Physics 560A
Lecture 15

The Heisenberg model of (anti) ferromagnetism

$$H = J \sum_{n.n.} \vec{S}_i \cdot \vec{S}_j$$

$$J = \frac{4t^2}{U} > 0$$

superexchange
↳ antiferromagnetism

$$J \sim -U < 0$$

exchange
↳ ferromagnetism

Mean field theory (spin S) (2

$$H_i = J \sum_{j=n.n.} \vec{S}_i \cdot \vec{S}_j - g\mu_B \vec{B} \cdot \vec{S}_i$$

$$\vec{B}_{eff} = \vec{B} - \frac{J}{g\mu_B} \sum_{j=n.n.} \langle \vec{S}_j \rangle$$

$$= \vec{B} - \frac{zJ}{g\mu_B} \langle \vec{S}_{n.n.} \rangle$$

Energy levels of atom i:

$$E_m = -g\mu_B B_{eff}\, m, \quad (\hbar = 1)$$

$$m = -S, -S+1, \ldots, S$$

$$Z_i = \sum_{m=-S}^{S} e^{-\beta E_m}$$

$$Z_i = \sum_{m=-S}^{S} \left(e^{\beta g \mu_B B_{eff}}\right)^m \qquad \boxed{3}$$

$$= \frac{\sinh\left[\left(S+\frac{1}{2}\right)\beta g \mu_B B_{eff}\right]}{\sinh\left(\frac{\beta g \mu_B B_{eff}}{2}\right)}$$

$$\langle S_{iz} \rangle = \hbar \langle m \rangle$$

$$\langle \mu_{iz} \rangle = g \mu_B \langle S_{iz} \rangle = \frac{1}{\beta} \frac{\partial \ln Z_i}{\partial B_{eff}}$$

$$= g \mu_B \left[\frac{\left(S+\frac{1}{2}\right) \cosh\left(S+\frac{1}{2}\right)\eta}{\sinh\left(S+\frac{1}{2}\right)\eta} - \frac{\frac{1}{2}\cosh}{\sinh\frac{\eta}{2}} \right.$$

$$\eta = \beta g \mu_B B_{eff}$$

or $\langle \mu_{iz} \rangle = g \mu_B S\, B_S(\eta)$

$$B_S(\eta) \equiv \frac{1}{S}\left[\left(S+\tfrac{1}{2}\right)\coth\left(S+\tfrac{1}{2}\right)\eta - \frac{1}{2}\coth\left(\frac{\eta}{2}\right)\right]$$

$$\langle S_{iz} \rangle = S\, B_S(\eta_i)$$

$$\eta = \beta g \mu_B B_z - z\beta J \langle S_z \rangle_{n.n.}$$

By symmetry, we expect

$$\langle S_{iz} \rangle = \begin{cases} -\langle S_z \rangle_{n.n.} & \text{for } J>0 \\ +\langle S_z \rangle_{n.n.} & \text{for } J<0 \end{cases}$$

So $\eta = \beta g \mu_B B + \beta z |J| \langle S_{iz} \rangle$

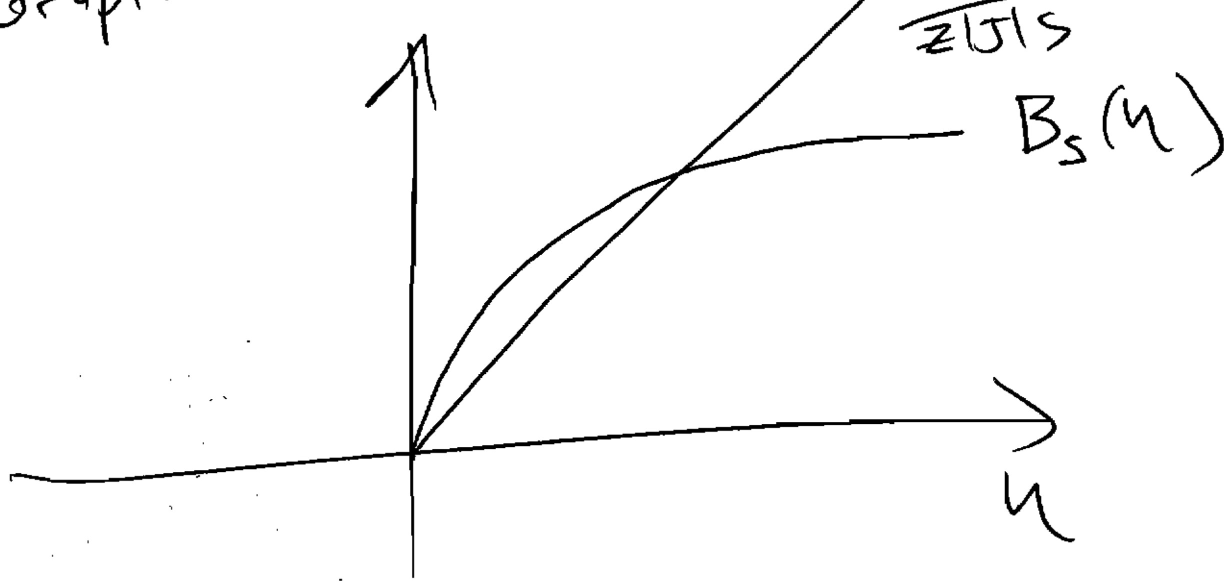
$$B_S(\eta) = \frac{k_B T}{z|J|S}\left(\eta - \frac{g\mu_B B}{k_B T}\right)$$

$\Rightarrow$ must be solved selfconsistently

For $B_{ext} = 0$, we have

$$B_S(\eta) = \frac{k_B T}{z|J|S}$$

Graphical solution:



Solution w/ $\langle S_{iz} \rangle \neq 0$ exists if

$$\left.\frac{dB_S}{d\eta}\right|_{\eta=0} > \frac{k_B T}{z|J|S}$$

For $\eta \ll 1$,

$$B_S(\eta) \approx \frac{1}{3}(S+1)\eta$$

$$\frac{1}{3}(S+1) > \frac{k_B T}{z|J|S}$$

or $T < T_c$, where

$$k_B T_c = \frac{z|J|S(S+1)}{3}$$

"Curie temperature"

Cases:

3D : MF ok

2D : AF exists only at $T=0$

1D : AF does not exist, even at $T=0$