Phys 460 Midterm 1
Solutions

1) $$dN = 2\,\frac{dx\,dp}{h}$$

$$N = \frac{2}{h}\int dx\,dp = \frac{2L}{h}\int_{-p_F}^{p_F} dp = \frac{4p_F L}{h}$$

$$N = \frac{4p_F L}{2\pi\hbar} = \frac{2k_F L}{\pi}$$

a) $$k_F = \frac{\pi}{2}\frac{N}{L} \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

b) $$E_0 = 2\int \frac{dx\,dp}{h}\,\frac{p^2}{2m} = \frac{L}{mh}\int_{-p_F}^{p_F} dp\; p^2$$

$$E_0 = \frac{2L\,p_F^3}{3mh} = \frac{1}{3}\,\frac{4Lp_F}{h}\,\frac{p_F^2}{2m} = \frac{1}{3}N\varepsilon_F$$

2) $$K = \frac{1}{T}\left[\mathcal{L}^{(2)} - \frac{(\mathcal{L}^{(1)})^2}{\mathcal{L}^{(0)}}\right]$$

$$\mathcal{L}^{(\nu)} = \frac{1}{h}\int d\varepsilon\, (\varepsilon-\mu)^{\nu}\, T(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right)$$

Perfect conductor (1D): $T(\varepsilon) = 1$

$\Rightarrow \mathcal{L}^{(1)} = 0$ by symmetry

$$K = \frac{1}{T}\mathcal{L}^{(2)}$$

let $x = \beta(\varepsilon-\mu)$

$$\mathcal{L}^{(2)} = \frac{1}{h}\int d\varepsilon\,(\varepsilon-\mu)^2\left(-\frac{\partial f}{\partial \varepsilon}\right) = \frac{(k_B T)^2}{h}\int \frac{dx\, x^2 e^x}{(e^x+1)^2}$$

$$= \frac{\pi^2}{3}\frac{(k_B T)^2}{h}$$

$$\boxed{K = K_0 = \frac{\pi^2}{3}\frac{k_B^2 T}{h}}$$