1–2) Griffiths 5.6, 5.7

3) **Velocity operator**

Define the velocity operator $v$ via

$$mv = p - qA/c.$$ 

Show that the Cartesian components of the velocity operator have the commutation rules

$$[v_x, v_y] = i(\hbar/m^2c)B_z,$$

$$[v_y, v_z] = i(\hbar/m^2c)B_x,$$

$$[v_z, v_x] = i(\hbar/m^2c)B_y.$$ 

This means that a charged particle in a magnetic field cannot simultaneously have definite values of the velocity components in all three directions.

4) **Lowest Landau level**

Using the results of problem 3, and the generalized uncertainty principle

$$\Delta A \Delta B \geq \frac{1}{2}|<[A, B]>|,$$

show that the kinetic energy of a particle moving in a constant magnetic field is bounded by $E \geq \hbar \Omega/2$, where $\Omega = eB/mc$ is the cyclotron frequency.