Practice problems for Physics 460 Midterm 2

Calculator and crib sheet (8.5"x11", 1 side) allowed

1) Bravais lattice & reciprocal lattice

a) Give a definition of a three-dimensional Bravais lattice.

b) Any function with the periodicity of the Bravais lattice may be expressed as a Fourier sum over a set of *reciprocal lattice vectors*. State the condition(s) which must be satisfied by a reciprocal lattice vector.

c) Consider an *orthorhombic* Bravais lattice, with fundamental translation vectors $\mathbf{a}_1 = a\hat{x}$, $\mathbf{a}_2 = b\hat{y}$, and $\mathbf{a}_3 = c\hat{z}$, with a, b, and c all different. What are the fundamental translation vectors of the reciprocal lattice? What is the first Brillouin zone?

2) Energy bands

a) State *Bloch's theorem* on the eigenstates of Schrödinger's equation in a periodic potential.

b) Sketch the three lowest energy bands for electrons in a weak periodic potential in one dimension.

c) What is the criterion which determines whether a material is an insulator or a metal within band theory?

3) One-dimensional tight-binding model

In the tight-binding approximation, the dispersion relation is given by

$$\varepsilon(\mathbf{k}) = E^*_{\text{at}} - \sum_{\mathbf{R}} t(\mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}},$$

where $\mathbf{R}$ are lattice vectors. Consider a one-dimensional lattice with nearest-neighbor coupling t only. The basis vector is $\mathbf{a}_1 = a\hat{x}$, and $t > 0$.

a) Find $\varepsilon(k_x)$.

b) Determine the group velocity and (inverse) effective mass as functions of k_x.

c) Suppose the material is doped so that there are an average of n valence electrons per atom in the lattice, where n is a real number between 0 and 2. Determine the Fermi energy ε_F and Fermi velocity v_F of the material.