Homework 1, Physics 528
Due in class Thursday September 1

1) Consider two large systems in thermal contact. The total energy of the combined system is E. The partitioning of energy between the two systems is governed by the number of accessible states

$$\Omega(E, E_1) = \Omega_1(E_1)\Omega_2(E - E_1).$$

a) Show that Ω can be expressed as a Gaussian function in the variable E_1, and determine the r.m.s. deviation of E_1 from the mean value $\bar{E}_1$.

b) Evaluate the r.m.s. deviation of E_1 for the special case when both systems are classical ideal gases.

2) Assuming that the entropy S and the number of accessible states Ω of a physical system are related through an arbitrary functional form

$$S = f(\Omega),$$

show that the additive character of S and the multiplicative character of Ω necessarily require that $S = k \ln \Omega$, where k is a constant.

3) Consider a classical gas of hard spheres of radius R. The spatial distribution of the particles is no longer uncorrelated. The presence of j particles in the system leaves only a volume $V - jv_0$ available for the $(j+1)$st particle, where $v_0 \propto R^3$. Assuming $Nv_0 \ll V$, determine the dependence of $\Omega(N, V, E)$ on V. Show that the equation of state of such a gas is similar to the ideal gas law, but with V replaced by $V - b$, where b is four times the actual volume occupied by the particles.

4) Using the fact that the entropy $S(N, V, E)$ of a thermodynamic system is an extensive quantity, show that

$$N \left.\frac{\partial S}{\partial N}\right|_{V,E} + V \left.\frac{\partial S}{\partial V}\right|_{N,E} + E \left.\frac{\partial S}{\partial E}\right|_{N,V} = S.$$

Note that this result implies that $(-N\mu + pV + E)/T = S$, that is, $N\mu = E + pV - TS$.

5) Kardar problem 1.4 (not graded, answer at the back of the book).

6) Kardar problem 1.6 (not graded, answer at the back of the book).