Lecture 23

Introduction to Nonequilibrium Green's functions

We have seen that a general (time-independent) observable one-body can be written in 2nd quantization as

$$\hat{Q} = \sum_{n,m} Q_{nm} \, d_n^\dagger d_m ,$$

where the sum runs over a complete single-particle basis. The most general time-dep. one-body observable has the form

$$\hat{A}(t_1, t_2) = \sum_{n,m} A_{nm}(t_1, t_2)\, d_n^\dagger(t_1)\, d_m(t_2).$$

Thus, the expectation values of all one-particle observables can be determined from the Green's functions

$$G^<_{nm}(t_1, t_2) = i\langle d_m^\dagger(t_2)\, d_n(t_1)\rangle$$

$$G^>_{nm}(t_1, t_2) = -i\langle d_n(t_1)\, d_m^\dagger(t_2)\rangle$$

Here $\langle\ \rangle$ implies the statistical and QM average.

How to compute $G^{\gtrless}$? (out of equilibrium!)

Interaction rep.

$$H = H_0 + V$$

$$|\psi(t)\rangle = e^{iH_0 t} e^{-iHt} |\psi(0)\rangle$$

$$\hat{O}(t) = e^{iH_0 t} \hat{O} e^{-iH_0 t}$$

Note $[H_0, V] \neq 0$, so

$$e^{iH_0 t} e^{-iHt} \neq e^{-iVt}$$

$$\langle \psi(t) | \hat{O}(t) | \psi(t) \rangle = \langle \psi(0) | e^{iHt} \hat{O} e^{-iHt} | \psi(0) \rangle$$

same as in Sch. rep./ Heisenberg rep.

Propagator

$$U(t) = e^{iH_0 t} e^{-iHt}$$

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$$

$$\frac{\partial}{\partial t}|\psi(t)\rangle = i\, e^{iH_0 t}(H_0 - H)e^{-iHt}\,|\psi(0)\rangle$$

$$= -i\,\hat{V}(t)\,|\psi(t)\rangle$$

$\Rightarrow$ time dep. of states determined by $\hat{V}(t)$; time dep. of operators determined by H_0.

$$U(0) = 1$$

$$\frac{\partial U(t)}{\partial t} = i e^{iH_0 t}(H_0 - H) e^{-iHt}$$

$$= -i e^{iH_0 t} V \left(e^{-iH_0 t} e^{iH_0 t}\right) e^{-iHt}$$

$$= -i \hat{V}(t)\, U(t)$$

$$U(t) = U(0) - i \int_0^t dt' \, \hat{V}(t') U(t')$$

the solution is

$$U(t) = T \exp\left[-i \int_0^t dt' \, \hat{V}(t')\right]$$

<u>S-matrix</u>

$$|\tilde{\psi}(t)\rangle = S(t, t')\, |\psi(t')\rangle$$

$$S(t,t') = U(t)U^{\dagger}(t')$$

Proof:

$$|\Psi(t)\rangle = U(t)\,|\Psi(0)\rangle = S(t,t')U(t')|\Psi(0)\rangle$$

Moreover

$$S(t,t') = T\left[\exp\left(-i\int_{t'}^{t} dt_1\, \hat{V}(t_1)\right)\right]$$

The crux of the matter

$$|\Psi(t)\rangle = S(t,-\infty)|\Psi(-\infty)\rangle$$