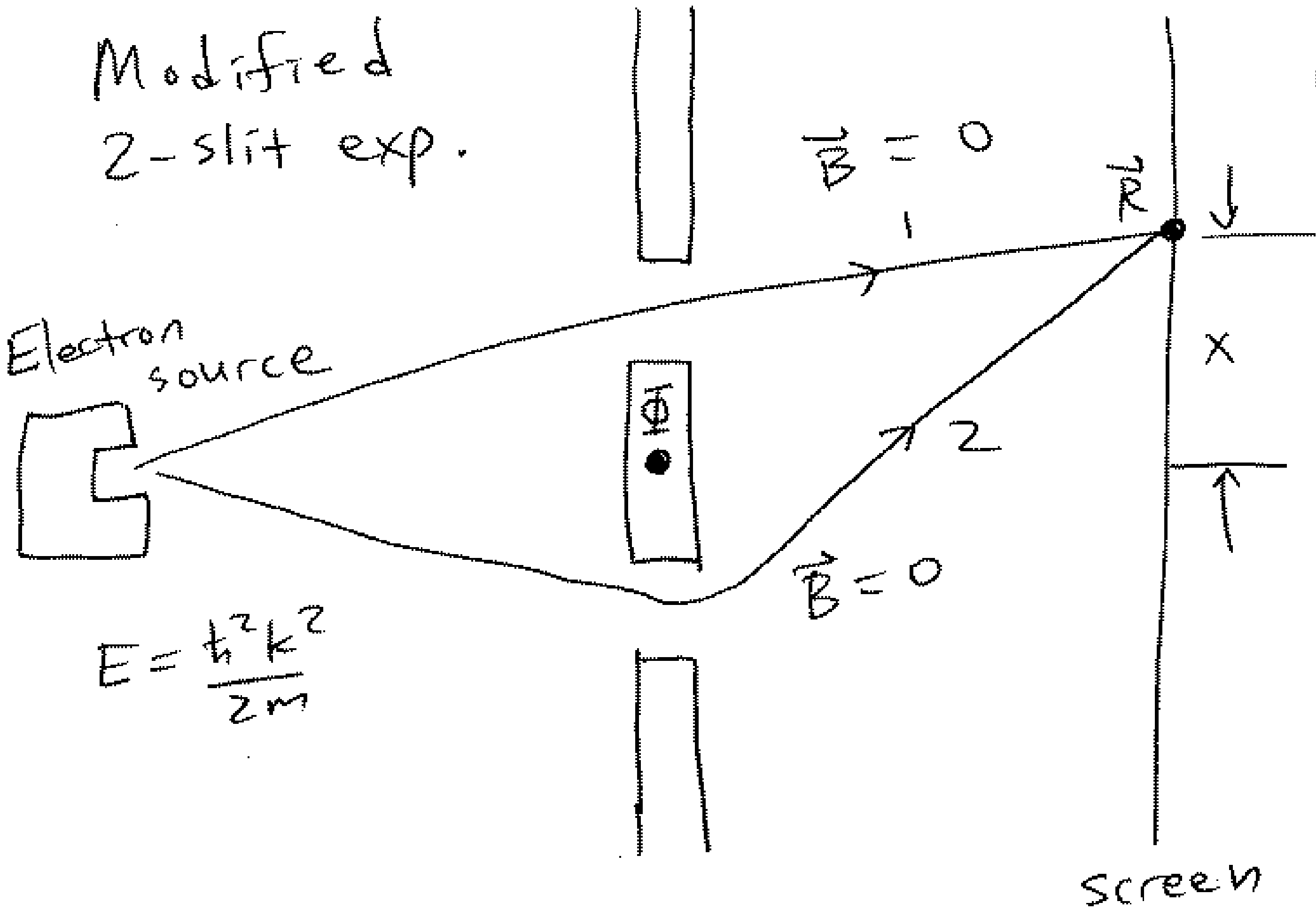
Physics 472 Lecture 2

The Aharonov-Bohm effect

In addition to the Meissner effect, another interesting consequence of the way the vector potential enters the Schrödinger equation is the Aharonov-Bohm effect, which describes the QM effect of the vector potential, even when $\vec{B} = 0$.

See Griffiths 10.2.3.

Modified 2-slit exp.



Magnetic flux Φ due to a solenoid within central barrier. $\vec{B} = 0$ everywhere the electrons can propagate.

<u>Time-indep. Sch. eq.</u>

$$E \Psi = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\vec{A}\right)^2 \Psi \qquad (V=0)$$

Outside barrier, $\vec{B} = 0$, so

$$\vec{A} = \nabla f(\vec{r}, t) = \frac{\hbar c}{q} \nabla \theta$$

$$E \psi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \hbar \nabla \theta \right)^2 \psi$$

Let $\psi = e^{i\theta(\vec{r})} \psi'$

ψ' satisfies free Sch. eq.

$$E \psi' = -\frac{\hbar^2}{2m} \nabla^2 \psi'$$

<u>Interference pattern</u>

$$\rho(\vec{R}) = |\psi_1(\vec{R}) + \psi_2(\vec{R})|^2$$

$$= |\psi_1'(\vec{R}) e^{i\theta_1} + \psi_2'(\vec{R}) e^{i\theta_2}|^2$$

$$\psi_1'(\vec{R}) = \sqrt{\rho_1}\, e^{ikL_1}, \quad \psi_2'(\vec{R}) = \sqrt{\rho_2}\, e^{ikL_2}$$

$$\rho(\vec{R}) = \rho_1 + \rho_2 + 2\sqrt{\rho_1\rho_2}\,\cos\left(k\Delta L + \theta_2 - \theta_1\right)$$

$\Delta L = L_2 - L_1$ = difference in path lengths

$$\theta_1 = \int_1 \frac{q}{\hbar c}\vec{A}\cdot\vec{dl}, \quad \theta_2 = \int_2 \frac{q}{\hbar c}\vec{A}\cdot\vec{dl}$$

$$\theta_2 - \theta_1 = \oint \frac{q}{\hbar c}\vec{A}\cdot d\vec{l}$$

$$= \frac{q}{\hbar c}\Phi = \frac{2\pi\,\Phi}{\Phi_0}$$

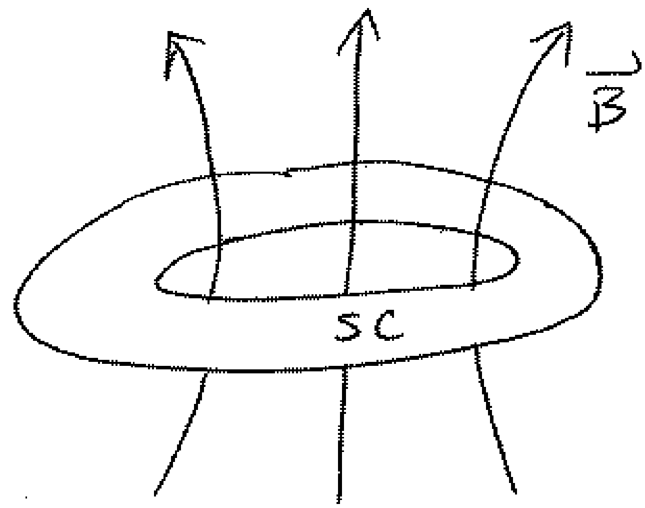
$\Phi_0 = \dfrac{hc}{q}$ = "flux quantum"

Interference pattern is shifted by magnetic flux in solenoid, even though $\vec{B}=0$ everywhere particle can propagate! Shift is periodic in Φ with period Φ_0.

Flux quantization

One consequence of the AB effect is the quantization of the magnetic flux piercing a SC ring.

The Meissner effect implies that $\vec{B}=0$ and $\vec{J}_e = 0$ in the interior of the superconductor:



$$0 = \vec{J}_e = \frac{n_s q}{m}\left(\hbar \nabla\theta - \frac{q}{c}\vec{A}\right)$$

$$\Rightarrow \quad \nabla\theta = \frac{q}{\hbar c}\vec{A} \qquad \left(\psi_s(\vec{r}) = \sqrt{n_s(\vec{r})}\, e^{i\theta}\right)$$

The wavefunction of the SC is single-valued, which means $\theta(\vec{r})$ must return to its original value on traversing the ring, or change by an integer multiple of 2π:

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi s, \qquad s \in \mathbb{Z}$$

$$\parallel$$

$$\oint \frac{q}{\hbar c} \vec{A} \cdot d\vec{l} = \frac{q}{\hbar c} \Phi = 2\pi s$$

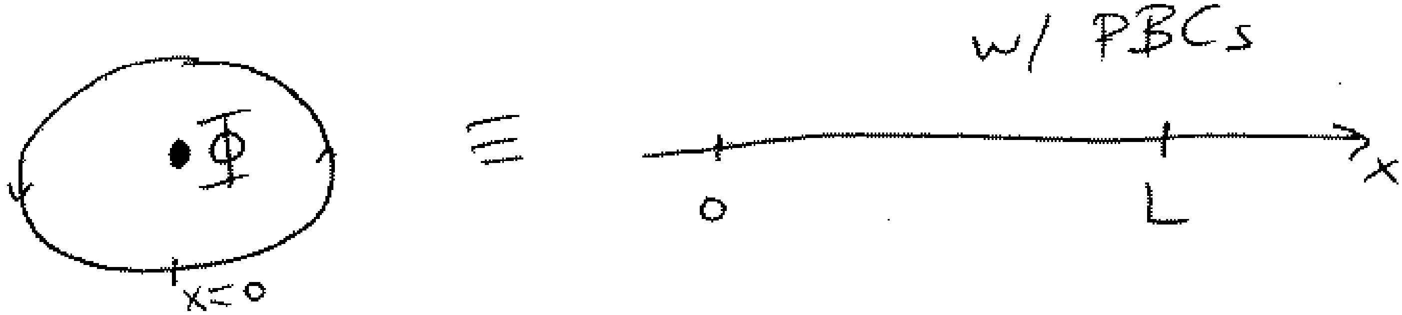
$$\boxed{\begin{array}{ll} \Phi = \dfrac{hc}{q} s = s\,\Phi_0, & \\ \Phi_0 = \dfrac{hc}{2e} & \text{superconducting flux quantum} \end{array}}$$

The superconducting surface currents adjust to fix the total magnetic flux through the ring to an integer times Φ_0. Such persistent currents have been observed to flow, undiminished for several years.

Normal Persistent Current

Nonsuperconducting rings also exhibit persistent currents at sufficiently low temperatures, or if they are sufficiently small.

As a model, let's consider free electrons in a one-dimensional ring:



$$E\,\psi(x) = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx} - \frac{q}{c}A_x\right)^2 \psi(x),$$

$$\psi(x+L) = \psi(x)$$

$\vec{B} = 0$ on ring, so

$$A_x = \frac{\hbar c}{q} \frac{\partial \theta}{\partial x}.$$

Let $\psi(x) = e^{i\theta(x)} \psi'$.

ψ' satisfies

$$E\psi' = -\frac{\hbar^2}{2m} \frac{d^2 \psi'}{dx^2},$$

but with BC

$$e^{i\theta(0)} \psi'(0) = e^{i\theta(L)} \psi'(L)$$

$$\psi'(L) = \psi'(0)\, e^{-i\Delta\theta}$$

$$\Delta\theta = \theta(L) - \theta(0) = \frac{q}{\hbar c} \oint \vec{A} \cdot d\vec{l} = \frac{q\Phi}{\hbar c}$$

For a normal system, $q = -e$.

$$\Delta\theta = +\frac{2\pi\,\Phi}{hc/e} = +\frac{2\pi\,\Phi}{\phi_0^{\text{normal}}}.$$

(Drop superscript, use $\phi_0 = \frac{hc}{e}$).

Solution of Schrödinger equation is $\psi'(x) = \frac{1}{\sqrt{L}}e^{ikx}$, $E = \frac{\hbar^2k^2}{2m}$.

BC: $$e^{ikL} = e^{i\Delta\theta} = e^{+i\frac{2\pi\Phi}{\phi_0}}$$

$$kL = 2\pi n + \frac{2\pi\,\Phi}{\phi_0}$$

$$k_n = \frac{2\pi}{L}\left(n + \Phi/\phi_0\right)$$

$$j_n = -\frac{e}{m}\,\mathrm{Re}\left\{(\psi')^* \frac{\hbar}{i}\frac{d}{dx}\psi'\right\}$$

$$= -\frac{e}{mL}\hbar k_n = -\frac{he}{mL^2}\left(n + \Phi/\phi_0\right)$$

For N noninteracting particles,

$$I = \sum_{i=1}^{N} j_{n_i} = -\frac{he}{mL^2}\sum_{i=1}^{N}\left(n_i + \Phi/\phi_0\right)$$

$$I = -\frac{Nhe}{mL^2}\frac{\Phi}{\phi_0} - \frac{he}{mL^2}\sum_{i=1}^{N} n_i$$

i) <u>N odd (neglect spin)</u>

$\sum_{i=1}^{N} n_i = 0$ in ground state

$$I = -\frac{Nhe}{mL^2}\frac{\Phi}{\phi_0}$$

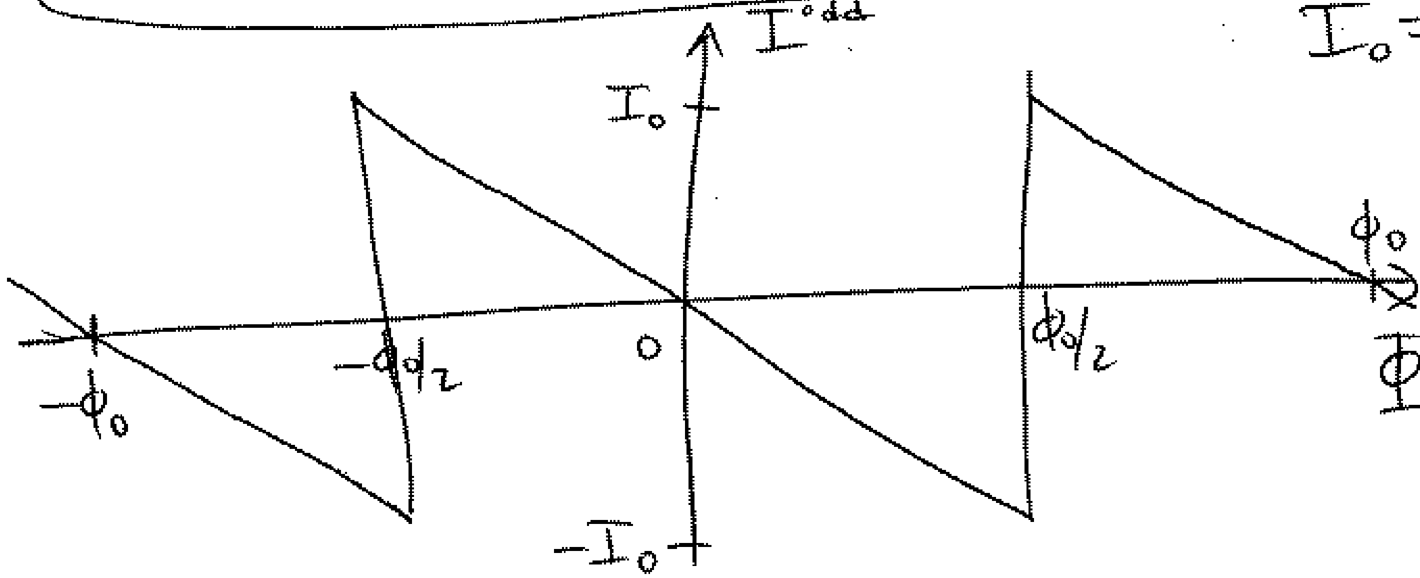
$$I = -\frac{e}{L}\frac{hN}{mL}\frac{\Phi}{\phi_0}$$

Highest occupied state has wavevector $k_F = \frac{\pi N}{L}$ and

$$V_F = \frac{\hbar k_F}{m} = \frac{hN}{2mL}, \quad \text{so}$$

$$\boxed{I^{odd} = -\frac{2eV_F}{L}\frac{\Phi}{\phi_0}, \quad N \text{ odd} \qquad -\phi_0/2 < \Phi \leq \phi_0/2}$$

$$I_0 = \frac{eV_F}{L}$$



However, this result of a diamagnetic persistent of maximum amplitude $I_0 = \frac{e v_F}{L}$ only holds for N odd!

Only then is $\sum_{i=1}^{N} n_i = 0$ in the ground state.

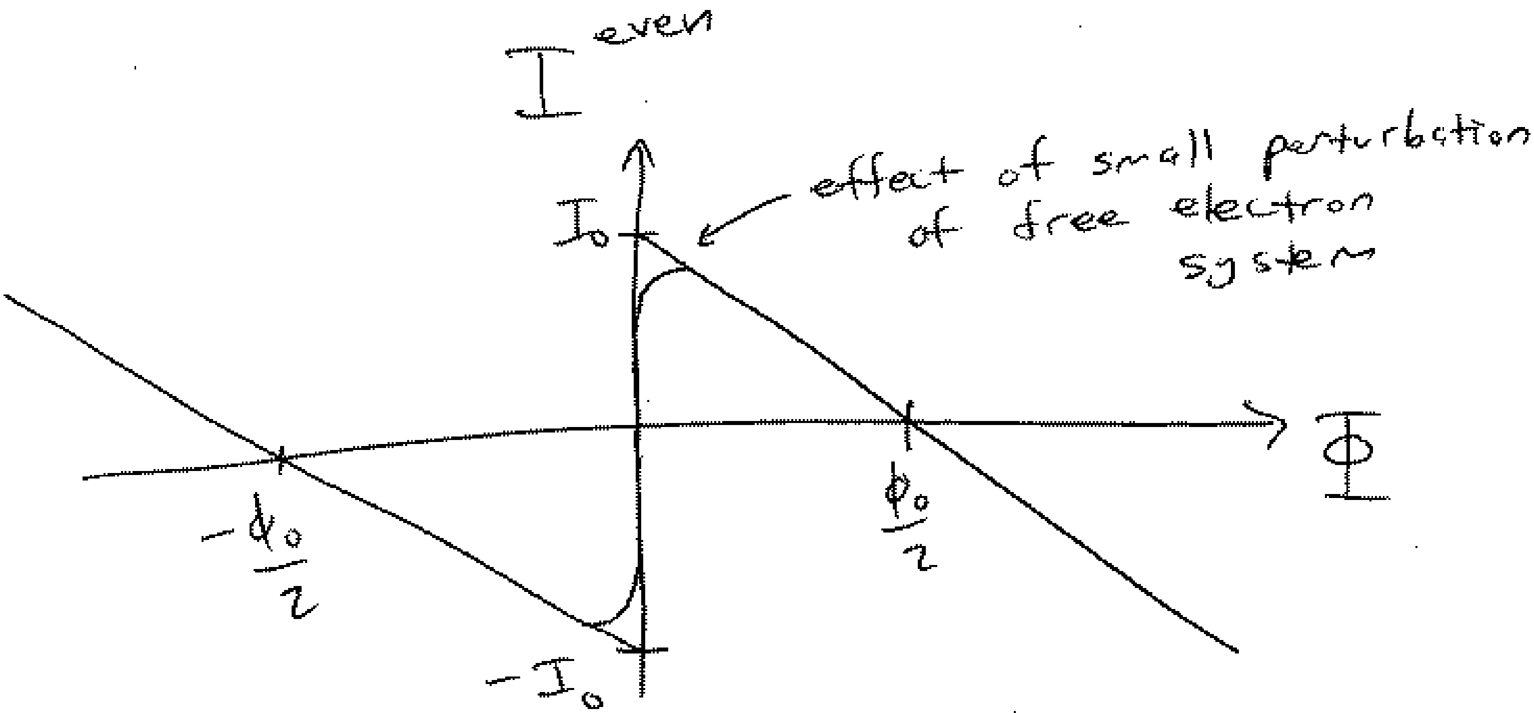
If N is even, then

$$\sum_{i=1}^{N} n_i = \pm \frac{N}{2}$$

and

$$I^{\text{even}} = -\frac{e}{L} \frac{hN}{mL} \left(\frac{\Phi}{\phi_0} \pm \frac{1}{2} \right)$$

$$= -\frac{2 e v_F}{L} \left(\frac{\Phi}{\phi_0} \pm \frac{1}{2} \right)$$



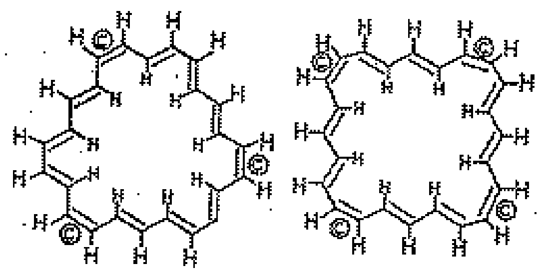
$$\Longrightarrow \quad \left.\frac{dI}{d\Phi}\right|_{\Phi=0} > 0 \qquad \underline{\text{paramagnetic}}$$

<u>Conclusion</u> For normal 1D ring, persistent current has maximum amplitude $I_0 = ev_F/L$, but is diamagnetic if N is odd and paramagnetic if N is even.

$\longrightarrow$ Spin

Persistent Current in a Microscopic Ring

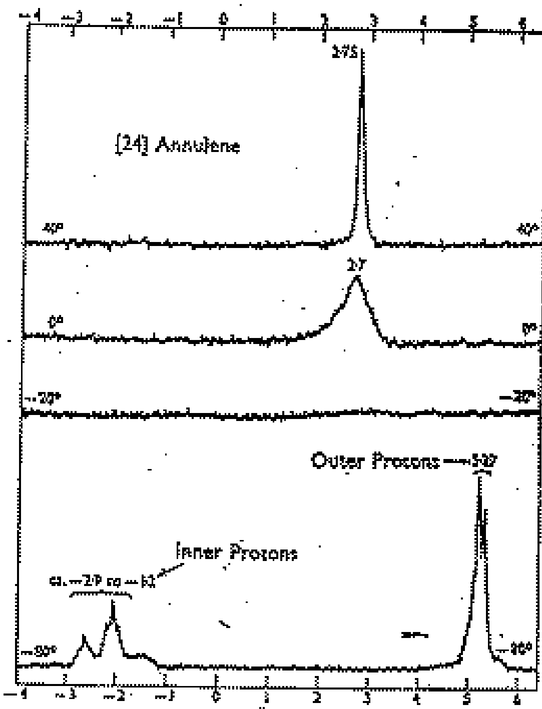
to allow a distinction to be made between structure (I) (inner : outer proton ratio, 37·5 : 62·5) and structure (II) (ratio, 33·3 : 66·7).



(I) (II)

The room temperature n.m.r. spectrum of [24]annulene evidently again represents an average, due to rotation of the carbon–carbon bonds (conformational isomerism). In addition, movement of the π-bonds (valence isomerism) may be involved, as has been postulated for cyclo-octatetraene[9] and [16]annulene.[4]

It is remarkable that in the low-temperature n.m.r. spectra of the $4n$ π-electron systems, [16]annulene and [24]annulene, the inner protons appear at low field and the outer protons at high field. This is a reversal of the behaviour of the $(4n + 2)$ systems, [14]annulene and [18]annulene. A similar reversal between $4n$ and $(4n + 2)$ systems in the dehydro-annulene series has already been observed,[10] and a theoretical explanation based on quantum-mechanical considerations has been advanced.[11]



FIGURE

N.m.r. spectrum of [24]annulene, in perdeuterotetrahydrofuran solution (100 Mc./sec., tetramethylsilane used as internal standard).

(*Received, October 20th*, 1966; *Com.* 794.)

Calder & Sondheimer 1966

Persistent Current in a Mesoscopic Au Ring

VOLUME 67, NUMBER 25 PHYSICAL REVIEW LETTERS 16 DECEMBER 1991

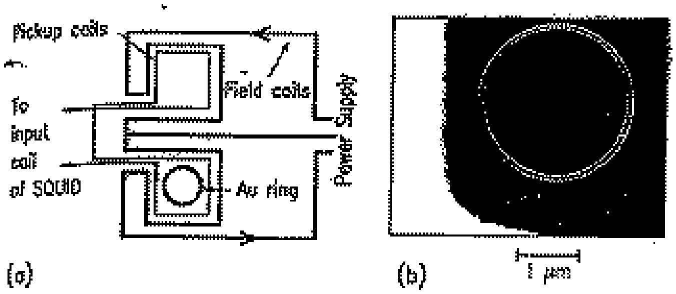
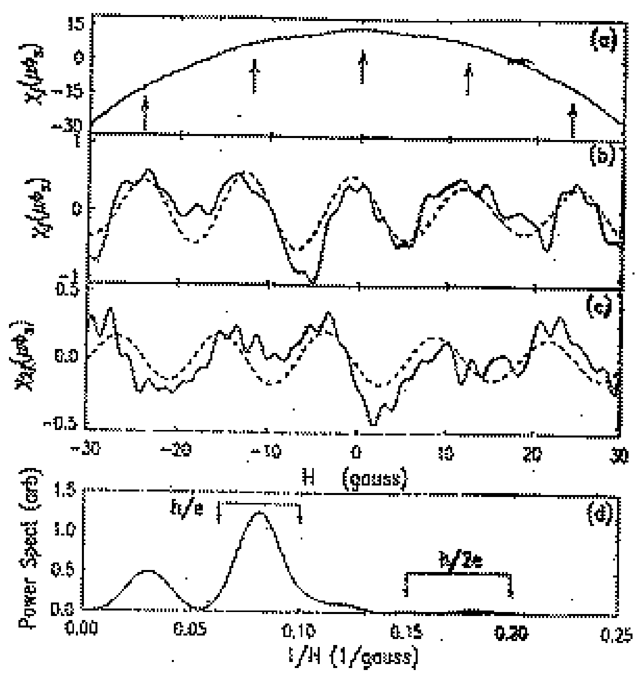


FIG. 1. (a) Schematic diagram of the pickup coil chip, illustrating the counterwound Nb pickup coils and the on-chip magnetic-field coils. (b) Micrograph of the 2.4-μm-diam Au ring in one corner of the 9-μm-inner-diam Nb pickup coil. The loop linewidth is 90 nm and the thickness is 60 nm.



Chandrasekhar et al. 1991