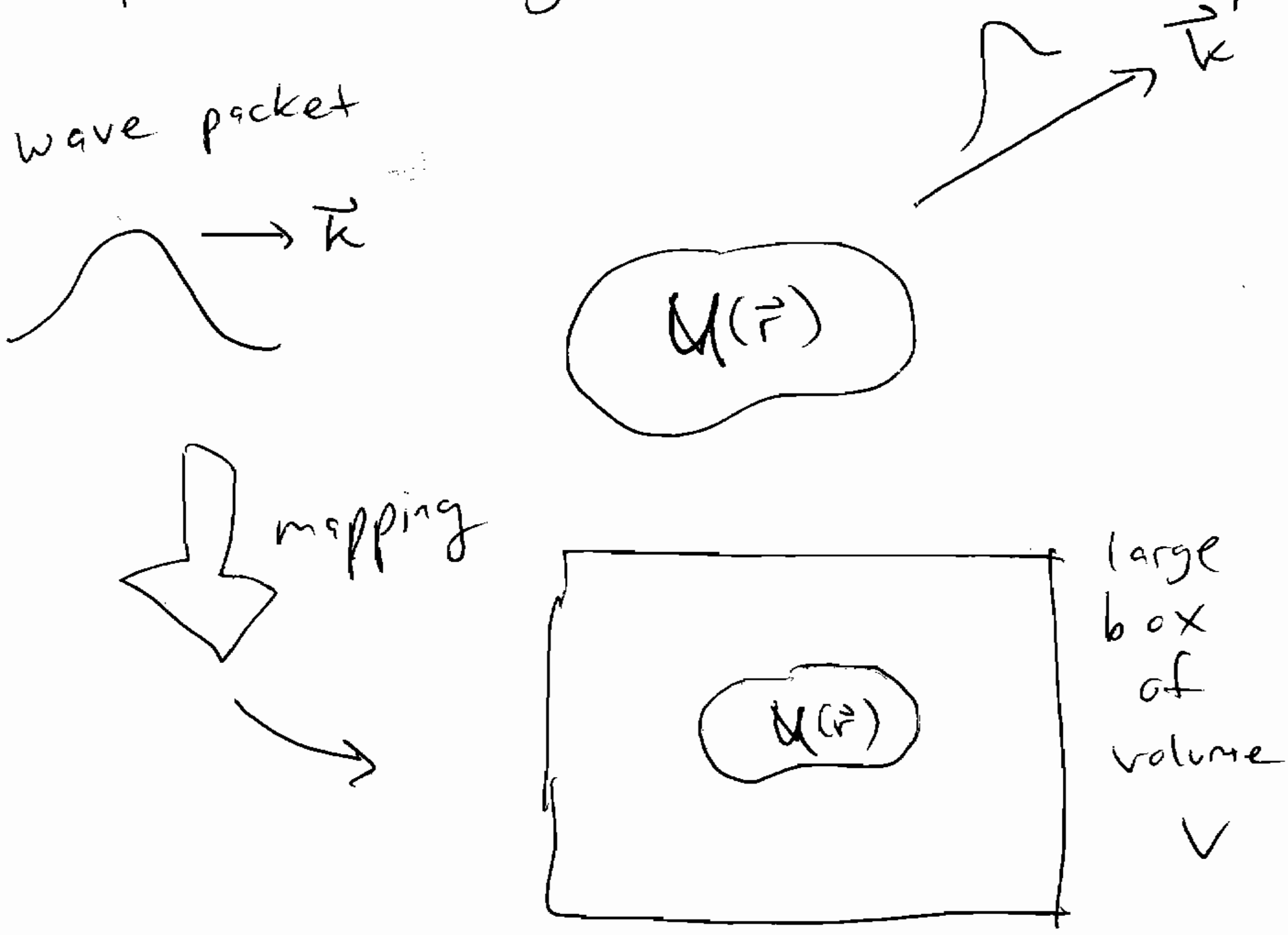
Scattering in 3D

(a brief introduction)

Three-dimensional scattering is treated in depth in Chapter 11 of Griffiths. To get a brief overview of the subject, let's consider scattering as an application of Fermi's golden rule:



What is the rate at which the potential $U(\vec{r})$ causes transitions from the initial plane wave of wave vector $\vec{k}$, $\psi_{\vec{k}}(\vec{r}) = V^{-1/2} e^{i\vec{k}\cdot\vec{r}}$, to a scattered wave $\psi_{\vec{k}'} = V^{-1/2} e^{i\vec{k}'\cdot\vec{r}}$?

$$\langle \vec{k}' | U | \vec{k} \rangle = \int \frac{d^3r}{V} e^{-i\vec{k}'\cdot\vec{r}} U(\vec{r}) e^{i\vec{k}\cdot\vec{r}}$$

$$= \frac{1}{V} \tilde{U}(\vec{k}' - \vec{k})$$

$$\Gamma_{\vec{k}\to\vec{k}'} = \frac{2\pi}{\hbar} |\langle \vec{k}' | U | \vec{k} \rangle|^2 \delta(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}})$$

<u>Fermi's golden rule</u>

Suppose we ask for the rate of scattering $d\Gamma$ into a small solid angle $d\Omega'$:

$$d\Gamma = \sum_{\vec{k}' \in d\Omega'} \Gamma_{\vec{k} - \vec{k}'}$$

There are

$$V \frac{d^3\vec{k}'}{(2\pi)^3} = \frac{V m |\vec{k}'|}{(2\pi)^3 \hbar^2} \, d\Omega' \, d\varepsilon_{k'}$$

states in the volume d^3k' of phase space, and therefore

$\frac{V m k'}{(2\pi)^3 \hbar^2}$ states per unit energy per unit solid angle.

$$\sum_{\vec{k}' \in d\Omega'} \longrightarrow d\Omega' \int_0^\infty \frac{V m k'}{(2\pi)^3 \hbar^2} \, d\varepsilon_{k'}$$

$$d\Gamma = \frac{2\pi}{\hbar} \frac{|\tilde{u}(\vec{k}'-\vec{k})|^2}{V^2} \frac{V m k}{(2\pi)^3 \hbar^2} \, d\Omega'$$

$$d\Gamma = \frac{d\Omega'}{V} \frac{mk}{4\pi^2 \hbar^3} |\tilde{u}(\vec{k}'-\vec{k})|^2$$

The incident flux of particles in state $\vec{k}$ is

$$\vec{J}_{in} = \frac{\hbar \vec{k}}{m} \frac{1}{V}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{d\Gamma}{|\vec{J}_{in}| \, d\Omega} = \frac{m^2}{4\pi^2 \hbar^4} |\tilde{u}(\vec{k}'-\vec{k})|^2}$$

differential scattering cross section

This result is refered to as the <u>Born approximation</u> for the differential scattering cross section.

See Griffiths, Ch. 11 for examples.