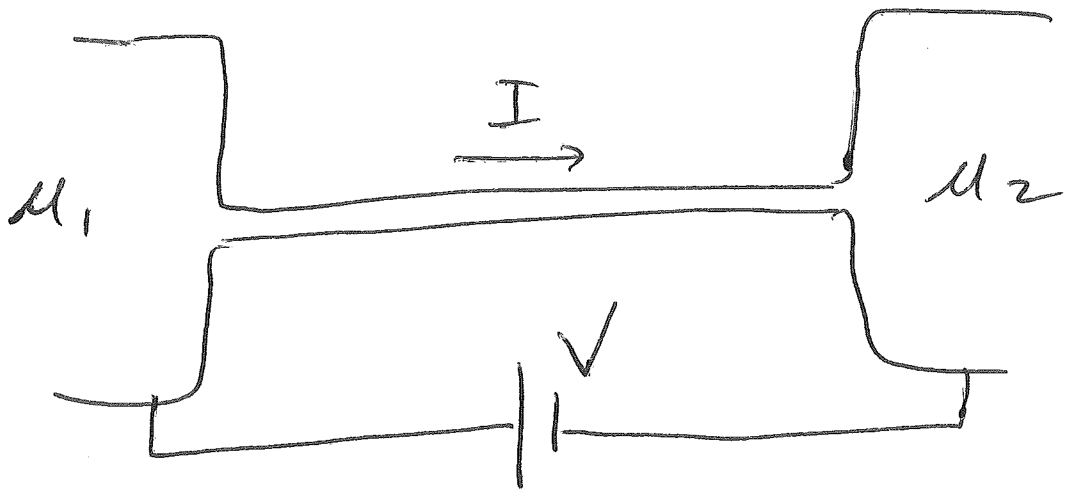
Phys. 528 Lecture 21

Q: What is the resistance of a perfect 1D wire?



$$I_2 = \frac{dQ_2}{dt} = -e\,\frac{dN_2}{dt}$$

$$dN = \frac{dx\,dp}{h} \quad (\times 2 \text{ for spin})$$

$$\frac{dN_2}{dt} = \frac{dx}{dt}\int_{p_{F_2}}^{p_{F_1}} \frac{dp}{h} = \frac{v_F}{h}\left(p_{F_1} - p_{F_2}\right)$$

$$p_{F_1} - p_{F_2} \simeq \frac{\mu_1 - \mu_2}{\partial\varepsilon/\partial p} = \frac{\mu_1 - \mu_2}{v_F}$$

$$\frac{dN}{dt} = \frac{v_F}{h}\,\frac{\mu_1 - \mu_2}{v_F} = \frac{\mu_1 - \mu_2}{h}$$

$$\mu_1 = \varepsilon_F - eV_1 \quad , \quad \mu_2 = \varepsilon_F - eV_2$$

$$\mu_1 - \mu_2 = -e(V_1 - V_2) = -eV$$

$$I_2 = -e\frac{dN_2}{dt} = \frac{e^2}{h}V = G_0 V$$

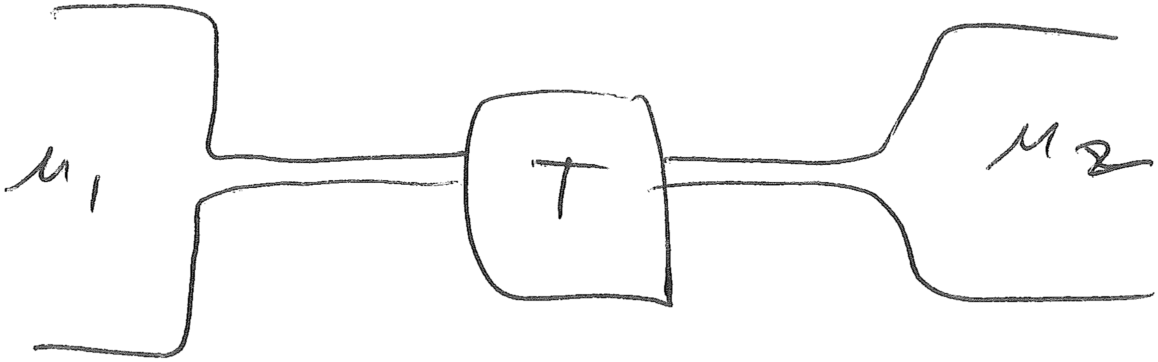
$$\boxed{R_0 = \frac{1}{G_0} = \frac{h}{e^2} = 25{,}812.8\,\Omega}$$

Resistance quantum

Including spin gives $\quad I_2 = 2G_0 V$

$$R = \frac{R_0}{2} = \frac{h}{2e^2}$$

Add a barrier:



$$I_2 = \frac{e^2}{h} T (V_1 - V_2)$$

Multiple channels

$$\boxed{I_2 = \left(\frac{e^2}{h} \sum_n T_n \right) V}$$

Landauer formula

T_n = transmission eigenvalue (incl. spin)

$$\sum_n T_n = \text{tr}\{T_{12}\}$$

T_{12} = transmission matrix

Effect of finite T :

$$T_n = T_n(\varepsilon)$$

$$I_2 = -\frac{e}{h}\int d\varepsilon \, \mathrm{tr}\{T_{12}(\varepsilon)\}\left[f_1(\varepsilon) - f_2(\varepsilon)\right]$$

Landauer-Büttiker formula

→ generalize to multiple term.

Heat current

$$T\,dS = dE - \mu\,dN \quad \text{(at fixed } V\text{)}$$

$$T_\alpha\,dS_\alpha = dE_\alpha - \mu_\alpha\,dN_\alpha \quad (\alpha = 1, 2)$$

$$I^Q_\alpha \equiv T_\alpha \frac{dS_\alpha}{dt} = \frac{dE_\alpha}{dt} - \mu_\alpha \frac{dN_\alpha}{dt}$$

$$I_\alpha^E \equiv \frac{dE_\alpha}{dt} = \frac{1}{h}\sum_\beta \int d\varepsilon \; \varepsilon \; \underbrace{\mathrm{tr}\{T_{\alpha\beta}(\varepsilon)\}}_{T_{\alpha\beta}(\varepsilon)} (f_\beta - f_\alpha)$$

$$I_\alpha^Q = \frac{1}{h}\int d\varepsilon \; (\varepsilon - \mu_\alpha) \sum_\beta T_{\alpha\beta}(\varepsilon) \left[f_\beta(\varepsilon) - f_\alpha(\varepsilon)\right]$$

Define

$$I_\alpha^{(\nu)} = \frac{1}{h}\int d\varepsilon \; (\varepsilon - \mu_\alpha) \sum_\beta T_{\alpha\beta}(\varepsilon) \left[f_\beta(\varepsilon) - f_\alpha(\varepsilon)\right]$$

Then

$$I_\alpha^{(1)} = I_\alpha^Q$$

$$I_\alpha = -e\, I_\alpha^{(0)}$$

<u>Linear response</u>

$$T_\beta - T_\alpha \ll \frac{T_\alpha + T_\beta}{2} = T_0$$

$$\mu_\beta - \mu_\alpha \ll \frac{\mu_\alpha + \mu_\beta}{2} = \mu_0$$

Taylor-expand Fermi functions:

$$f_\alpha(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu_\alpha}{k_B T_\alpha}}+1}$$

$$f_\alpha(\varepsilon) \simeq f_0(\varepsilon) + \frac{\partial f_0}{\partial \mu}(\mu_\alpha - \mu_0)$$
$$+ \frac{\partial f_0}{\partial T}(T_\alpha - T_0)$$

$$\frac{\partial f_0}{\partial \mu} = -\frac{\partial f_0}{\partial \varepsilon}$$

$$\frac{\partial f_0}{\partial T} = \frac{\varepsilon - \mu_0}{T_0}\left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$f_\alpha(\varepsilon) \simeq f_0(\varepsilon) + \left[\mu_\alpha - \mu_0 + (\varepsilon - \mu_0)\frac{T_\alpha - T_0}{T_0}\right]$$
$$\times\left(-\frac{\partial f_0}{\partial \varepsilon}\right)$$

$$f_\beta - f_\alpha \simeq \left(-\frac{\partial f_0}{\partial \varepsilon}\right)\left[\mu_\beta - \mu_\alpha + (\varepsilon - \mu_0)\frac{T_\beta - T_\alpha}{T_0}\right]$$

$$I_\alpha^{(\nu)} = \sum_\beta \left[\mathcal{L}_{\alpha\beta}^{(\nu)}(\mu_\beta - \mu_\alpha) + \mathcal{L}_{\alpha\beta}^{(\nu+1)}\frac{T_\beta - T_\alpha}{T_0}\right]$$

$$\boxed{\mathcal{L}_{\alpha\beta}^{(\nu)} = \frac{1}{h}\int d\varepsilon\,(\varepsilon - \mu_0)^\nu\, T_{\alpha\beta}(\varepsilon)\left(-\frac{\partial f_0}{\partial \varepsilon}\right)}$$

Onsager coefficient

$$\begin{pmatrix} I^{(0)} \\ I^{(1)} \end{pmatrix} = \begin{pmatrix} \mathcal{L}^{(0)} & \mathcal{L}^{(1)} \\ \mathcal{L}^{(1)} & \mathcal{L}^{(2)} \end{pmatrix}\begin{pmatrix} \Delta\mu \\ \frac{\Delta T}{T_0} \end{pmatrix}$$

$G = e^2 \mathcal{L}^{(0)}$ electrical cond.

Fluctuation-dissipation theorem

$$\int_{-\infty}^{\infty} d\tau \, \langle I_\alpha^{(0)}(t) \, I_\beta^{(0)}(t+\tau) \rangle e^{-i\omega\tau}$$

$$= \mathcal{L}^{(0)}_{\alpha\beta} \frac{\hbar\omega/2}{e^{\beta\hbar\omega} - 1}$$

cf. Johnson-Nyquist noise

$$\int_{-\infty}^{\infty} d\tau \, \langle I_\alpha^{(0)}(t) \, I_\beta^{(1)}(t+\tau) \rangle e^{-i\omega\tau}$$

$$= \mathcal{L}^{(1)}_{\alpha\beta} \frac{\hbar\omega/2}{e^{\beta\hbar\omega} - 1}$$

$$\int_{-\infty}^{\infty} d\tau \, \langle I_\alpha^{(1)}(t) \, I_\beta^{(1)}(t+\tau) \rangle e^{-i\omega\tau}$$

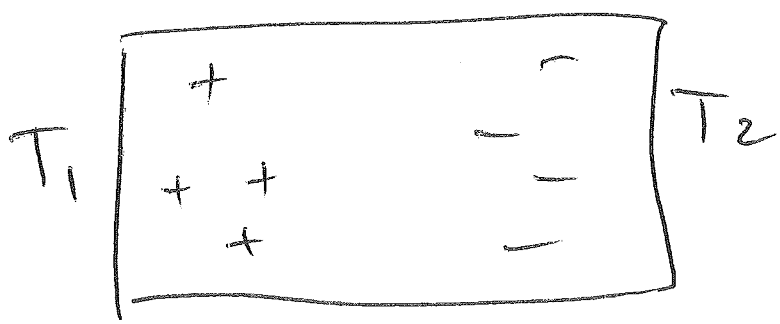
$$= \mathcal{L}^{(2)}_{\alpha\beta} \frac{\hbar\omega/2}{e^{\beta\hbar\omega} - 1}$$

Micro-reversibility (see Pathria 15.7)

$$\mathcal{L}^{(\nu)}_{\alpha\beta} = \mathcal{L}^{(\nu)}_{\beta\alpha} \qquad (\vec{B} = 0)$$

$$\mathcal{L}^{(\nu)}_{\alpha\beta}(\vec{B}) = \mathcal{L}^{(\nu)}_{\beta\alpha}(-\vec{B})$$

Seebeck effect



$I^{(0)} = 0$

open electric circuit

$$0 = I^{(0)}_1 = \mathcal{L}^{(0)}_{12}(\mu_2 - \mu_1) + \mathcal{L}^{(1)}_{12}\frac{T_2 - T_1}{T_0}$$

$$\mu_2 - \mu_1 = -e(V_2 - V_1) = -\frac{\mathcal{L}^{(1)}_{12}}{\mathcal{L}^{(0)}_{12}}\frac{T_2 - T_1}{T_0}$$

$$\Delta V = \frac{1}{eT_0} \frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}} \Delta T \equiv -S \Delta T$$

$$\boxed{S = -\frac{1}{eT_0} \frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}} = \text{Seebeck coeff.}}$$

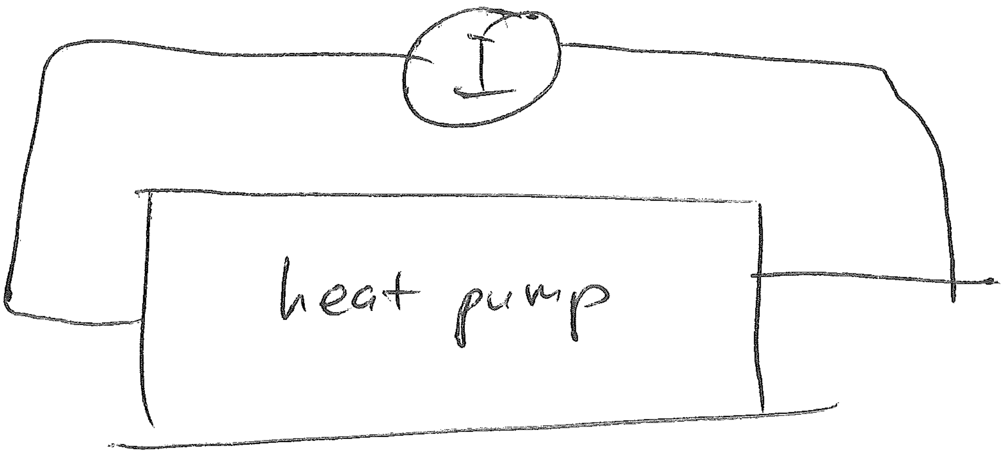
= thermoelectric power

= thermopower

$$S = \frac{\frac{1}{h} \int d\varepsilon \, \frac{\varepsilon - \mu_0}{T_0} \, T(\varepsilon) \left(-\frac{\partial f_0}{\partial \varepsilon}\right)}{-\frac{e}{h} \int d\varepsilon \, T(\varepsilon) \left(-\frac{\partial f_0}{\partial \varepsilon}\right)}$$

= entropy per unit charge transported

Peltier effect



$T_1 = T_2 = T_0$

$I^{(0)} \neq 0$

$$I^{(1)} = \mathcal{L}^{(1)} (\mu_2 - \mu_1)$$

$$I^{(0)} = \mathcal{L}^{(0)} (\mu_2 - \mu_1)$$

$$I^{(1)} = \frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}} I^{(0)} = \frac{\mathcal{L}^{(1)}}{-e\mathcal{L}^{(0)}} I$$

$$\equiv \Pi I$$

$$\boxed{\Pi = T_0 S \quad \text{Peltier coeff.}}$$

Thermal conduction

Open electric circuit $I^{(0)} = 0$

$$I^{(1)} = K \Delta T$$

$$0 = I^{(0)} = \mathcal{L}^{(0)} \Delta\mu + \mathcal{L}^{(1)} \frac{\Delta T}{T_0}$$

$$I^{(1)} = \mathcal{L}^{(1)} \Delta\mu + \mathcal{L}^{(2)} \frac{\Delta T}{T_0}$$

$$\Delta\mu = -\frac{\mathcal{L}^{(1)}}{\mathcal{L}^{(0)}} \frac{\Delta T}{T_0}$$

$$I^{(1)} = \left(\mathcal{L}^{(2)} - \frac{[\mathcal{L}^{(1)}]^2}{\mathcal{L}^{(0)}} \right) \frac{\Delta T}{T_0}$$

$$\boxed{K = \frac{1}{T_0} \left(\mathcal{L}^{(2)} - \frac{[\mathcal{L}^{(1)}]^2}{\mathcal{L}^{(0)}} \right) \quad \text{thermal conductance}}$$