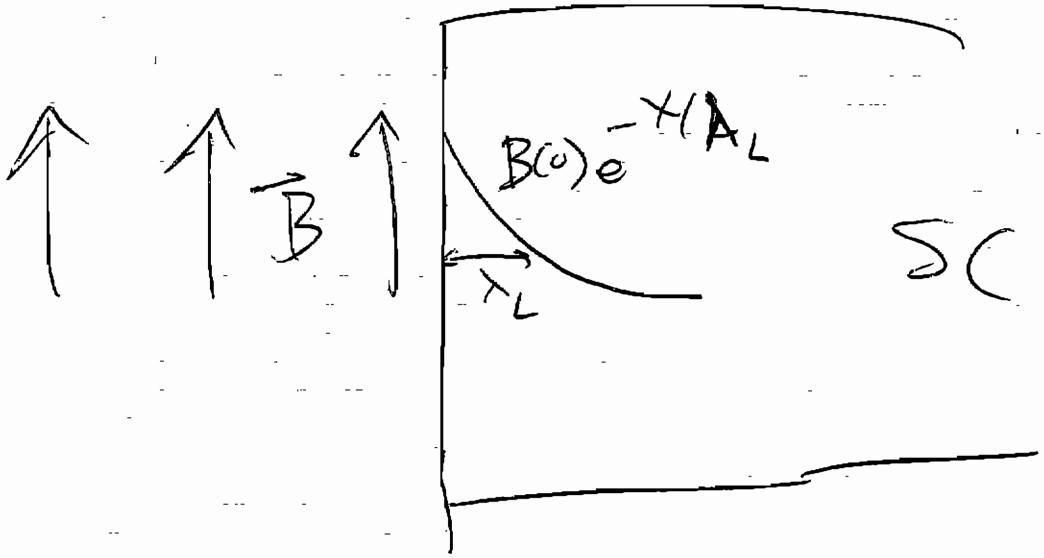
$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} \qquad \text{gives}$$

$$\nabla \times \nabla \times \vec{B} \left(= -\nabla^2 \vec{B}\right) + \frac{4\pi n_s q^2}{mc^2} \vec{B} = 0$$

$$\nabla^2 \vec{B} = \frac{4\pi n_s q^2}{mc^2} \vec{B}$$

Defining the <u>penetration depth</u> $\lambda_L = \sqrt{\frac{mc^2}{4\pi n_s q^2}}$,

$$\nabla^2 \vec{B} = \frac{1}{\lambda_L^2} \vec{B}$$



- <u>Magnetic Energy</u> (small correction)

To calculate the energy cost of the screening currents which shield the interior of a superconductor from a magnetic field, consider the work done on the superconductor as an external field is increased from zero to $\vec{H}$:

$$W = -\int_0^{\vec{H}} \vec{M} \cdot d\vec{H}'$$

But $0 = \vec{B} = \vec{H}' + 4\pi \vec{M}$

so $\vec{M} = -\dfrac{\vec{H}'}{4\pi}$

$$W = \frac{1}{4\pi} \int_0^{\vec{H}} \vec{H}' \cdot d\vec{H}'$$

$$= \frac{\vec{H}^2}{8\pi}$$

Thus the free energy of a superconductor is increased in an applied field:

$$F_S(H) = F_S(0) + \frac{H^2}{8\pi}$$

- Critical field H_c

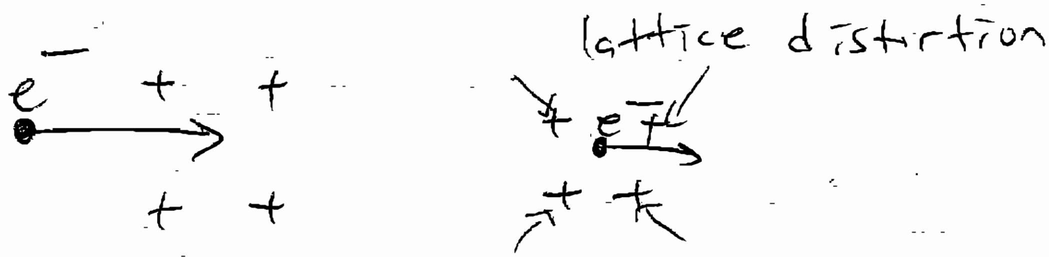
superconductivity is destroyed if $F_S(H) > F_N(H) \approx F_N(0)$

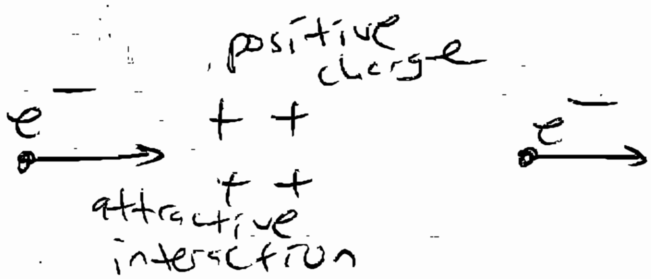
$$F_N = F_S(0) + \frac{H_c^2}{8\pi}$$

$$\boxed{\frac{H_c^2}{8\pi} = \Delta F}$$ stabilization energy of superconductor

Cooper pairs

The electron-phonon-electron interaction is attractive, and can overcome the screened Coulomb repulsion of the electrons:

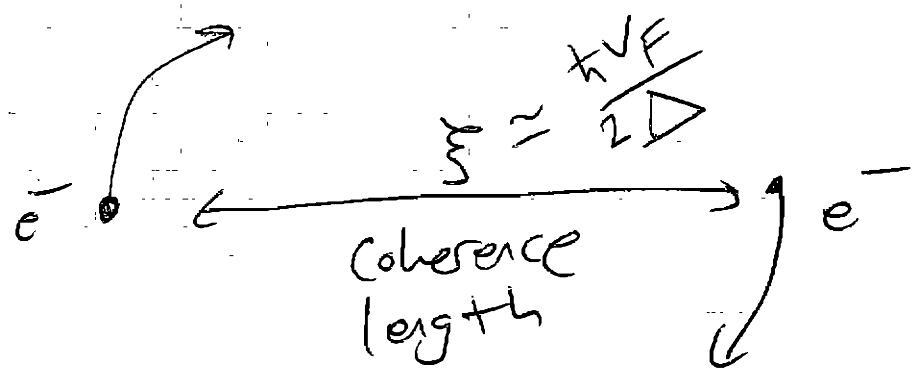




retarded, attractive interaction

This effective attractive interaction gives rise to two-electron bound states — Cooper pairs.

However, Cooper pairs are not independent of each other — as many as 10^6 electrons may be present in the volume occupied by a single cooper pair.



The formation of these bound states is responsible for the lowering of the energy at the superconducting state compared to the normal state, and leads to an energy gap 2Δ at the Fermi surface.

- <u>superconducting order parameter</u>

Cooper pairs typically have a singlet spin state and orbital angular momentum zero:

<u>S-wave pairing</u>

Many high-temperature supercondcuctors, on the other hand (like YBCO), have Cooper pairs with a spin singlet wavefunction and orbital angular momentum 2:

<u>d-wave pairing</u>.

For this case, the superconducting order parameter is more complicated, and we won't discuss it further.

Because Cooper pairs are bosons, a macroscopic number of them can condense into the ground state orbital, yielding a macroscopic density $n_s(\vec{r})$ of pairs. In a semiclassical approximation, the probability amplitude for cooper pairs is

$$\psi(\vec{r}) = \sqrt{n_s(\vec{r})}\, e^{i\theta(\vec{r})}$$

$$n_s(\vec{r}) = |\psi(\vec{r})|^2.$$

The velocity of a particle is

$$\vec{v} = \frac{1}{m}\left(\vec{p} - \frac{q}{c}\vec{A}\right) = \frac{1}{m}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\vec{A}\right)$$

The particle flux is

$$\psi^* \vec{v} \psi = \frac{n_s}{m}\left(\hbar \nabla \theta - \frac{q}{c}\vec{A}\right)$$

(assuming $n_s(\vec{r}) \cong$ const.)

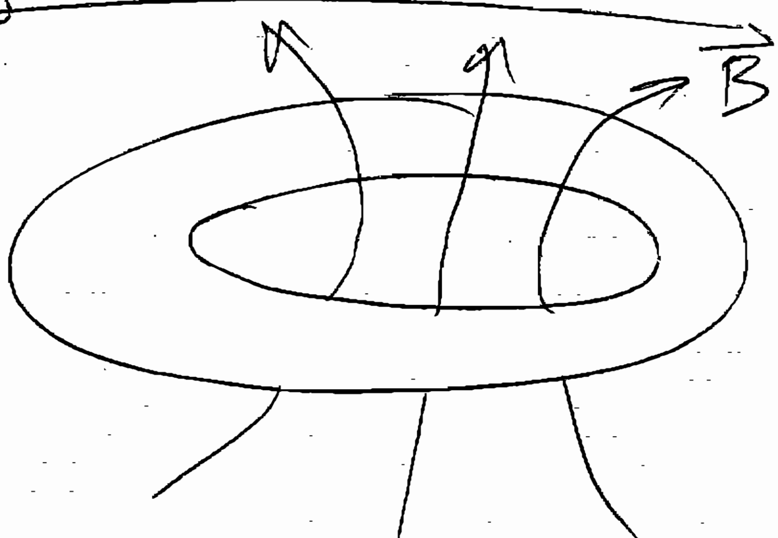
The current is

$$\vec{J}_e = q \psi^* \vec{v} \psi = \frac{n_s q}{m}\left(\hbar \nabla \theta - \frac{q}{c}\vec{A}\right)$$

$$\boxed{\nabla \times \vec{J}_e = -\frac{n_s q^2}{mc}\vec{B} \qquad \text{London equation}}$$

Thus we see that the London eq and Meissner effect follow directly from the assumption that the superconducting state is a Bose condensate of cooper pairs. It follows also that $q = -2e$ and $m = 2m_e$.

Flux quantization



Consider a multiply-connected

superconductor in the form of a ring. The Meissner effect implies that $\vec{B}$ and $\vec{J}_e$ are zero in the interior of the superconductor:

$$0 = \vec{J}_e = \frac{n_s q}{m}\left(\hbar \nabla\theta - \frac{q}{c}\vec{A}\right)$$

$$\Rightarrow \quad \nabla\theta = \frac{q}{\hbar c}\vec{A}$$

The wave function of the Cooper pairs is single-valued, which means

$\theta(\vec{r})$ ~~can~~ must return to its original value on traversing the ring, or increase by a multiple of 2π:

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi s \qquad s \in \mathbb{Z}$$

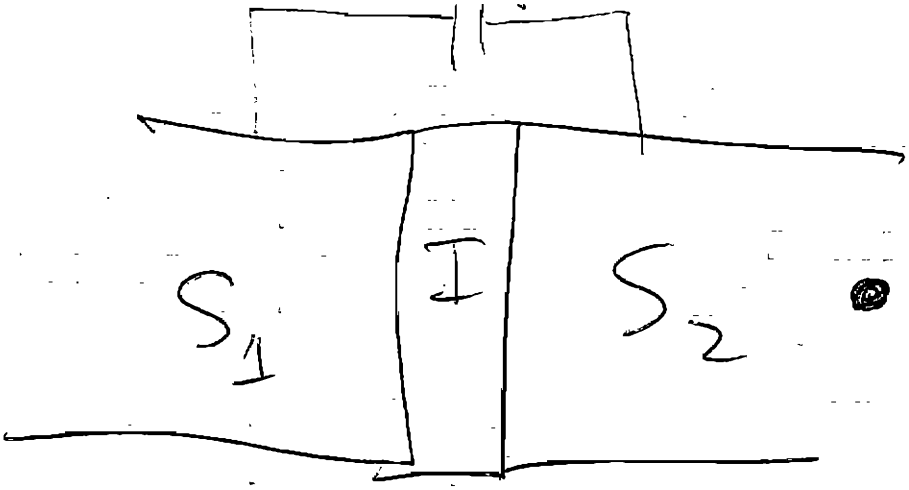
$$\|$$

$$\oint \frac{q}{\hbar c} \vec{A} \cdot d\vec{l} = \frac{q}{\hbar c} \underset{\text{magnetic flux}}{\Phi} = 2\pi s$$

$$\boxed{\Phi = \frac{hc}{q} s = s\,\phi_0}$$

$$\boxed{\phi_0 = \frac{hc}{2e} \quad \text{superconducting flux quantum}}$$

Thus the superconducting eddy currents adjust to fix the total magnetic flux through the ring to be an integer times the flux quantum. Such <u>persistent currents</u> have been observed to flow undiminished for several years. $\Rightarrow$ $\boxed{16'}$ <u>Type II</u>



- Josephson Effect

— SIS junction

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = \begin{pmatrix}\varepsilon_1 & T \\ T & \varepsilon_2\end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

ε_i = energy of ground state orbital in superconductor i

T = tunneling matrix element through insulator

DC Josephson effect

If S_1 and S_2 are identical

materials and there is no applied voltage, we can take $\varepsilon_1 = \varepsilon_2 = 0$

$$\Rightarrow \quad i\hbar \frac{\partial \psi_1}{\partial t} = T\psi_2$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = T\psi_1$$

$$\frac{\partial \psi_1}{\partial t} = \frac{1}{2} n_1^{-1/2} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i\psi_1 \frac{\partial \theta_1}{\partial t}$$

$$\frac{\partial \psi_2}{\partial t} = \frac{1}{2} n_2^{-1/2} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i\psi_2 \frac{\partial \theta_2}{\partial t}$$

$$\psi_1^* \frac{\partial \psi_1}{\partial t} = \frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t}$$

$$\psi_2^* \frac{\partial \psi_2}{\partial t} = \frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t}$$

sch. eq:

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{T}{i\hbar}\sqrt{n_1 n_2}\, e^{i\delta}$$

$$\delta = \theta_2 - \theta_1$$

$$\frac{1}{2}\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = \frac{T}{i\hbar}\sqrt{n_1 n_2}\, e^{-i\delta}$$

equating real and imaginary parts gives:

$$\frac{\partial n_1}{\partial t} = \frac{2T}{\hbar}\sqrt{n_1 n_2}\sin\delta$$

$$\frac{\partial n_2}{\partial t} = -\frac{2T}{\hbar}\sqrt{n_1 n_2}\sin\delta$$

$$\frac{\partial \theta_1}{\partial t} = -\frac{T}{\hbar}\sqrt{\frac{n_2}{n_1}}\cos\delta$$

$$\frac{\partial \theta_2}{\partial t} = -\frac{T}{\hbar}\sqrt{\frac{n_1}{n_2}}\cos\delta$$

If $n_1 \simeq n_2$ as for identical superconductors, we have

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \qquad \frac{\partial}{\partial t}(\theta_2 - \theta_1) = 0$$

and

$$\frac{\partial n_1}{\partial t} = -\frac{\partial n_2}{\partial t}$$

The current flowing through the barrier from $1 \rightarrow 2$ is

$$J = q\frac{\partial n_2}{\partial t} = -q\frac{\partial n_1}{\partial t}$$

$$J = \frac{4eTn_s}{\hbar}\sin(\theta_2 - \theta_1)$$

$$= J_0 \sin(\theta_2 - \theta_1)$$

with no applied voltage, a dc current between $-J_0$ and J_0 will flow across the junction, according to the phase difference $\delta = \theta_2 - \theta_1$.

- AC Josephson effect

If a voltage V is applied then $\varepsilon_1 - \varepsilon_2 = qV = -2eV$

Let $\varepsilon_1 = -eV$,

$\varepsilon_2 = eV$

$$\Rightarrow \quad \frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{ieV}{\hbar} n_1 + \frac{T}{i\hbar}\sqrt{n_1 n_2}\, e^{i\delta}$$

Re: $$\frac{\partial n_1}{\partial t} = \frac{2T}{\hbar}\sqrt{n_1 n_2}\,\sin\delta$$

Im: $$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - \frac{T}{\hbar}\sqrt{\frac{n_2}{n_1}}\,\cos\delta$$

Similarly,

$$\frac{\partial n_2}{\partial t} = -\frac{2T}{\hbar}\sqrt{n_1 n_2}\,\sin\delta$$

$$\frac{\partial \theta_2}{\partial t} = -\frac{eV}{\hbar} - \frac{T}{\hbar}\sqrt{\frac{n_1}{n_2}}\,\cos\delta$$

$n_1 \simeq n_2 \quad \Rightarrow \quad \frac{\partial}{\partial t}(\theta_2 - \theta_1) = -\frac{2eV}{\hbar}$

$$\delta(t) = -\frac{2eV}{\hbar} t + \delta(0)$$

$$J = J_0 \sin\delta$$

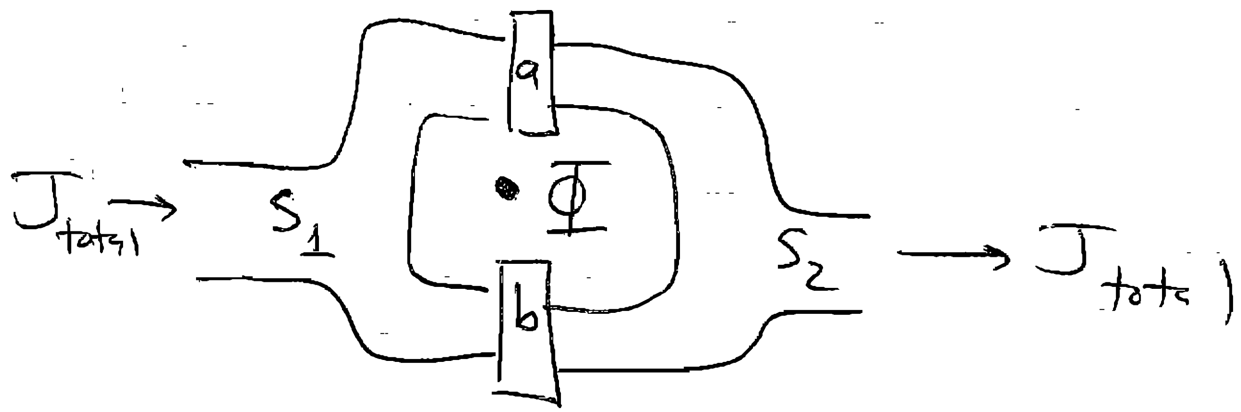
$$J(t) = J_0 \sin\left[\delta(0) - \frac{2eVt}{\hbar}\right]$$

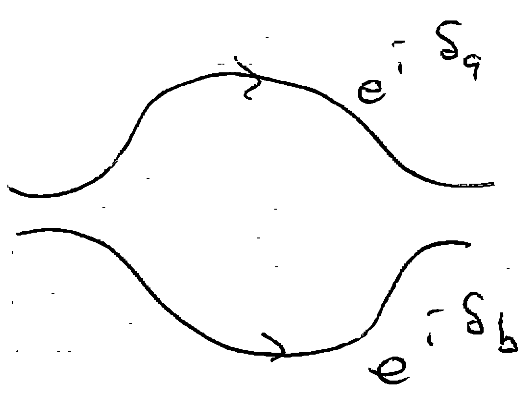
The current oscillates with frequency

$$\boxed{\omega = \frac{2eV}{\hbar}}$$

This implies that a photon of energy $\hbar\omega = 2eV$ is emitted or absorbed each time a cooper pair crosses the barrier. By measuring V and ω, it is possible to obtain a very precise value of $e/\hbar$.

- **SQUID** = Superconducting Quantum Interference Device.





$$\delta_b - \delta_a = \frac{2e}{\hbar c}\Phi$$

$$\delta_b = \delta_0 + \frac{e\Phi}{\hbar c}$$

$$\delta_a = \delta_0 - \frac{e\Phi}{\hbar c}$$

$$J_{total} = J_0 \sin \delta_a + J_0 \sin \delta_b$$

$$= J_0 \left[\sin\left(\delta_0 + \frac{e}{\hbar c}\Phi\right) + \sin\left(\delta_0 - \frac{e\Phi}{\hbar c}\right) \right]$$

$$= 2\left(J_0 \sin \delta_0\right) \cos \frac{e\Phi}{\hbar c}$$

The current is periodic with period $\underline{\underline{2\phi_0}} = \frac{hc}{e}$.

SQUIDS are the most sensitive devices for measuring magnetic fields. accuracy $\sim 10^{-15}$ T.