Physics 460/560 Lecture 26

Mesoscopic Physics I:

Persistent currents

Realizations:

1) superconducting ring
macroscopic persistent current,
flux quantization $\phi_0^{sc} = \frac{hc}{2e}$

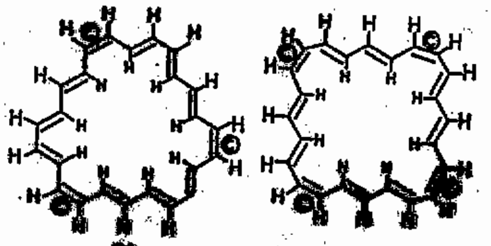
2) aromatic molecules
persistent current, parity effect

3) mesoscopic normal metal rings

periodicity: $\frac{hc}{e} = \phi_0$

Persistent Current in a Microscopic Ring

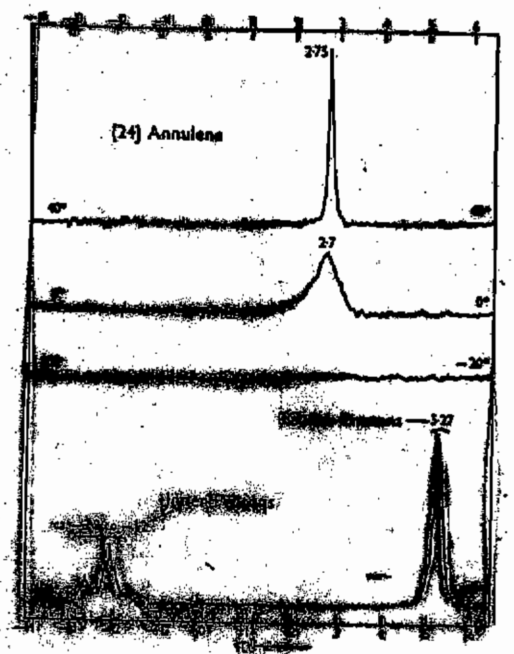
to allow a distinction to be made between structure (I) (inner : outer proton ratio, 37·5 : 62·5) and structure (II) (ratio, 33·3 : 66·7).



(I) (II)

The [illegible] n.m.r. [illegible] of [24][illegible] average, [illegible] bonds [illegible] movement of the [illegible] may be [illegible] cyclo-oct[illegible]

It is [illegible] n.m.r. spectra of the [illegible] [16]annulene and [24]annulene, the [illegible] appear at low field and the outer [illegible] field. This is a reversal of the [illegible] $(4n + 2)$ systems, [14]annulene and [illegible]. A similar reversal between $4n$ and $(4n + 2)$ systems in the dehydro-annulene [illegible] already been observed, and a theoretical [illegible] tion based on quantum-mechanical considerations has been advanced.



[illegible], in [illegible] Mc./sec., [illegible]

([illegible], October 20th, 1966; Comm. [illegible])

Calder & Sondheimer 1966

Persistent Current in a Mesoscopic Au Ring

VOLUME 67, [illegible] 25 PHYSICAL [illegible] 16 [illegible]

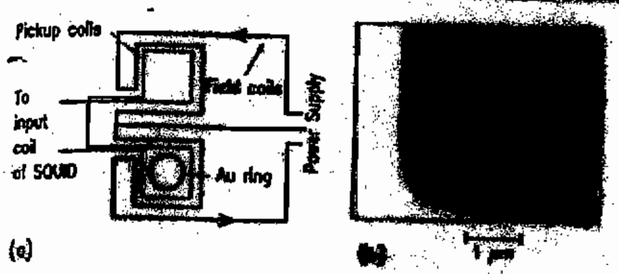
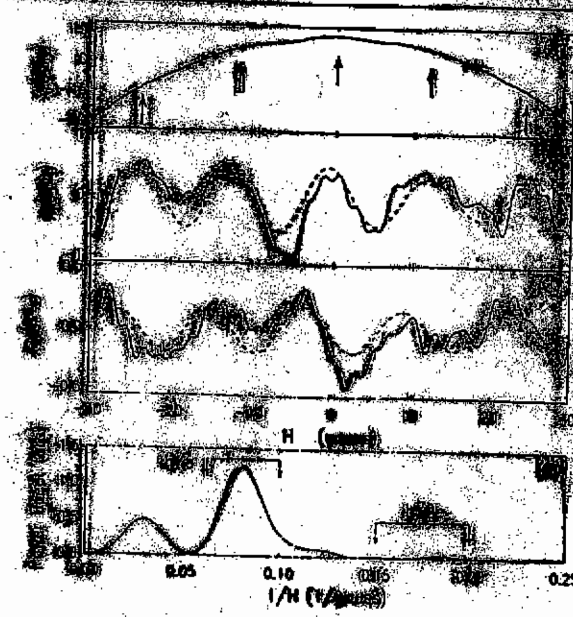


FIG. 1. (a) Schematic [illegible] of the pickup coil chip, [illegible] the [illegible] pickup coils and the [illegible] magnetic-field coils. (b) [illegible] of the [illegible] Au ring in one corner of [illegible]. The loop linewidth is 90 [illegible] the thickness is [illegible].

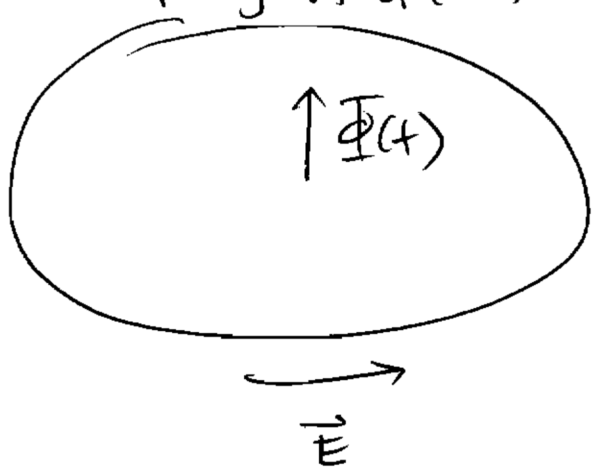


Chandrasekhar et al. 1991

1. Lenz's Law

- Diamagnetism of a perfect classical conductor.

ring of circumference L



$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{1}{c}\frac{d\Phi}{dt}$$

- $$m\frac{dv}{dt} = qE = -\frac{q}{cL}\frac{d\Phi}{dt}$$

$$I = \frac{Nqv}{L} = -\frac{Nq^2}{mcL^2}\Phi + \text{const.}$$

N = # of charge carriers

Superconductor

- $$I = -\frac{2N_s e^2}{mcL^2}\left(\Phi - n\phi_0^{sc}\right)$$

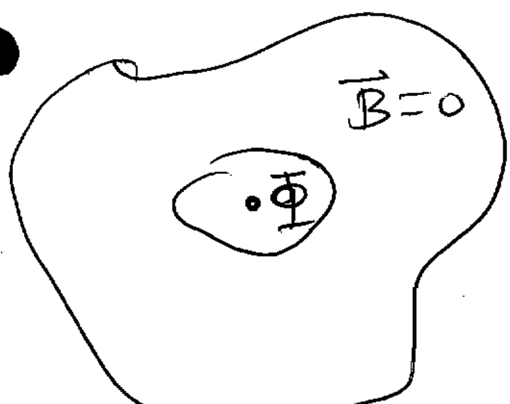
N_s → total # of cooper pairs

multiple valued

Persistent current in a supercond. ring is <u>macroscopic</u> and always <u>diamagnetic</u>.

2. <u>General considerations</u>

Byers + Yang Phys. Rev. Lett. <u>7</u>, 46 (1961)



Metal ring threaded by a magnetic flux $\underline{\Phi}$.

$q=-e$ kinetic energy

$$H = \sum_{j=1}^{N} \left\{ \frac{1}{2m} \left[\frac{\hbar}{i} \nabla_j + \frac{e}{c} \vec{A}(\vec{r}_j) \right]^2 + U(\vec{r}_j) \right\}$$

$$+ \sum_{i<j} V(\vec{r}_i, \vec{r}_j)$$

surface potential, disorder

interparticle interactions

Since $\vec{B}=0$ in metallic region,

- $\nabla \times \vec{A} = 0. \implies \vec{A} = \nabla \chi$

χ is not single-valued:

$$\Delta \chi = \oint \vec{A} \cdot d\vec{\ell} = \Phi.$$

Schrödinger equation

- $H \, \psi(\vec{r}_1, \ldots, \vec{r}_N) = E \, \psi(\vec{r}_1, \ldots \vec{r}_N)$

Let $\tilde{\psi}(\vec{r}_1, \ldots, \vec{r}_N) = \psi(\vec{r}_1, \ldots, \vec{r}_N) e^{\frac{ie}{\hbar c} \sum_j \chi(\vec{r}_j)}$

$\Rightarrow \tilde{H} \tilde{\psi} = E \tilde{\psi} \qquad \tilde{H} = H(\vec{A}=0)$

Boundary conditions

- ψ single-valued $\Rightarrow \tilde{\psi} \to \tilde{\psi} e^{i \frac{e}{\hbar c} \Phi}$ when one particle is taken around the ring.

Theorem 1

Energy eigenvalues $E_n(\phi)$ are periodic ~~in~~ Φ with period

$\frac{hc}{e}$.

Theorem 2

$E_n(-\Phi) = E_n(\Phi)$

follows from $\hat{H}\Psi^* = E\Psi^*$

$$\Psi^* \rightarrow \Psi^* e^{-\frac{ie}{\hbar c}\Phi}$$

Theorem 3

$Z(\Phi) = Z(-\Phi)$ is also periodic with period ϕ_0.

$$Z = \sum_n e^{-\beta E_n(\Phi)}$$

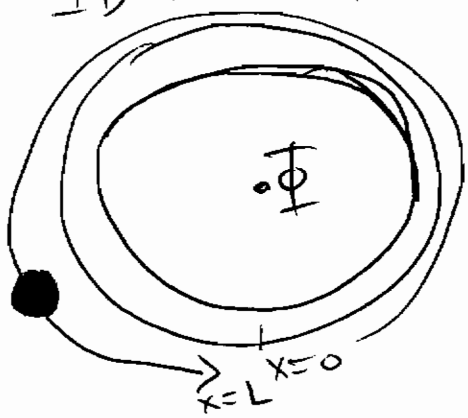
corollary : same holds for $F = -k_B T \ln Z$

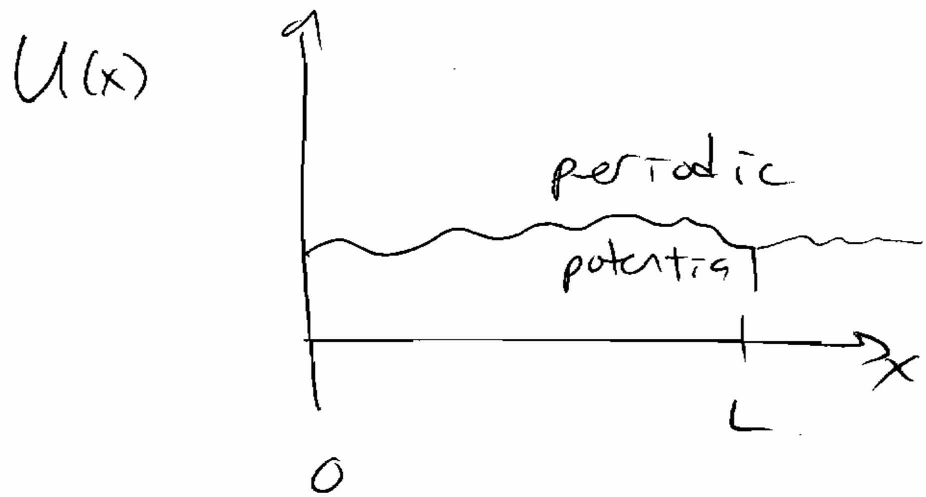
Analogy with Bloch's theorem

$$\tilde{\Psi}(\vec{r}_1, \dots \vec{r}_N) = \psi(\vec{r}_1, \dots \vec{r}_N)\, e^{\frac{ie}{\hbar c}\sum_j \chi(\vec{r}_j)}$$

ψ periodic function of each coordinate

1D idealization:





$$e^{\frac{ie\Delta x}{\hbar c}} = e^{\frac{ie\Phi}{\hbar c}} = e^{ikL}$$

k = Bloch wavevector

$$= \frac{e}{\hbar c}\frac{\Phi}{L}$$

Equilibrium current

- $$\langle I \rangle = \sum_n \frac{e^{-\beta E_n}}{Z} I_n$$

$$I_n = \frac{e}{L} \sum_{j=1}^{N} v_j^{(n)} = \frac{e}{L} \sum_{j=1}^{N} \frac{1}{\hbar} \frac{d\varepsilon_j^{(n)}}{dk}$$

(neglecting interparticle interactions)

- But $k = \frac{e}{\hbar c} \frac{\Phi}{L}$

$$I_n = -c \sum_{j=1}^{N} \frac{d\varepsilon_j^{(n)}}{\partial \Phi} = -c \frac{dE_n}{d\Phi}$$

(current in an eigenstate).

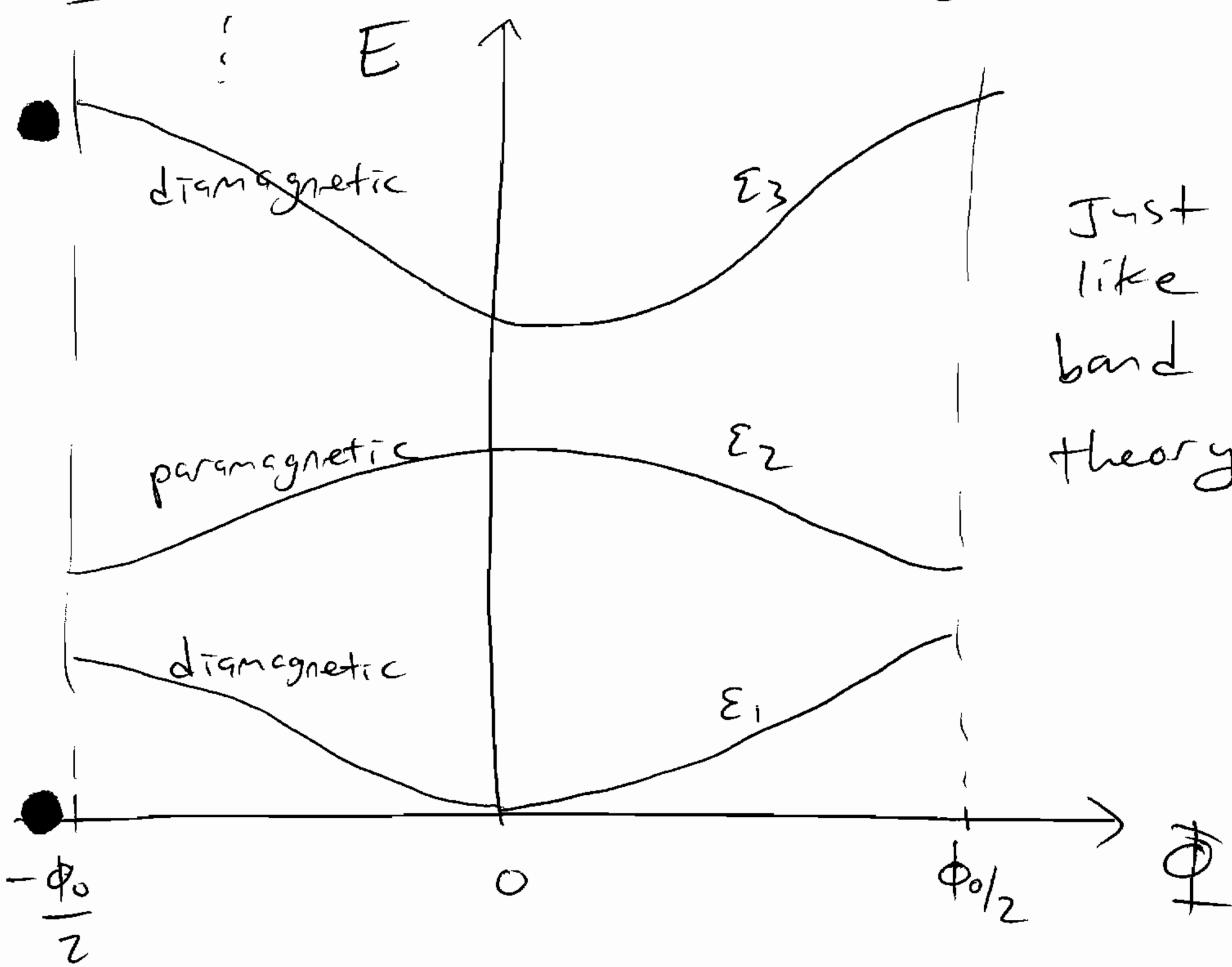
$$\boxed{\begin{aligned} \langle I \rangle &= -c \sum_n \frac{dE_n}{d\Phi} \frac{e^{-\beta E_n}}{Z} \\ &= -c \frac{\partial F}{\partial \Phi} \end{aligned}}$$

The result

- $\langle I \rangle = -c \dfrac{\partial F}{\partial \Phi}$ is more general than the derivation we have given.

1D case

single-particle energies:



Just like band theory

Case 1) Nearly free electrons

$U(r_i)$ weak.

$$\varepsilon = \frac{\hbar^2 k^2}{2m} \qquad e^{ikL} = e^{\frac{ie\Phi}{\hbar c}}$$

$$kL = 2\pi n + \frac{2\pi \Phi}{\phi_0}$$

$$k = \frac{2\pi}{L}(n + \Phi/\phi_0)$$

$$\varepsilon_n(\Phi) = \frac{h^2}{2mL^2}(n + \Phi/\phi_0)^2$$

$$E = \sum_{j=1}^{N} \varepsilon_{n_j}(\Phi)$$

$T=0$: $$I = -c\frac{\partial E_0}{\partial \Phi} = -\frac{ch^2}{mL^2}\sum_{j=1}^{N}\frac{1}{\phi_0}\left(n_j + \frac{\Phi}{\phi_0}\right)$$

- $$I = -\frac{e h^2}{mL^2\phi_0}\left(N\frac{\Phi}{\phi_0} + \cancel{\sum_{j=1}^{N} n_j} \to 0\right)$$

for $-\frac{\phi_0}{2} < \Phi < \frac{\phi_0}{2}$

(when N is odd)

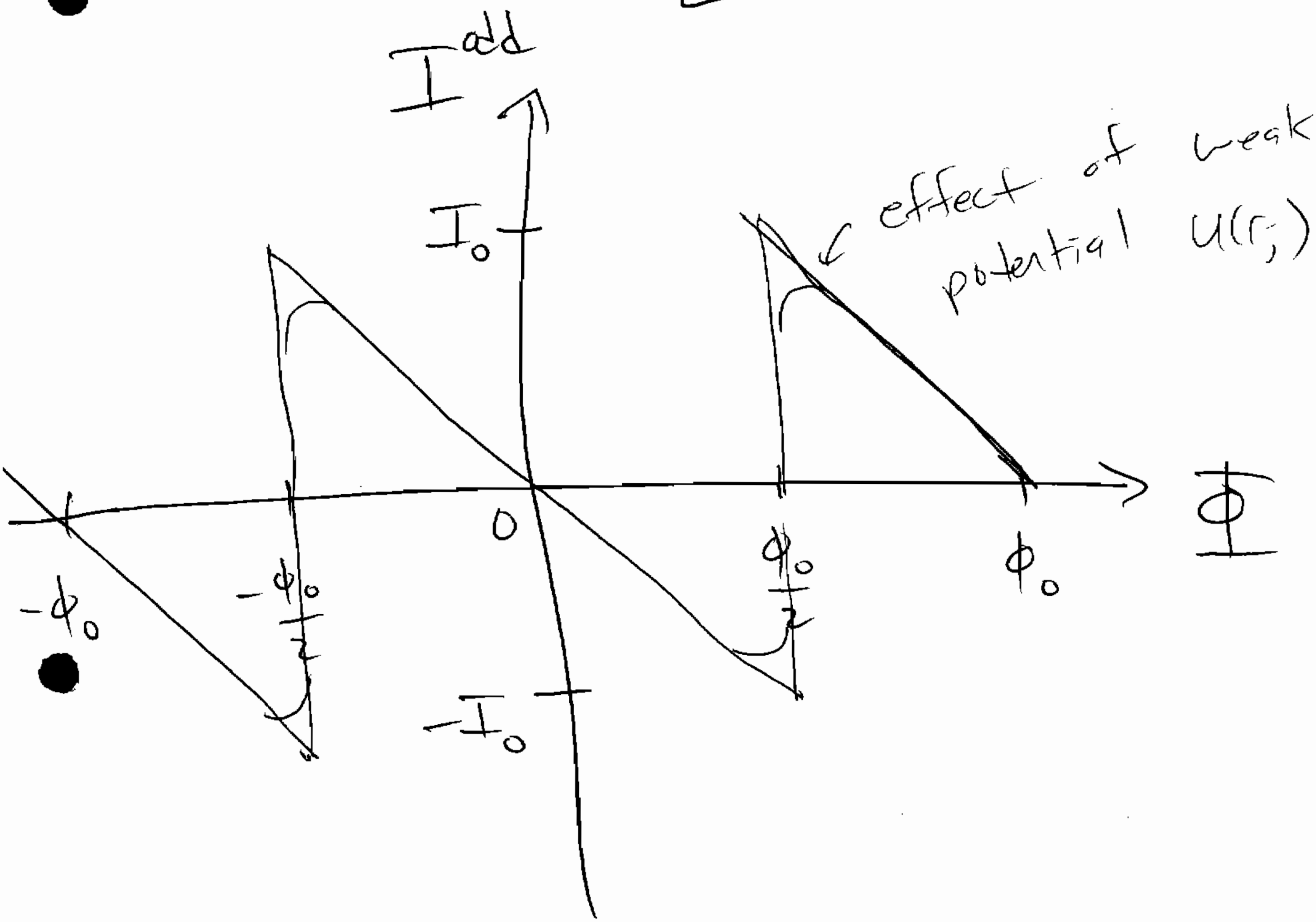
$$I = -\frac{e}{L}\,\frac{hN}{mL}\,\frac{\Phi}{\phi_0}$$

- But $k_F = \frac{\pi N}{L}$ in 1D (neglecting spin)

So $$\frac{hN}{mL} = \frac{h k_F}{\pi m} = \frac{2\hbar k_F}{m} = 2v_F$$

$$\Rightarrow \boxed{I = -\frac{2ev_F}{L}\,\frac{\Phi}{\phi_0} \qquad -\frac{\phi_0}{2} < \Phi < \phi_0/2}$$

Let $I_0 = \frac{e v_F}{L}$



Because $F(\Phi) = F(-\Phi)$, $I(-\Phi) = -I(\Phi)$

and because $F(\Phi + \phi_0) = F(\Phi)$,

$I(\Phi + \phi_0) = I(\Phi)$.

However, this result of a diamagnetic persistent current of maximum amplitude $I_0 = \frac{e v_F}{L}$ only holds for N odd!

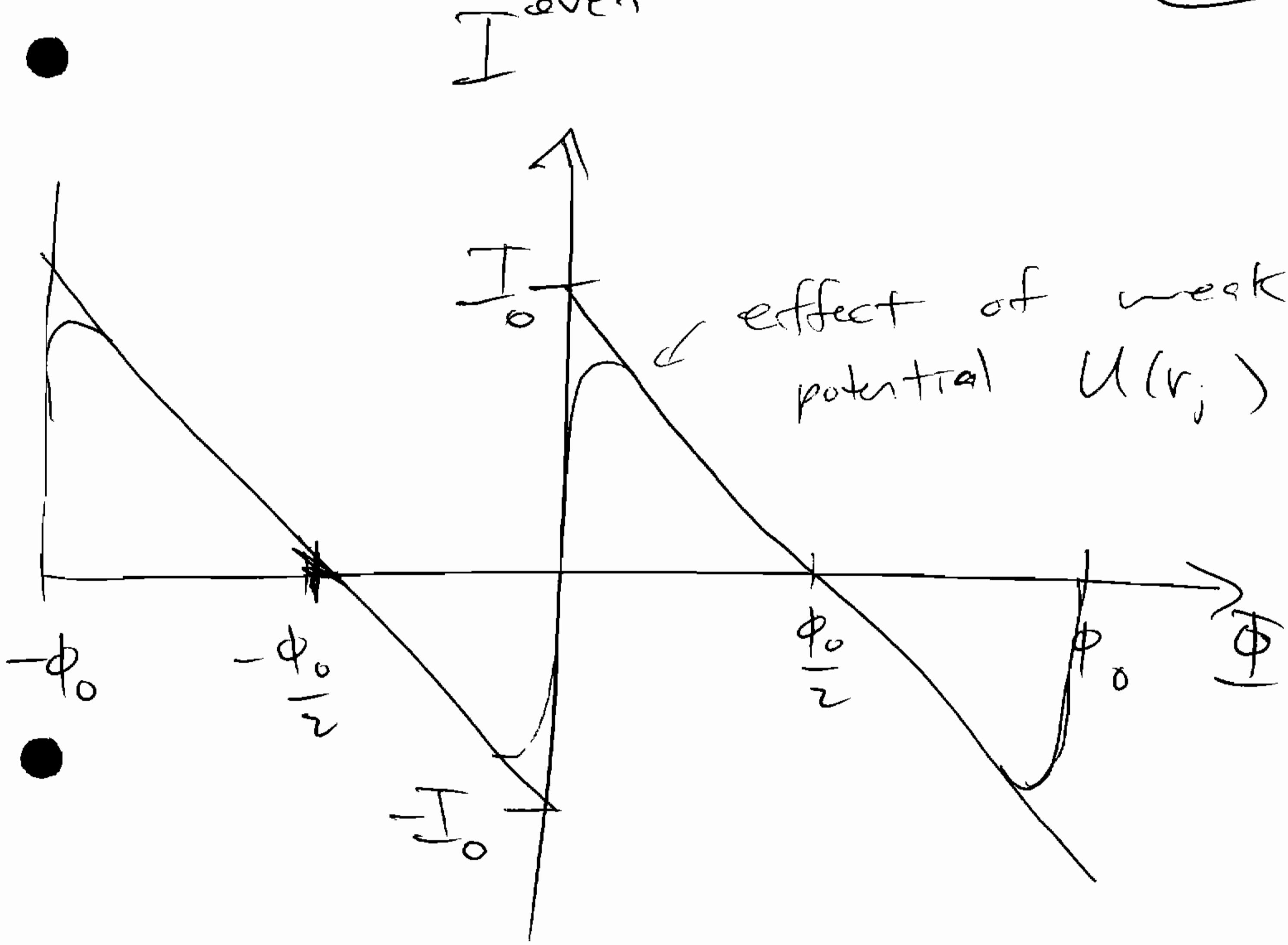
only then is $\sum_{j=1}^{N} n_j = 0$

for $n_j \in \mathbb{Z}$, $n_{j+1} - n_j = 1$.

If N is even, then

$$\sum_{j=1}^{N} n_j = \pm \frac{N}{2}$$

and $I^{even} = -\frac{e}{L} \frac{hN}{mL} \left(\frac{\Phi}{\phi_0} \pm \frac{1}{2} \right)$



$$\Rightarrow \quad \left.\frac{dI}{d\Phi}\right|_{\Phi=0} > 0 \qquad \underline{\text{paramagnetic}}$$

Time to encircle ring

$$\tau_0 = \frac{L}{v_F} \quad \text{(no scattering)}$$

$$\Rightarrow \quad I_0 = \frac{e}{\tau_0}$$

Case 2:
Strong scattering

$U(r_j)$ not small.

Let $\ell \ll L$ be the mean free path. An electron encircling the ring does a random walk,

- with step size ℓ. Steps are equally likely forward and backward. The net r.m.s. displacement after N_s steps is $\ell\sqrt{N_s}$.

- $L = \ell\sqrt{N_s}$ typical time to encircle ring: τ_D

$$N_s = \frac{L^2}{\ell^2}$$

$$\tau_D = N_s \frac{\ell}{v_F} = \frac{L^2}{v_F \ell}$$

- $\frac{h}{\tau_D} = \frac{h v_F \ell}{L^2} = \frac{h D}{L^2} =$ Thouless energy

- $D = v_F \ell$ = 1D diffusion coefficient

Expected persistent current

$$I \sim \frac{e}{\tau_D} = \frac{e v_F}{L} \frac{\ell}{L}$$

- still diamagnetic if N = odd
  paramagnetic if N = even

(for spin-1/2, if $\frac{N}{2}$ = odd/even)

$\Rightarrow$ E v. Φ from p. 10

measured current in small

- metal rings somewhat larger!

Effects of pairing

Nearly free electrons:

$$\varepsilon_n(\Phi) = \frac{h^2}{2mL^2}\left(n + \Phi/\phi_0\right)^2$$

$$\varepsilon_n + \varepsilon_{-n} = \frac{h^2}{mL^2}\left(n^2 + \frac{\Phi^2}{\phi_0^2}\right)$$

$$F \underset{T\to 0}{=} N_s \frac{h^2}{mL^2}\frac{\Phi^2}{\phi_0^2} + \text{const.}$$

$$I = -c\frac{\partial F}{\partial \Phi} = -\frac{2N_s h^2 c}{mL^2\phi_0^2}\Phi$$

$$= -\frac{2N_s e^2}{mcL^2}\Phi$$

c.f. p. 4

$\Rightarrow$ macroscopic diamagnetic current!

For a superconducting ring with $\Phi = \phi_0^{sc} = \frac{hc}{2e}$ enclosed, Cooper pairs are formed from pairing

$$\mathcal{E}_n + \mathcal{E}_{-n-1} = \frac{\hbar^2}{2mL^2}\left[\left(n + \frac{e\Phi}{\hbar c}\right)^2 + \left(n+1-\frac{e\Phi}{\hbar c}\right)^2\right]$$

$$= \frac{\hbar^2}{mL^2}\left[\left(n+\frac{1}{2}\right)^2 + \left(\frac{e\Phi}{\hbar c} - \frac{1}{2}\right)^2\right]$$

$$F \underset{T\to 0}{=} N_s \frac{\hbar^2}{mL^2}\left(\Phi - \phi_0^{sc}\right)^2 \frac{1}{\phi_0^2} + \text{const.}$$

$$I = -c\frac{\partial F}{\partial \Phi} = \frac{-2N_s e^2}{mcL^2}\left(\Phi - \frac{hc}{2e}\right).$$

⋮

For an enclosed flux of

$\Phi = s\frac{hc}{2e}$, Cooper pairs

formed from $\mathcal{E}_n + \mathcal{E}_{n-s}$.

~~Note $\Phi_{total} \neq \Phi$ = externally applied flux.~~

~~The total flux is quantized; the external flux is not!~~

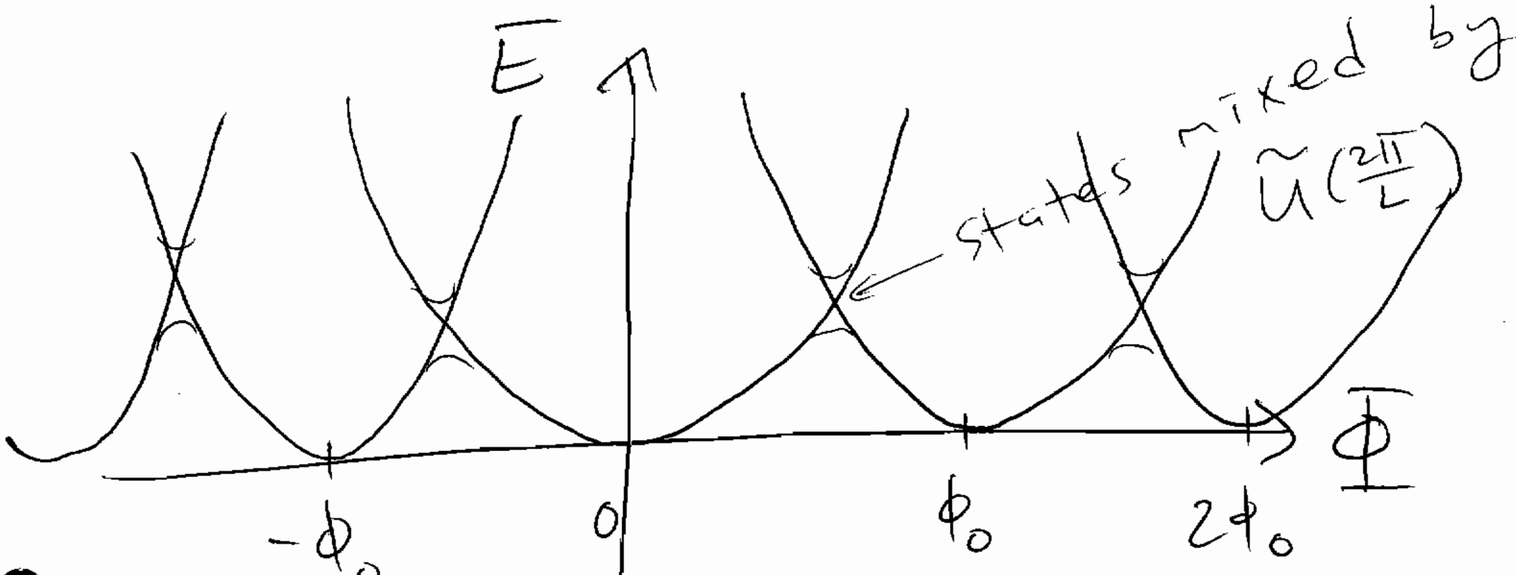
Normal vs. Superconducting persistent current

3D ring ≈ ensemble of 1D rings

1D Normal metal ring at $T=0$:

$$E_0(\Phi) = \sum_{j=1}^{N} \frac{h^2}{2mL^2}\left(n_j + \frac{\Phi}{\phi_0}\right)^2$$

$$= \text{const.} + \frac{Nh^2}{2mL^2}\left(\frac{\Phi}{\phi_0} + \frac{1}{N}\sum_{j=1}^{N} n_j\right)^2$$

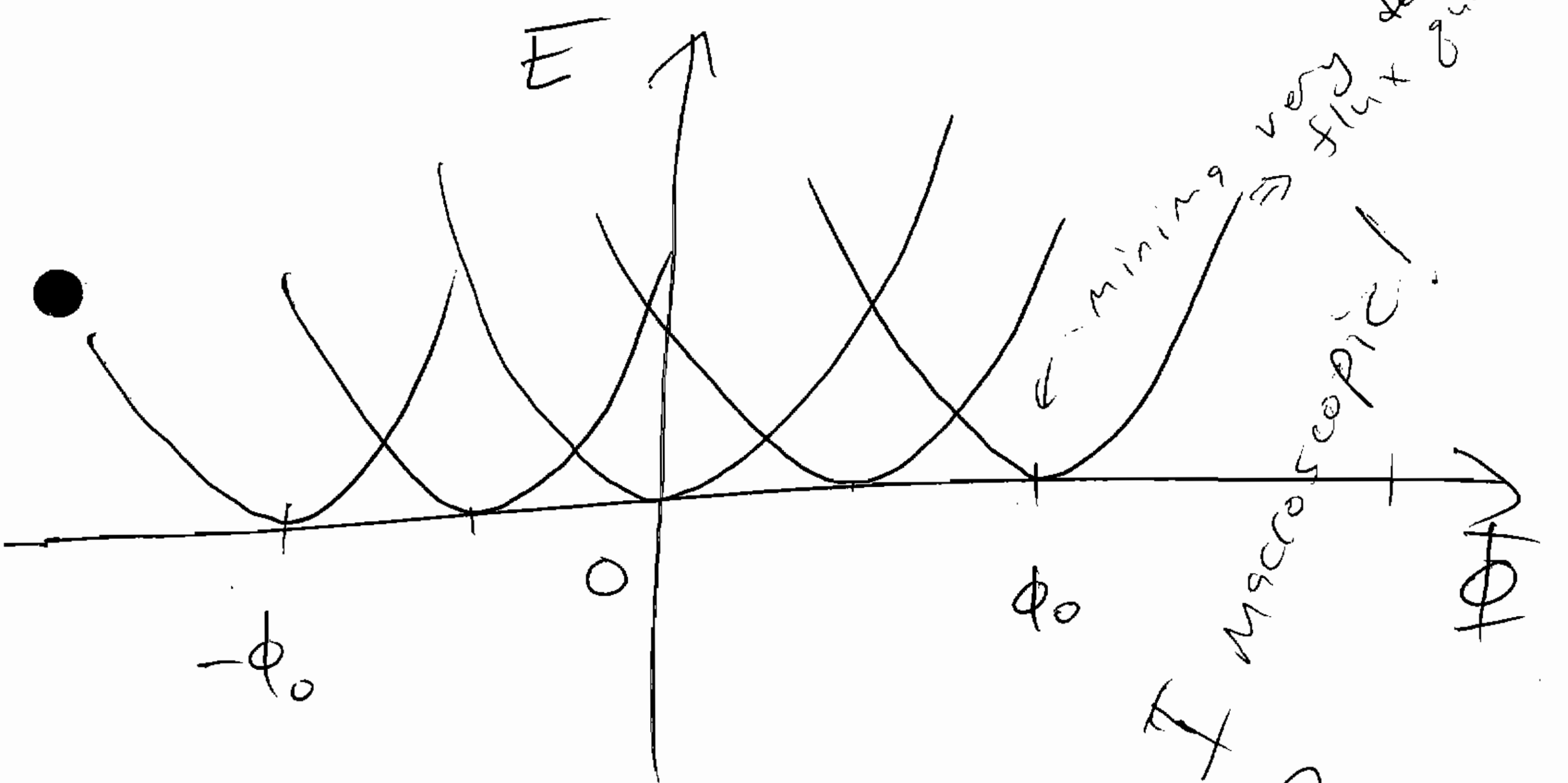


Consecutive energy parabolas differ only by the transfer of a single electron from $-k_F$ to $k_F \Rightarrow I$ mesoscopic!

- Superconducting ring at $T=0$:

$$E_0(\Phi) = \frac{N_s h^2}{mL^2}\left(\Phi - n\frac{hc}{2e}\right)^2$$

$n \in \mathbb{Z}$

- 

  - periodicity halved
  - different parabolas not mixed

- because mixing two states involves breaking all cooper pairs and reforming them → equivalent to tunneling a vortex through ring.