Exercises for Physics 472

Problem Set 2; Due 4pm Friday September 12

1–2) Griffiths 5.6, 5.7

3) Velocity operator

Define the velocity operator $\mathbf{v}$ via

$$m\mathbf{v} = \mathbf{p} - q\mathbf{A}/c.$$

Show that the Cartesian components of the velocity operator have the commutation rules

$$[v_x, v_y] = i(e\hbar/m^2c)B_z,$$
$$[v_y, v_z] = i(e\hbar/m^2c)B_x,$$
$$[v_z, v_x] = i(e\hbar/m^2c)B_y.$$

This means that a charged particle in a magnetic field cannot simultaneously have definite values of the velocity components in all three directions.

4) Lowest Landau level

Using the results of problem 3, and the generalized uncertainty principle

$$\Delta A \Delta B \geq \frac{1}{2}|\langle [A, B] \rangle|,$$

show that the kinetic energy of a particle moving in a constant magnetic field is bounded by $E \geq \hbar\Omega/2$, where $\Omega = eB/mc$ is the cyclotron frequency.